Journal of Forensic and Investigative Accounting
Volume 17: Issue 1, January–June 2025

Discounts: Present Value Theory Applied

James A. Lisi

Introduction

Because most non-controlling equity interests in a private company cannot be sold independently, these securities are stranded assets. Stranded assets are investments that cannot be used by their owner or no longer produce their full economic return. Common practice to value stranded equity interests is to appraise a company's current equity value and discount it with a discount for lack of marketability (DLOM). A DLOM reduces *pro rata* company value that possesses absolute control to a liquidation value[1] for lack of full ownership rights and lack of a marketplace to sell the interest. However, this hypothetical discount method does not have a valid scientific basis. Discounting mismatches an immediate sale of the company with a prospective exit from the security. In a privately held company, this prospective exit most likely takes the form of a future company sale, although larger start-up companies will give investors opportunities to exit at future investment rounds.

Like the value of the whole company or any other asset, the market value of a private security is delivered by its prospective cash flows. Given a clean attempt at appraising the market value of common stock and other company securities, fundamentals would direct an analyst to identify the cash distributions received by the owner of the security and apply a market discount rate to the future cash flows. There are two typical cash distributions: an interim dividend or interest payment and proceeds from a future sale of the company. Assessment of these two distributions is a technique used by professional investors and syndicates to determine their entry price into real estate, start-up, and other private companies.

Most often, the eventual exit is the main driver of value because it is the larger cash distribution, which means that the earnings growth rate during the holding period is a key variable. When growth is fast and the exit to cash happens in a relatively short time period, investors participate. And when growth is slow with a long or undetermined holding period, no investors can be found. In either the fast-growth case or the slow-growth case, the investment analysis is the same: an evaluation of benefits and costs received over time versus the desired return on investment for the risk.

Appraisal standards require that information applied to a valuation be known, knowable, or foreseeable. So, a security's market value is found based on the expected exit path, not on speculative options. The exit of the security owner—via company liquidation, business broker, private equity acquisition, venture capital round, or initial public offering (IPO)—will differ based upon the earnings stage, growth rate, and size of the business opportunity. The exit may be Series A shares taken out in Series B. The exit may be closing and winding up the firm. For most of the 30,000,000 businesses in the U.S., the likely path is through a sale of the company through a business broker. For market value analysis, the exit path applied is the one a hypothetical buyer of the security, a prospective investor, would model.

Although financial buyers are most common, the synergistic acquisition may be the proper model to find the market value of the security. The "foreseeable" rule says that assumptions must be probable, so if the synergistic acquirer is identified, then modeling a financial buyout is an error. For example, when the potential acquirer is an existing shareholder and holds a seat on the board of directors, which is common for mature start-ups having a large potential market, exit multiples should represent the potential acquirer's acquisition behavior.

Now, the exit model brings in the question of the matching holding period. Holding period and foreseeability are linked. To have an equitable estimate of market value at the present time, the best holding period would be for the time where the current character of the company is effective. For a family-owned real estate holding company, the holding period can be decades. For a start-up company raising money from friends and family, the hypothetical holding period may be measured at only a year or two, using a prospective seed round valuation for exit value. Modeling three or four prospective rounds of development is certainly permissible but may be speculative. If the longer path is modeled, techniques to evaluate possible outcomes exist. Multiple exit scenarios can be modeled and formed into an exit value with probability-weighted expected return models or Monte Carlo simulations.

In practice, the investor's discount rate is not distinguished from the company's discount rate, leaving the question of investor's market-rate risk unexamined. Yet, for a security model to provide a valid result, the investor (buyer) perspective is needed. In contrast to market rate, investment values are personalized for a particular investor and risk rates do not require empirical support. Cash distribution projections for investment value may also consider improbable exit scenarios.

The main objective of this paper is to enable the common practitioner to value non-controlling interests with an income approach, where value is set by future cash distribution expectations discounted back to the present for investor perceived risk. Any security's distributions can be modeled: common stock, preferred stock, warrants, options and promissory notes. The investor discount rate described here is an estimate of the market rate of return explained by arms-length transaction data. So, it is intended to represent the marginal rate of return where buyer demand and seller supply intersect.

Predominant Models Fail

In essence, the DLOM solution (i.e., reducing current company value by some percentage) is backward-looking and based upon false premises. Current company value is not relevant because the company value is set in the present and the security's cash flows come in the future. The company and investor cash flows are wholly different, with the security dependent upon dividend policy, growth/decline in revenue, growth/decline in company capitalization rate, company capital structure, and the exit plan. Discounts occur when the projected cash returns fail to meet an investor's desired rate of return.

One error common to discounting models is that they ignore real-world frictions such as the impact of exit costs. A second, related error is the use of perfect-market frameworks instead of an intrinsic value framework. Another issue is that appraisers are trained to omit unknown conditions, so most DLOM analyses ignore the holding period, which is the fundamental cause for discounting in the first place.

Public company data is widely used to determine discount rates because the auction mechanism of public markets is a good ideological fit for perfect market thinking. Valuation models using public data take price volatility as the measure of risk. The math used in volatility-based models is not flawed; the flaw is that share price movement in an auction is not connected to intrinsic value. First, asset prices found in public markets fluctuate substantially, even if the underlying asset is not risky or changing in risk. These fluctuations are absent from intrinsic value. Second, short-term price movements around a central price do not reveal whether the asset is mispriced. And third, comparing shares in large, diversified, financially engineered public companies to small private companies is problematic for the sheer number of variables introduced.

Put option models are also used for discounting. Option models presume that the price of heavily traded assets follows a geometric Brownian motion with constant drift and volatility. Black-Scholes option models are based upon the time value of money, risk-free rate (RFR),[2] option period, and price volatility. These models have been adapted to include the effect of a dividend and determine the value of a right to sell a share of stock at the current price for an estimated holding period. The option value is the cost to buy this insurance and quantifies a discount to the current company value.

In the option pricing method (OPM), current company equity is reallocated among security types based upon a Black-Scholes model in order to incorporate the idea of a future exit at a higher price than the current company value. Holding total equity constant, payoff thresholds that will be encountered by each class of security are modeled with an expected exit date. For the OPM principle to make sense, all security values would rise with a sufficiently large exit price, but instead, common stock rises in value while preferred stock falls.

In spite of being conceptually strong, OPM and put option models have the same issue as DLOM: current company value is not a relevant basis because the actual cash distributions to the investor are not addressed. Even the basic reasoning behind using OPM implies that security prices are disconnected from current company value. Now, by incorporating the variables of RFR, holding period, and dividend payment, Black-Scholes ideas are clearly better concepts than DLOM; nevertheless, both option models miss the mark.

One proprietary discounted cash flow (DCF) model has found acceptance in the profession. The model discounts investor cash flows over a holding period to an exit. Its faults include pairing a company discount rate to investor cash distributions, ignoring exit costs, and commingling company equity issues with investor issues. The company discount rate is not a match for investor distributions, so the two cannot be paired.

