THE COLLEGIAN

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C O N T E N T S

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Editor . . . . David Lachterman
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It is my sincere hope that the Collegian has been of value to the community during the past year. My endeavor has been to present the best writings from every area of the Program along with outstanding independent work. In addition, an effort was made to encourage creative writing, specifically in the medium of the short story. The meagerness of the response to the announced 'Short Story Contest' led me to give greater credence to the oft-expressed complaint that the requirements of the Program enervate literary energies. In the hope that the summer vacation might witness a revival of those energies, Miss Brann and I have decided to extend the contest until the first week of the coming school year. Stories may be submitted to either of us through the college mail. The prize is still $20.00 for use in the Bookstore.

DAVID LACHTERMAN

A RATIONAL EXTENSION OF THE FIFTH BOOK OF EUCLID'S ELEMENTS

David H. Stephenson

"<u>A ratio</u> is a sort of relation in respect of size between two magnitudes of the same kind." "When of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first (magnitude) is said to <u>have a greater ratio</u> to the second than the third has to the fourth."

In comparing these two definitions, one cannot help but be struck by the comparative adjective "greater" (μείζονα) applied to ratios, for it presupposes that ratios themselves have size. And if they have size, is it not possible to determine a ratio of ratios consistent with Euclid's fifth book?

Definition 7, quoted above, is apparently included by Euclid to give a name to one class of ratios which are not the same, but also to distinguish this class from the other class of unequal, or rather "not same", ratios. That is, given any ratio, there are innumerable ratios other than it, and these fall into two classes: those which Euclid would call greater than it, and those than which it is greater, i.e., those less than it. These three classes, the same, greater, and less, could of course have been distinguished by adjectives without quantitative connotations, such as same, light, and dark. Therefore the use of the word, "greater", to describe a relationship of ratios implies a common conception of them in some way akin to the conception of magnitudes.

This kinship can be explained in two ways. First, Euclid and his contemporaries may have felt that in the case of ratios to the same, as in propositions 7-10 of Book V, the relative size of the two antecedent magnitudes obviously determines the relationship of any two such ratios. Thus a greater magnitude should be said to have a greater ratio to the same than the less has. Since all ratios can be compared by finding proportionals to the same, they can be called greater, less, or same according to this reasoning.

Secondly, the definition of ratio quoted above implies more about the size of a ratio than is immediately apparent in the translation. In particular κατὰ πηλικότητα , which Heath prefers to translate "in respect of size" gives insufficient indication of what kind of relation between two magnitudes constitutes their ratio. There are two possible ways in which the size of two unequal magnitudes can be basically compared: one can exceed the other by a certain magnitude, or one can be a multiple of the other. Regardless of any ambiguity in the term πηλικότης , Euclid could not have meant a relation with respect to difference of size as in the former comparison. Therefore a ratio is in some way dependent on the <u>quantuplicative</u> relationship of two magnitudes.

This is clear in the case of numbers and commensurable magnitudes, so that Euclid does not bother to define either ratio or greater ratio for numbers. Such magnitudes (or numbers) obviously have ratios greater than, the same as, or less than each other according as the first antecedent magnitude contains more, as many, or fewer parts of its consequent magnitude than the other antecedent contains of its consequent. (That is if the parts taken are equal to the largest common measure.)

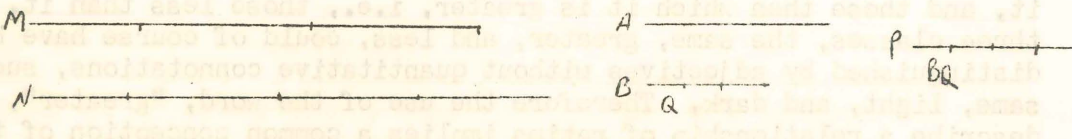
Moreover the definitions of same and greater ratio for incommensurables in Book V hardly differ at all from the above definition for commensurables. Definition 5, for example, can be restated in the following way:

Magnitudes are said to be in the same ratio, the first to the second and the third to the fourth, when the first magnitude contains as many parts (submultiples) of the second that the third contains of the fourth for all equisubmultiples of the third and fourth.

That this is equivalent to Euclid's definitions is evident from the fact that according as the multiple M of one magnitude A exceeds, equals, or is less than another multiple N of a second magnitude B, the first magnitude A itself must exceed, equal or be less than a multiple p of a part of the second magnitude B, where the part BQ is the same submultiple of the second magnitude B that the first magnitude A is of its multiple M, and P is the same multiple of BQ that N is of B. In modern notation :

$$\text{if } mA \gtreqless nB \text{ then}$$

$$A \gtreqless \frac{nB}{m} = n \frac{B}{m}$$



But this means that the relationship of two ratios is determined by the relationship of numbers of parts of the consequent magnitudes contained by the antecedent magnitudes. If the numbers are the same for all equisubmultiple parts then the two ratios are the same. If in some instance the number of containable parts in the first ratio exceeds the number in the second ratio, should we not say that the first ratio is larger than the second? (This is equivalent to Euclid's definition 7.)

Although the above arguments are implicit in Euclid's Fifth Book and are justifications for his assumption that ratios have size, they also point to other similar assumptions which Euclid nevertheless refused to admit. For if the fact that ratios have size is deduced from the quantitative relationships between magnitudes or numbers inherent in different ratios, then their "relation in respect of size", viz. the quantuplicative relation of these ratios should also be prescribed by these numbers or magnitudes. So it should make sense to speak of a multiple of ratios or a ratio of ratios.

Ratios of Ratios

Consider the following ratio of ratios: (A:B):(C:B) and the ratio of magnitudes A:C (A,B, C therefore being of the same kind). Then if A equals C, it is immediately apparent that (A:B):(C:B)::A:C since A:B::C:B, therefore A:B has the same size as C:B, i.e., the magnitude A:B equals the magnitude C:B. By dint of the common notions equimultiples of equal magnitudes are equal; larger multiples of equals being larger than smaller multiples. And Definition 5 establishes the above proportion.

However if A does not equal C, (A:B):(C:B) may be greater than the same as, or less than A:C. For instance, assume (A:B):(C:B) exceeds A:C for all A greater than C. Then this is true in particular when A is itself a multiple of C. By Euclid's Definition 7 there are equimultiples -- say W;X of A:B the same multiple as K of A, and Y:Z of C:B the same multiple as L is of C, -- such that W:X exceeds Y:Z but K does not exceed L. Symbolically depicted:

$$\begin{array}{l} W{:}X{::}(A{:}B + A{:}B + \dots + A{:}B) > (C{:}B + C{:}B + \dots + C{:}B){::}Y{:}Z \\ \quad K = (\;A \;+\; A \;+ \dots +\; A\;) \leq (\;C \;+\; C \;+ \dots +\; C\;) = L \end{array}$$

Then since A is a multiple of C so is K. But K is less than or equal to L. Therefore L is a greater multiple of C than is K, or else L and K are equimultiples. Whatever multiple K is of C let that multiple be taken of C:B, and call it P:Q. Then K and P:Q are equimultiples; so are L and Y:Z. Since L exceeds or equals K, Y:Z exceeds or equals P:Q, and W:X must exceed P:Q since it exceeds Y:Z. Let S:T be the same multiple of C:B that A is of C. Then since K and P:Q are equimultiples of C and C:B, they will be equimultiples of the equimultiples A and S:T. And W:X and K are also equimultiples of A:B and A, so W:X and P:Q are equimultiples of A:B and S:T. Therefore since W:X exceeds P:Q, A:B exceeds S:T.