Present Value Solution

The best solution to finding the value of an asset that will be held for a time is found in the well-known DCF analysis that has been used since the early 1900s. The necessary elements of a private security DCF are the investor holding period, interim dividends, exit proceeds, and investor discount rate. The investor discount rate is discovered here with a "study of historical studies" to show the investor discount rate components.

The first step is examining the continuum of holding periods that occurred as Securities and Exchange Commission regulations changed. Looking at the variations between studies shows a natural relationship between initial costs, time, risk, and discount size. The studies compare a share of stock without a time restriction to the same stock with a restriction from sale. This analysis is an apples-to-apples comparison that provides information on the investor discount rate. Because these studies were completed at different times and holding periods, they show a clear linkage between holding period and discount size.

Like the simplified premise that the only relevant difference between the price of a restricted stock and a share sold contemporaneously in the market is holding period risk, the studies below have only one relevant difference between them: holding period length. For the early restricted stock studies, the hold was for two years. It changed in April 1997 to a one-year period and in February 1998 to six months with provisions that allow stock to dribble out in small quantities under certain conditions. Table 1 summarizes some of the supporting restricted stock studies:

Table 1: Commonly Cited Discount Studies

Restricted Stock Study	Sample Size	Years Studied	Holding Period	Average Discount
Stout Risius Ross[3]	98	2005–2010	4 months	10.9%
FMV Opinions[4]	N/A	1997–2005	~1 year	21.6%
Willamette Management Assoc.[5]	33	1981–84	2 years	31.2%
Silber[6]	69	1981–88	2 years	33.8%
Management Planning[7]	49	1980–96	2 years	27.1%

The data shows that with each reduction in the holding period, the average private-placement discount declined. For an average holding period of two years, discounts averaged 31%. For a one-year holding period, the discounts averaged 23%, and for an average four-month holding period, 11%. A 2007–2008 study by Trugman Valuation Associates, Inc. (TVA) shows a more steeply rising discount curve (Table 2).[8] TVA's study and the other studies have the same pattern: a high starting discount and a more linear discount rate applied as time goes on.

Table 2: Trugman Valuation Associates Discount Study

Holding Period	Average Discount	95% Confidence Range
1–31 days	11.6%	0% to 27%
32–63 days	14.6%	0% to 36%
64–185 days	20.4%	0% to 57%
185+ days	26.9%	0% to 64%

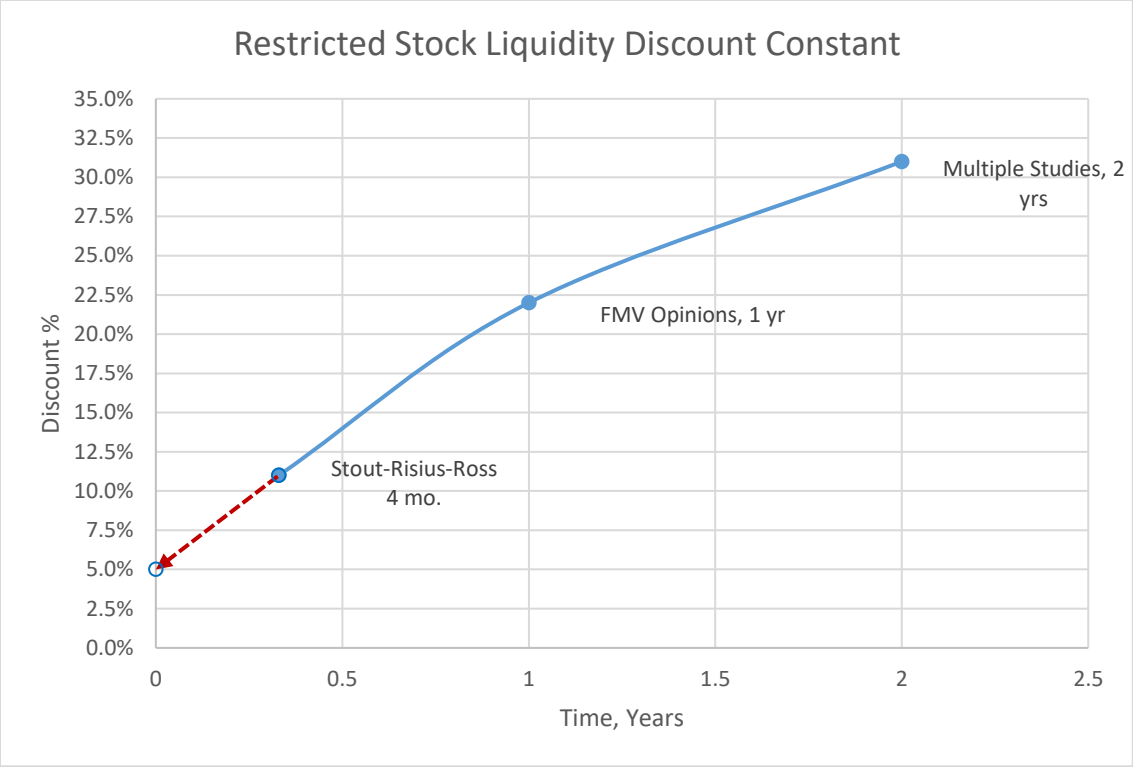
The first present value (PV) principle demonstrated by the group of studies is that as the holding period increases, the discount increases. The second principle established in this data is that discounts have a positive "time zero" value (i.e., a starting discount that is not zero).

Initial Cost

A fundamental part of any acquisition process are the buyer's costs related to due diligence and broker fees. These considerations are assumed away by perfect-market theories and disguised in averages. In a study for a single holding period, these costs are hidden within the price, but in a comparison of multiple time periods, the set-up costs within the discount are exposed when the relationship is plotted. This economic/mathematical behavior occurs whenever fixed costs are mixed with time-based rates.

So, discount behavior is not reliably found with simple averages. Discount behavior must be unpacked acknowledging the real-world friction costs that show up in the data. The relationship between discount and time is a first-order linear equation that has a constant and a rate. The investors' discount rate excludes "set-up costs." By plotting the different median results versus their holding periods, the fixed cost and annual change in rate are explained.

Figure 1: Restricted Stock Liquidity Discount and Fixed Cost



Thus, liquidity discounts are explained in the form of a constant plus a rate:

$$f(t) = C + (1 - (1 - d)^t)$$

where C is a fixed cost related to underwriting costs at time zero, d is the discount rate per period, and t is the number of time periods for the restriction from sale. The graphical solution of public company studies in Figure 1 points to a 5% initial discount, while TVA's data in Table 2 points to an 8% initial discount.[9] Eliminating the initial cost from the averages leaves an annual average liquidity discount rate of 6%–10% per year, depending on the time horizon selected.

More Data Points

Unfortunately, researchers cannot test different holding periods of their choosing; they can only test the periods made available in the market. Because the two-year holding period is the longest imposed by regulators, the Figure 1 discount curve is truncated. If the general behavior of discounts is to be explained, a complete liquidity discount curve must be plotted that include points beyond two years. Next, additional inputs are applied to extend the curve.

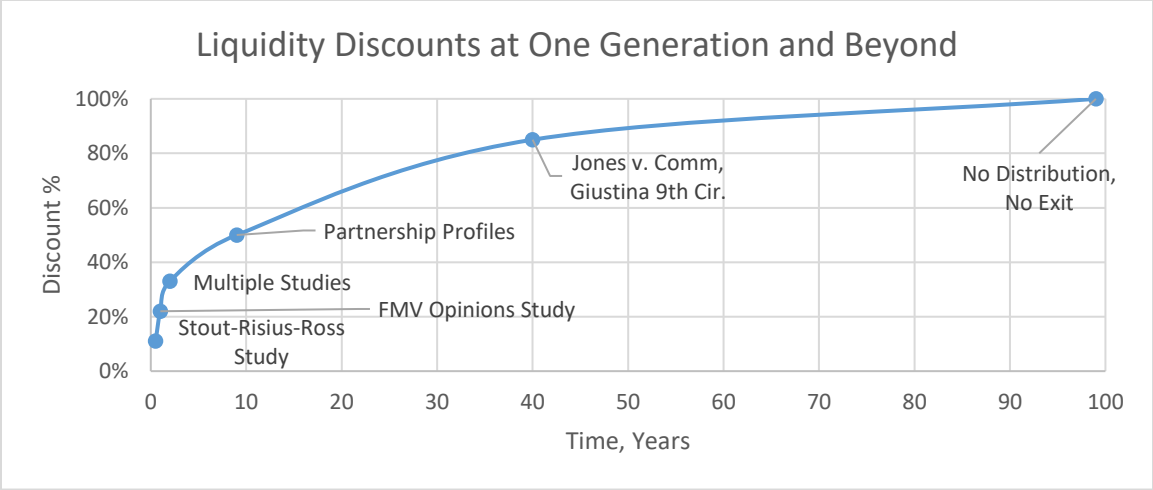
Partnership Profiles Inc.'s (PPI) Re-Sale Discount Study has research that adds an inference. PPI measures discounts on non-controlling interests in real property companies. As in the different restricted stock studies, holding periods in the past were longer—in this case, much longer. Annual studies on interests in companies holding real property also behaved according to PV theory. As holding periods fell, so did discounts. In 1993 and 1994, the expected holding period on PPI investments was 8–10 years, with an average price-to-value discount of 46%–48%. Although the PPI companies earned income from real property instead of intangible property, this transaction data helps explain discount behavior for expected holding periods beyond two years. (PPI discounts are expected to be lower than the public company discounts because the downside for real property investments is substantially less than for companies generating income from intangible assets. Real property companies have an important value floor, while operating companies can disappear completely.)

Two unmeasurable, inherent data points are also relevant to the discount curve. PV theory says, and the Internal Revenue Service (IRS) and professional texts agree, that with ready cash conversion available via a marketplace, no discount exists. The liquidation value and market value are the same. So, the point on the graph representing immediate cash receipt (zero time to exit) and having a 0% discount is relevant. The corollary is relevant as well: when an investor receives nothing (zero

cash distributions) over an infinite holding period, the discount is 100%. The security is worthless because no buyer in the marketplace will pay a positive value without expecting a future cash benefit.

Instead of guessing at a line or curve between 47% and 100% discounts, we have qualitative court rulings that give us another implied data point. Two court cases dealt with companies whose principal investment was stranded and whose exit was not foreseeable. *Estate of Giustina*[10] and *Estate of Jones*[11] concern companies with real, fixed, and intangible assets far greater than their in-use value. In each case, the courts found that non-controlling interests could not unlock this value. Using an income approach, the Tax Court assessed the interest at 15% of the highest and best asset value (the current value if the company and its real estate holdings were sold in the open market). This 85% discount is plotted on the discount curve, assuming a change in control occurs after 40 years (the estimated time when the next generation of owners could come into control and unlock the higher asset value). Combining these data points, a theoretical discount curve is plotted in Figure 2, with the endpoint set for convenience at 99 years.

Figure 2: Liquidity Discounts to 100 Years



The data does not suggest a market-clearing supply-and-demand relationship for private securities or a preset discount that can be applied to private securities. Instead, the data shows that a discount starts at a constant of about 5% and increases with holding period to 100%, at which point a security is worthless.

How Does the Curve Move?

It would be premature to conclude from this data that all company values eventually discount to zero value at an 8% annual discount rate. The security's cash flows and risk have not been examined. The discount rate will vary based on the behavior of these variables. The curve in Figure 2 is merely an illustration of the concept and not a definitive representation.

Dividend Effect

Restricted stock studies also show that dividend payments have a material effect on value. This finding is not a surprise, but averages can be deceiving because they combine different subpopulations. When an independent variable, such as dividend payments, splits a population, using the average is a major error. A basic PV analysis explains the behavior of the two exclusive populations. The example below uses a DCF analysis for a company that accumulates cash versus an otherwise identical company that pays dividends.

The relevant cash flows consist of two types of events: dividend distributions and the return of principal/appreciation when the company is sold. A single discount rate and the same total cash distribution are used to demonstrate the effect. Table 3 presents a DCF for a security without a dividend. The present value calculations use mid-year cash flows for both the dividend and exit, a 10% discount rate, and a total distribution of $5 million. Discounts in Table 3, Table 4, Table 7, and Table 8 for each year are calculated with the formula $1- (1/(1 + d)^t)$, where d is the discount rate per period and t is the number of periods. The investor cash flow model without dividend payments is shown in Table 3.

Table 3: Equity Interest, No Dividend, Average Risk

DISCOUNTED CASH FLOW ANALYSIS NO DIVIDEND COMPANY EQUITY INTEREST						
FISCAL YEAR	**2022**	**2023**	**2024**	**2025**	**2026**	**TOTAL**
PRESENT VALUE OF CASH FLOWS						
Discount Rate	10.0%					
Projected Cash Distribution	0	0	0	0	5,000,000	
Discount	4.7%	13.3%	21.2%	28.4%	34.9%	
Present Value of Projected Cash Flow	0	0	0	0	3,256,139	3,256,139

Add a regular dividend, and the equity holder in the dividend-paying company sees the flows in Table 4 instead:

Table 4: Equity Interest, With Dividend, Average Risk

DISCOUNTED CASH FLOW ANALYSIS DIVIDEND COMPANY MEDIAN EQUITY INTEREST						
FISCAL YEAR	**2022**	**2023**	**2024**	**2025**	**2026**	**TOTAL**
PRESENT VALUE OF CASH FLOWS						
Discount Rate	10.0%					
Projected Cash Distribution	250,000	250,000	250,000	250,000	4,000,000	
Discount	4.7%	13.3%	21.2%	28.4%	34.9%	
Present Value of Projected Cash Flow	238,366	216,696	196,996	179,088	2,604,911	3,436,057

As expected, the interest value increases with an early cash distribution, by about $180,000 with these inputs.

De-Risking Effect of Dividends

Dividend payments actually have two effects. Early payment has a time value, as shown above, but data also shows that dividend payment affects the investor's risk assessment. Liquidity studies demonstrate that the risk rate changes when investor dividend expectations are unmet, met, or exceeded.

Dividend expectations fall into two groups. In the first group, investors are in companies generating earnings from intangible assets (the typical business). They do not view a dividend as essential. Many of these businesses reinvest earnings to grow faster, looking toward a higher exit price for return on investment. On the other hand, the second group, investors in real property, ordinarily see a dividend as an essential feature of the security. While exit is necessary to maximize value, current income is a strong motivator for real estate investors. In both cases, when the investor's dividend expectation is not met, the discount change is greater than expected at the original discount rate. The magnitude of change indicates that the discount rate changed when the dividend expectation was violated. The difference in dividend risk can then be calculated for the two populations.

The findings of the Stout Risius Ross (SRR) 2005–2010 study of public company stocks are shown in Table 5. Dividend payers are discounted less than average, while non-payers are discounted more than average.

Table 5: Stout Risius Ross Discount Study

SRR Study 2005–2010	Average Discount	Discount Change
All	10.9%	
Dividend payer	7.4%	−32%
Non-dividend payer	11.9%	+9%

PPI data identifies the same pattern at a different magnitude with real estate holding companies, as shown in Table 6. From 2017 to 2021, investments in real estate companies paying a dividend traded at an average 23% discount, while non-dividend-paying programs averaged a 41% discount.

Table 6: Partnership Profiles Discount Study

Partnership Profiles 2017–2021	Average Discount	Discount Change
All	26.0%	
Dividend payer	23.0%	−12%
Non-dividend payer	41.0%	+58%

Adding a dividend improves security value by about 32% from the average for public companies. For the asset holding company, eliminating an expected dividend reduces its security value by about 58%. Failure to assess this condition violates USPAP Rule 9-4(d), which requires analysis for interim benefits during the holding period. Because dividend payment is an exclusive condition, dividends are either paid or not paid; the average finding of a restricted stock study should never be used in an appraisal.

Dividend Payer Risk

The model in Tables 5 and 6 demonstrates the de-risking behavior of a dividend. Since the cash flows are constant, PV theory leads us to infer that the risk rate falls when a dividend is issued. Said another way, investors pay more for dividend-paying securities and less for those without dividends, matching the investor behavior seen in the SRR and PPI studies. To demonstrate de-risking, the discount rate used in Tables 3 and 4 is adjusted to a 12% discount rate for owners who do not pay dividends and 7% for those who pay dividends. The adjusted scenarios are shown in Table 7 and Table 8, respectively. The rate differential explains the risk reduction phenomena seen in the data.

Table 7: Equity Interest, No Dividend, Risk Premium

DISCOUNTED CASH FLOW ANALYSIS NO DIVIDEND COMPANY EQUITY INTEREST						
FISCAL YEAR	**2022**	**2023**	**2024**	**2025**	**2026**	**TOTAL**
PRESENT VALUE OF CASH FLOWS						
Discount Rate	12.0%					
Projected Cash Distribution	0	0	0	0	5,000,000	
Discount	5.5%	15.6%	24.7%	32.7%	39.9%	
Present Value of Projected Cash Flow	0	0	0	0	3,002,541	3,002,541

Table 8: Equity Interest, With Dividend, Risk Reduction

DISCOUNTED CASH FLOW ANALYSIS DIVIDEND COMPANY MEDIAN EQUITY INTEREST						
FISCAL YEAR	**2022**	**2023**	**2024**	**2025**	**2026**	**TOTAL**
PRESENT VALUE OF CASH FLOWS						
Discount Rate	7.0%					
Projected Cash Distribution	250,000	250,000	250,000	250,000	4,000,000	
Discount	3.3%	9.7%	15.6%	21.1%	26.2%	
Present Value of Projected Cash Flow	241,684	225,873	211,096	197,286	2,950,075	3,826,014

When both early cash flow and risk reduction are incorporated, the expected market value for securities with and without dividends differs by about $800,000, compared to the $180,000 difference seen with an unadjusted risk rate.

Earnings Stage Risk

While dividends are shown to reduce discounts in restricted stock studies, another cash flow condition deserves attention but has not been studied previously. This condition relates to early-stage companies that consume cash. Restricted stock data shows very high discounts for companies that have negative cash flow and require cash infusions. Venture capital and private equity investments similarly show a very high return on investment demand for seed and early-stage companies.

Conceptually, the early-stage firm's cash generation profile is the opposite of that of dividend payers. Because early-stage firms require regular cash contributions to continue operating, they are going concerns only when they have access to additional capital. When companies consume investor cash instead of provide cash for use, risk increases, and value is expected to fall. Companies with no revenue are expected to have higher discounts than those that have revenue but are unprofitable, and profitable companies are expected to have the lowest discounts.

Pluris DLOM restricted stock data supports the hypothesis that discounts fall as company earnings mature from losses toward self-sustaining cash flow. The database of over 4,800 transactions available as of November 2023 shows that discounts decrease as cash flow increases. In the same way dividends are either paid or not paid, the conditions shown in Table 9 are mutually exclusive; only one of the three conditions is the correct match for a given company. So, the correct condition must be chosen and matched to the condition of the subject company. The three company cash flow conditions are no revenue, revenue with negative EBITDA[12] and positive EBITDA. With the three independent conditions having median discounts of 12%, 20%, and 29%, a 20% discount is not applied to a profitable company because profitable public companies carry 12% discounts.

Table 9: Liquidity Discount by Earnings Stage

LIQUIDITY DISCOUNT BY EARNINGS STAGE **PLURIS DLOM HISTORICAL DATA** **ALL HOLDING PERIODS THRU NOVMEBER 2023**				
Condition	**Count**	**Std Dev**	**Median**	**Mean**
No Revenue	1,060	27.70%	28.90%	28.30%
Revenue, Negative EBITDA	1,827	28.60%	19.50%	20.60%
Positive EBITDA	1,054	29.80%	11.80%	14.20%
All	4,814	28.60%	19.60%	21.60%

The data in Table 9 is for all holding periods over many years, so the output only provides a general inference. Also, the Pluris data does not have searchable fields for net income or dividend-paying status, which would give us two more strata to examine.

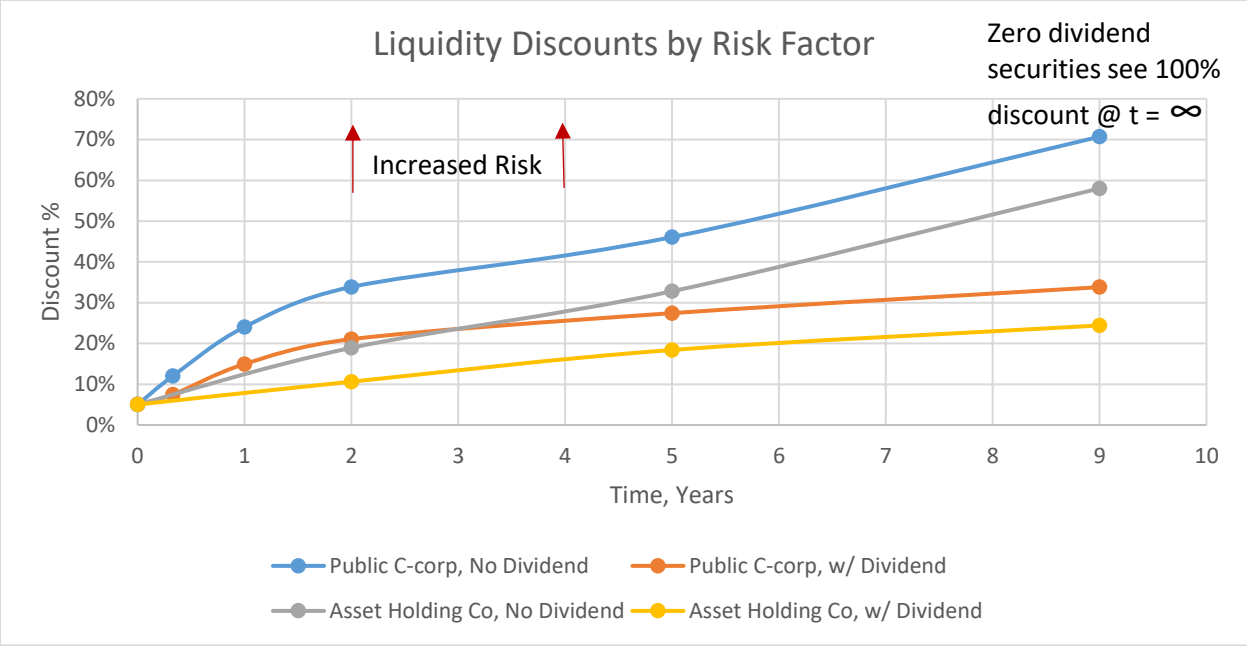
Based on the distinct independent variables presented, the liquidity discount has at least four dimensions to assess: initial cost, base discount rate, dividend risk, and cash flow risk. Discount rate behavior likely follows a pattern where dividend payers receive an average 7% discount rate; interests in self-sustaining, non-dividend companies have an average 9% discount rate; interests in cash-consuming companies with revenue have a 14% discount rate; and pre-revenue companies have an average discount rate approaching 30%.

Value Floor Effect

Liquidity studies reveal one more discount driver: the value floor. Comparing real estate holding companies and operating businesses shows a different magnitude of risk between the two types of businesses. Tangible assets are known to provide a positive value floor, reducing cost of failure. In an operating business, failure is likely to destroy all value, giving a value floor of zero. Accordingly, the higher liquidation floor of the tangible asset business reduces risk.

Figure 3 compares discount behavior over time for an operating company and a real estate holding company, both with and without dividends. A real estate holding company paying a dividend is expected to be the lowest risk of the four scenarios because the investment is backed by tangible assets and returns cash during the holding period. An operating company without a dividend is expected to be at the highest risk because the investment has no risk-reducing characteristics. Plotting the relationship between business type and dividend behavior over time shows the effect of each of these discount drivers. The plot in Figure 3 is shown for nine years. Data for public companies is restricted stock study averages, adjusted for the dividend effect shown in the SRR study; data for asset holding companies is published by Partnership Profiles.

Figure 3: Liquidity Discounts by Dividend and Industry



The data supports the hypothesis that a higher value floor reduces discounts. The value floor concept extends beyond real estate to securities such as preferred stock because a preference to be paid first raises the value floor and reduces the risk of losing principal. Thus, preferred stock is expected to carry less risk than common stock. Interestingly, the value floor effect is weaker than the effect of dividend payments, suggesting that holding risk is principally driven by cash events and less by business risks. Also note that securities receiving dividends always have positive value and never become worthless. Conversely, securities that do not receive a dividend advance toward a 100% discount over time.

(The graphic's goal is to understand liquidity discount rate behavior, not predict behavior specifically. These curves are the author's best stratification of the natural phenomena, acknowledging that study data was collected at different times, composed of different companies' interests, and traded in different markets. Real estate discounts were adjusted for DLOC coarsely. New studies could be materially different.)

Interpreting Lack-of-Control Data

Beyond the issues exposed in restricted stock data, the other major element of risk is the lack of company control for a non-controlling interest. Multiple studies on control changes have substantiated the effect of gaining or losing control of a public company. Under the "bundle of rights" concept advanced by Shannon Pratt, losing control of how to monetize the investment is a deficiency that demands a significant negative adjustment. Simply put, if an investor can acquire 100% of an asset and control all its aspects for a particular number of dollars, when control rights are absent, that same number of dollars must buy a larger percentage than its *pro rata* equity stake. The higher potential economic return compensates for the lost rights.

Significant debate surrounds the size of control premiums and discounts. Numerous academic studies point to the average discount between public company non-control and control falling between 17% and 24%.[13] At this point, care is needed to identify which elements of control are captured in company issues and which fall to the investor. The company equity basis used here for a starting point is a community interest in the company, which can be thought of as in-use value or a form of investment value. Company equity value includes the effect of control interests on the group of equity holders. Diversions of cash flow to control interests, rules established in company documents such as bylaws, and the ability to secure working capital and debt at favorable rates affect both risk and cash flows for the investor (equity holder) community. Many appraisers mix these community issues with individual investor issues. To have a sound independent variable structure, community issues are expressed at the company level, not at the investor level.

With current strategy, agency issues and management quality assessed at the company level, lack-of-control risk for the non-controlling interest is concerned with diminished ownership rights.[14] When an owner does not have control, the lack of

control risks are related to being unable to change company behavior in the face of an external change in business environment. The primary risks for the missing ownership rights are the lost abilities to:

- Declare and pay dividends, and make cash distributions to investors
- Sell equity to investors, or acquire equity from investors
- Borrow, or not borrow
- Change governing documents including organizing articles and operating rules
- Set policy and change course of the business
- Elect directors and appoint management
- Determine management compensation and perquisites
- Make acquisitions and divestitures
- Liquidate, dissolve, sell out or recapitalize the company
- Register the company's stock for a public offering

Now, to be relevant to a discount rate, change-of-control data must be explained over a time period. For this purpose, the control premia seen in public company acquisitions are converted to a rate with an assumed payback period on the acquisition. The payback period is based on the time horizon needed to materialize the expected return on investment. Korn Ferry is the world's leading executive search firm.[15] Their data provides insights on management tenure. In 2016, their clients' average CEO tenure was 8.0 years, and in 2020, tenure fell to 6.9 years. Since remaining CEO tenure is a likely limit for acquisition return assumptions, a payback period for gaining control between three and ten years seems reasonable. Using 20% for a transactional control discount and a seven-year planning horizon, the lack-of-control risk rate is calculated to be a 3.2% annual rate, which is a reasonable investor response to the reduction in ownership rights.

The application of a control discount based on holding period makes sense. No control discount exists when the time horizon is zero because control is not an issue when the interest is immediately convertible to cash. The lack-of-control discount increases with time because the risk of the company running off course becomes larger with time. With no starting friction to consider, a 3.2% central tendency has a discount curve intersecting (0, 0):

Figure 4: Control Discount Behavior

During the holding period, the risk to the minority investor due to lack of control varies from this central tendency primarily based on 1) management quality, 2) owner to supply any needed capital, and 3) strength of minority rights. This concept of the value of control is consistent with the teaching of NYU Leonard N. Stern School of Business professor Aswath Damodaran.[16] He advocates that a control premium or discount varies as a function of management quality, which includes the effect of company systems and financial controls. Good, trustworthy management lowers the potential need for ownership intervention. Unknown, out-of-control management raises the risk.

Modeling Costs to Convert to Cash

For a prospective investor to enter into a deal with unencumbered cash, the offer is based on the return of similarly unencumbered cash. So, exit costs must be assessed in the cash flow projection. The premise that exit costs, such as taxes, apply is evidenced by the fact that for comparably rated bonds, the tax-exempt bond receives a higher price than the comparable taxable bond,[17] and also by the behavior of sellers and companies "going public" to minimize brokerage costs and taxes. The investor looks to the net cash return as a benefit. At a minimum, $1,000 invested must return more than $1,000 after receiving the sale proceeds and paying taxes, or the investment will not be made.

In DCF, company exit costs and taxes are modeled in the cash flow projection.[18] Remember that a price paid for the company in the future may be at a different multiple, or the capital structure may deliver uneven change to common equity. For example, if EBITDA grows, a higher exit multiple may be appropriate, increasing exit equity value and reducing the "discount" from the company's current value. Using DCF analysis for these conditions is a suitable method and USPAP-compliant, but projections of future exit cash proceeds can be speculative when good information is unavailable.

"Going Concern" Modeling

For a stranded asset, accepting a "going concern" assumption with no future exit implies that the asset is permanently stranded and there is no cash for the investor outside of dividends. The model used to convert a linearly growing dividend to value is a capitalization of cash flows (CCF). This sunk cost scenario is encountered in *Estate of Giustina* and *Estate of Jones*. The company's "never sold" assumption usually diminishes the value of the non-controlling interest.

Investor Discount Rate

With the identified investor behaviors, the investor rate of return is assembled from the identified risk components and return demands. With exit costs recognized in the cash flow projection, the risk rate has two components: a liquidity risk rate and a lack-of-control risk rate.

As with Black-Scholes reasoning, the base liquidity risk of survival to the end of the holding period and exit is the RFR, which is modified by the three main behaviors seen in restricted stock studies: business type, earnings stage, and dividend. An illustration is a mature operating business with positive cash flow that pays a 2.0% dividend and has seven years to exit. Table 10 illustrates the construction of the liquidity discount rate for periods one through seven.

Table 10: Discount Rate for Lack of Liquidity

INVESTOR DISCOUNT RATE LIQUIDITY		
Risk Free Rate		**3.0%**
Adjustments to Subject	**Adjustment**	**Cumulative Discount**
Industry, Intangible Assets	70%	5.1%
Earnings Stage, Cash Flow Positive	0%	5.1%
Dividend. Yes	−32%	3.5%
Adjusted Risk Free Rate		**3.5%**

Determining the Lack-of-Control Discount Rate

To the non-controlling interest, the risk posed by lack of absolute control is the potential for the company to deviate from current expectations during the holding period. In comparison to a public company baseline, private companies tend to have fewer controls and more management risk, so an adjustment from the study of public companies is incorporated.

To estimate the risk difference between public companies and private companies, three studies of public vs. private companies set the risk difference for private firms between 18% and 37%.[19] Other factors commonly evaluated for non-controlling interests are the influence of the block size being examined and any demonstrated conflicts of interest. (This list could have more factors; for example, a non-voting security would be further impaired.) In Table 11, the 3.2% central tendency found earlier is adjusted for the lack-of-control discount rate.

Table 11: Discount Rate for Lack of Control

INVESTOR DISCOUNT RATE

CONTROL		
Public Company DLOC		**3.2%**
Adjustments to Subject	**Adjustment**	**Cumulative Discount**
Planning, Systems and Controls v. Public Co.	18%	3.8%
Control in Block, None	0%	3.8%
Conflict of Majority Interest, None	0%	3.8%
Control Discount Indicator		**3.8%**

Determining the Investor Discount Rate

Consider that the rate of return to be developed is a minimum or threshold rate of return used to find the security's market value. The remaining piece of the discount rate is the desired rate of return for investments without these non-controlling interest risks. The investment return inputs are the industry's long-term growth rate of earnings (LTGR) and the industry's dividend rate. Industry rates are appropriate for setting market rates, which are different from what an individual wants for a return. Assuming a 7% LTGR and a 2% dividend, the example investor discount rate for a DCF is shown in Table 12.

Table 12: Discount Rate for DCF Calculation

INVESTOR DISCOUNT RATE	
	Discount Percentage
LTGR, Industry	7.0%
Dividend Rate, industry	2.0%
DLOL	3.5%
DLOC	3.8%
Non-controlling interest Discount Rate	**16.2%**

The investor discount rate is applied to the projection of dividends and exit cash disbursements. Now the model will operate such that if the cash flows grow faster than the investor discount rate, a premium to value will be realized. And if the opposite is true, a reduction in value will be realized.

Feasibility

Angel and venture capital investments with high exit multiples are examples of discount rates being applied to investor cash events. Early-stage investment is typically made on the basis of predicting the exit proceeds over a time horizon of five to seven years, with a discount rate appropriate for each stage. These investments are ordinarily made with preferred equity, having priority over common equity and some additional voting rights as a class. These preferred securities are lower risk than their affiliated common securities.

Using data from the Pepperdine Capital Markets Survey, the CBRE North American Cap Rate Survey, and the SBA Underwriting Guide, rough median discount rates for these non-controlling investments are shown in Figure 5. Operating risk increases with maturity and decreases with the size of EBITDA. Comparatively, common equity is expected to have greater discount rates than shown here because of the lack of preferences and voting rights.

Figure 5: Investor Discount Rate Survey Data

APPROXIMATE DISCOUNT RATE RANGE STAND-ALONE COMPANIES		
DISCOUNT RATE*	**BUSINESS STAGE**	**SUBJECT**
0%	**No Risk, Guaranteed Future Profit**	**MAXIMUM VALUE**
5%	Regulated Monopoly, Quality Real Estate	
12%	Mature Companies with dominant trade position and $5MM EBITDA	
16%	Mature Companies with highly predictable future and $5MM EBITDA	
22%	Mature Companies with low entry barriers or under $5MM EBITDA	
35%	VC Expansion Stage	
38%	Angel,Early Stage	
40%	Angel & VC Start-up	
43%	Angel & VC Seed	
80-90%	Concept	
100%	**Zero Future Profit**	**MINIMUM VALUE**

* Adapted from Pepperdine Capital Markets Survey, CBRE Survey & SBA Underwriting Guide

The discount rate determination presented here results in rational and reasonable rates compared to survey data.

Conclusions

Models used today to value stranded non-controlling interests in closely held companies fail to match the valuation problem on one point or another. They may confuse the present with the future, use market variables that do not relate to intrinsic value, or use company variables where investor variables are necessary. Use of average discounts produces even greater error because the population of securities has a number of mutually exclusive conditions. An average discount is in error by 10% to 50% on dividend paying behavior alone because the two dividend payment populations (payer vs. nonpayer) have significantly different market risks. Current models provide erroneous conclusions because:

1. Payout proceeds to investors have both fixed and variable issues,
2. Risk falls when cash is delivered to the investor during the holding period,
3. Company exits may be at a valuation different from the current valuation, and
4. Capital structure and value floor affect the future cash payouts to each security.

Fundamentally, no link exists between a private company's current value and the cash proceeds delivered over time to a security's owner. Consequently, discounting current company value attaches to the wrong cash flow. The nature of the valuation problem points to the income approaches for a solution based upon the control interest's dividend policy and exit plans. The cash flow model takes one of two forms: a capitalization of cash flows (CCF) applied to a dividend persisting to infinity, or a DCF applied to a projection of dividends and exit proceeds over a holding period. The corresponding investor discount rate needed for these methods is strongly driven by a company's tendency to consume its investors' cash, reinvest cash into the company, or distribute cash to its investors.

The biggest questions for appraisers of private securities are whether a company will be sold, when the sale will occur, and what the expected selling price will be. While speculative to a degree, these are the same basic assumptions that professional investors make when acquiring a stake in a private company. No other course provides the relevant connection to the investor.

Appendix: Assembling a DLOM

This paper began by stating that investor interests are only remotely related to current company value. This disconnect is true. However, with assumptions similar to capitalizing cash flow, current company value can be a proxy for future exit proceeds' present value if exit costs, taxes, and holding periods are considered in the DLOM.

Where the subject company is not changing rapidly, a linear projection of current company value to future value may be a safer alternative than speculating on the exit cash flow. DLOM, when applied to current company value, is also an idea that courts and other constituents grasp more easily than discounting cash projections. A relevant DLOM is assembled considering the two-part nature of the discount: exit costs plus the holding risk (liquidity and control) until an assumed exit event.

Taking a discount from the current value implies that the appreciation and discount rates over the holding period are linear and equal. Discounting from current company value also implies that shares sold separately will have the same value if sold as a 100% block. These two assumptions prevent consideration of the predictable situation where a company's cash flows grow into a larger exit multiple and results in a larger discount than warranted.

Because exit costs are not part of current company cash flows, marketing, selling, and company tax costs are considered as a separate discount element called the discount for lack of salability (DLOS).[20] Marketing and selling costs can be determined based on current company value, under the equal appreciation and discount rate assumption. On the other hand, company paid taxes for the exit need to be projected and discounted back to present value. Deducting these costs from the current value leaves the residual amount delivered to investors from a future sale. For brevity's sake, these costs are stipulated and converted to percentages. An example is shown in Table 13.

Table 13: Discount for Lack of Salability Calculation

INVESTOR DISCOUNT DLOS	
Brokerage Fee on Operating Asset Value	5.0%
PV of Company Taxes on Change in Equity Value	3.0%
DLOS	**8.0%**

Discount for Lack of Marketability

As in the DCF method, the starting point is equity value, which includes agency costs and other issues related to the community interest in the company. For the DLOM example, the investor DLOL of 3.5% found earlier is used to determine the seven-year holding discount of 21.9% in Table 14. The investor discount for lack of control (DLOC) of 3.8% found earlier determines the seven-year holding discount of 23.6% in Table 15.

Table 14: Discount for Lack of Liquidity Calculation

INVESTOR DISCOUNT DLOL	
Adjusted Risk Free Rate	3.5%
Exit Assumption, years	7
DLOL	**21.9%**

Table 15: Discount for Lack of Control Calculation

INVESTOR DISCOUNT DLOC	
Adjusted Risk Free Rate	3.8%
Exit Assumption, years	7
DLOC	**23.6%**

Assembling the DLOM

As restricted stock studies show, the natural structure of DLOM is a first-order equation. DLOS is the initial cost. Liquidity and control discount rates are time related: (1 – d) for each discount is multiplied together for holding period t_1 to t_7. DLOM follows the form:

$$f(t) = C + (1 - (1 - d)^t)$$

or

$$DLOM = DLOS + (1 - (1 - DLOL)(1 - DLOC))$$

An example is completed in Table 16.

Table 16: DLOM Calculation

DISCOUNT FOR LACK OF MARKETABILITY DLOM			
	Discount Percentage	**Cumulative Discounted Value**	**Cumulative Discount**
DLOS, t_0	8.0%	92.0%	8.0%
plus			plus
DLOL, t_1 to t_7	21.9%	78.1%	
DLOC, t_1 to t_7	23.6%	59.7%	40.3%
Non-controlling interest DLOM			**48.3%**

The DLOM is applied to current company value to find a non-controlling interest value.

Variable Independence

The DLOS, DLOC, and DLOL are not interrelated and affect the discount independently. Cash costs stand separate from time-related risk rates. Control studies are made at a particular transaction date and have no liquidity element. Restricted stock studies are studies of non-controlling interests with identical company control. Since these variables do not intersect, each value is independent, and using all three elements does not double-count an impact.

Footnotes

[1] Liquidation value here is the plain definition that means converting an asset to cash at a certain date. Market value is the price received for an asset in the marketplace. When an asset is provided sufficient market exposure time prior to sale and obtains a full complement of buyers, liquidation value is market value. Liquidation value can be subject to two constraints, a lack of buyers or a lack of time. If a complex asset has no buyer as a whole system, the complex asset is sold in pieces, which when marketed without a time constraint has been called orderly liquidation. The imposition of time restriction on cash conversion has been called forced liquidation. The time restriction reduces the number of buyers available in the market and compels a seller to accept the best offer available at the deadline.

[2] The most common measure of the risk-free rate is the ten-year treasury.

[3] The Stout Risius Ross Restricted Stock Study: A Recent Examination of Private Placement Transactions from September 2005 through May 2010. Aaron M. Stumpf, CPA/ABV, Robert L. Martinez, and Christopher T. Stallman, Business Valuation Review, Volume 30 • Number 1, © 2011, American Society of Appraisers.

[4] FMV Introduces Detailed Restricted Stock Study, by Espen Robak, CFA, Shannon Pratt's Business Valuation Update, November 2001, pp. 1–3.

[5] Willamette Management Associates study (unpublished).

[6] Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices, Silber, William L., Financial Analysts Journal, July–August 1991, pp. 60–64.

[7] Robert P. Oliver and Roy H. Meyers, "Discounts seen in private placements of restricted stock: the Management Planning, Inc., long-term study (1980–1996)," published in Chapter 5 of Robert F. Reilly and Robert P. Schweihs, Eds. The Handbook of Advanced Business Valuation (New York: McGraw Hill, 2000).

[8] Trugman Valuation Associates, Inc. (TVA), Restricted Stock Study, William Harris, Business Valuation Review, Volume 28 • Number 3, © 2009, American Society of Appraisers.

[9] 185+ days with a 26.9% average discount is an undefined range. If the discount for six months (180 days) is assumed to be 24%, then the monthly rate of increase in the discount is 3.6%. The first period discount, which we similarly assume reasonably represents one month, is 11.6%, and 11.6% − 3.6% = 8.0%.

[10] EBITDA is Earnings before Interest, Tax, Depreciation and Amortization. EBITDA is a measure of cash flow generated by company operating assets.

[11] Estate of Giustina v. Commissioner, 586 Fed. Appx. 417 (9th Cir. 2014)

[12] Estate of Jones v. Commissioner, T.C. Memo 2019-101

[13] Based on voting rights research, Jay B. Abrams estimates that the typical public company control premium falls between 21% and 28% (17% to 22% discounts). See Quantitative Business Valuation, 2001, p. 291. He adds that oppressed minorities in public companies are a good comparison to private minorities and that a 46% discount is demonstrated in these situations. Paul Hanouna, Atulya Sarin, and Alan C. Shapiro measured control value as the premium paid between comparable minority and majority transactions. They found that the median control premium for publicly listed U.S. stocks is 22% to 32% (18% to 24% discounts). See Value of Corporate Control: Some International Evidence, 2001, p. 28. Christopher Z. Mercer in A Brief Review of Control Premiums and Minority Interest Discounts concluded an implied median control discount of 19% to 22%.

[14] The value of a property depends on the anticipated utility or income that will accrue to the property owner in the future. See Shannon Pratt, p. 548, Valuing a Business, Third Edition.

[15] Korn Ferry is a global organizational consulting firm.

[16] Professor of Finance at the Stern School of Business at New York University (Kerschner Family Chair in Finance Education), where he teaches corporate finance and equity valuation.

[17]As noted by Roger Grabowski when he was Managing Director at Duff & Phelps, LLC: "Income tax treatment of distributions received by holders of a security does matter. At the simplest level, we see differences in pricing of comparably rated bonds which are fully taxable compared with those whose interest payments are tax exempt."

[18] Because the discount rate is considered to be a pre-tax rate, investor level tax costs are excluded from the cash flow model. The model assumed for exit proceeds is based upon a C-corporation in an asset sale because asset sales are the norm for M&A transactions. With an asset sale, the corporation bears a tax liability on certain profits. The company tax is included as an exit cost because the tax occurs prior to the investor's cash flow. Switching the assumption to a stock sale eliminates this complexity. Switching the entity type to a pass-through shifts the tax cost from the company to the investor, confusing the situation, but the company tax is not eliminated; tax is only passed to the investor. The investor now bears a higher tax than represented in returns at the risk-free rate and other rate of return benchmarks. These latter options are not zero-sum assumptions and can have large-value impacts.

[19] The Private Company Discount, by John Koeplin, Atulya Sarin, and Alan C. Shapiro, 2000; A New Examination of The Private Company Discount: The Acquisition Approach, by Maher Kooli, Mohamed Kortas, and Jean-François L'Her, 2003; The Existence of, and Earnings Quality Explanations for, the Private Company Discount, by Gus De Franco, Ilanit Gavious, Justin Jin, and Gordon D. Richardson, 2006.

[20] Lari Masten, Crossing the Bridge to Liquidity—DLOM and DLOL. The salability discount, named at the 2019 NACVA-CTI Annual Consultants' Conference, is the analysis of delivering unencumbered cash to the investor at their exit. The return-of-cash-to-the-investor concept is not only a key part of the Mandelbaum decision and others prior; following cash events is also fundamentally sound economic methodology. Salability is also recognized in the Stout Restricted Stock Study Companion Guide (2023).

References

Abrams, Jay B., (2010) Quantitative Business Valuation, 2nd Edition, Wiley.

Damodaran, Aswath, (2006). Damodaran on Valuation, 2nd Edition, Wiley.

Damodaran, Aswath, (2009). Valuing Financial Service Firms, Stern School of Business.

Damodaran, Aswath (2012). Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2012 Edition, Stern School of Business.

Fama, Eugene (1970) The Efficient Capital Markets: A Review of Theory and Empirical Work, Journal of Finance.

Finnerty, John D. and Emery, Douglas R. (2004) The Value of Corporate Control and the Comparable Company Method of Valuation, Financial Management, Spring 2004.

Grant, Eugene L., Ireson, W. Grant and Leavenworth, Richard S. (1990), Principles of Engineering Economy, Eighth Edition, Wiley.

Lisi, James A., (2021) Business Value in Use: Differentiating Value to the Owner from Value in Exchange, Value Examiner, March/April 2021.

Lisi, James A., (2019) Business Valuation Terms Need Restructuring Based on Value Drivers, Business Valuation Update, Vol. 25, No. 8.

Marks, Kenneth H., Slee, Robert T., Blees, Christian W. and Nall, Michael R., (2012) Middle Market M&A, Wiley.

Mercer Capital (2008) The QMDM companion, version 4.0, Explanatory Text to Accompany the Quantitative Marketability Discount Model.

Pratt, Shannon P., Reilly, Robert F., and Schweihs, Robert P. (1996), Valuing a Business, Third Edition.

Smith, Adam (1776), An Inquiry into the Utility and Causes of the Wealth of Nations, Book 1, Chapter 4, Of the Origin and Use of Money, W. Strahan and T. Cadell.

Stegemoller, Michael A. (2002) Public and Private Takeover Markets: An Empirical Analysis, The University of Georgia.

Taleb, Nassim Nicholas, (2012). Antifragile, Random House.

Trugman, Gary R., (2017) Understanding Business Valuation, Wiley.

Uniform Standards of Professional Appraisal Practice (USPAP), effective January 1, 2020, through December 31, 2021.