Hence a necessary condition that the ratio of two ratios (with the same consequent) exceed the ratio of their antecedents whenever the first antecedent exceeds the second is that the multiple of a ratio be always less than the ratio of the same multiple of its antecedent to its (unmultiplied) consequent. That is, if (A:B):(C:B) > A:C for all A greater than C then E:F > G:H whenever E is the same multiple of some magnitude K that G:H is of K:F. (Algebraically, $E{:}F{::}mK{:}F > m(K{:}F){::}G{:}H$ for $m > 1$)

It also follows from this that if A is less than C, this condition requires that (A:B):(C:B) < A:C. In other words (A:B):(C:B) exceeds, is the same as, or less than A:C according as A exceeds, equals, or is less than C. Such ratios exist already in Euclid's work: the duplicate ratio or ratio of the squares on two magnitudes, for example. Thus since the duplicate ratio of A:C does exceed, equal, or become less than A:C as A exceeds, equals, or is less than C we might simply equate or define the ratio of two ratios having the same consequent, (A:B):(C:B), as the duplicate ratio of their antecedents, Duplicate (A:C). But any such definition presumes the necessary condition stated above concerning multiples of ratios.

Such a condition raises the following question: when one ratio is greater than another, <u>how much greater is it</u>? The multiple of any magnitude, including ratios, is so many times (viz. double, triple, etc.) as great as the magnitude of which it is a multiple; if then one ratio is a multiple of another, the larger ratio, considered as a magnitude, must be so many times as large as the smaller.

To answer this question is it not most consistent to continue in the direction begun by Euclid in the seventh definition of Book V? That is, as was pointed out earlier in this article, Euclid's assumption that a ratio can be greater than another is based on the fact that some magnitudes (in the limited sense, i.e., excluding ratios) numbers, or multiples of magnitudes are greater than others. In particular it probably struck Euclid as obvious that, as proposition 8 states, "the greater (magnitude) has to the same a greater ratio than the less has". This proposition along with the preceding one therefore imply that <u>the relative size of two ratios with the same consequents is determined by the relative size of their antecedents</u>. And thus a ratio R:S, where R is a multiple of another magnitude Q, is as much larger than Q:S as R is larger than Q, i.e., R:S is the same multiple of Q:S that R is of Q.

Accepting this as a logical consequence of Euclid's definitions and propositions, one must also agree that (A:B):(C:B) is not greater than A:C for all A greater than C and less than A:C for all A less than C, because we have just proved that this would require that for any R a multiple of Q, R:S must be the same multiple of Q:S, and the ratio of the multiple R of the antecedent Q to the consequent -- (R:S) -- was supposed to exceed the same multiple of the ratio (also R:S), which is impossible.

With multiples of ratios so defined Euclid's propositions in Book V can be easily proved, and in addition the following four propositions relate ratios of ratios and ratios of simple magnitudes.

<u>Proposition 1</u>

The ratio of two ratios having the same consequent is the same as the ratio of their antecedent magnitudes.

That is given any A,B,C, magnitudes of the same kind (A:B):(C:B)::A:C for if not (A:B):(X:B) is greater (or less) than A:C. Hence if greater some multiple of A:B, say P:M, exceeds another multiple of C:B, say Q:N, but the first multiple of A, say K, does not exceed the second multiple of C, say L. But since K and P:M are equimultiples, P:M::K:B. Similarly Q:N::L:B. Hence K:B exceeds L:B and by Euclid's Proposition 10, K exceeds L, which contradicts the hypothesis. Therefore, etc.

Similarly (A:B):(A:C)::C:B Q.E.D.

<u>Proposition 2</u>

The ratio of the ratio compounded of two ratios to either component ratio is the same as the other component ratio; and if the ratio of one ratio to a second ratio is the same as a third ratio, the first is the same as the ratio compounded of the second and the third. That is given any six magnitudes A,B,C,D,E,F, such that (A:B comp C:D)::E:F, I say that (E:F):(A:B)::C:D and (E:F):(C:D)::A:B.

As Euclid demonstrates in Book VI, to any three magnitudes a fourth proportional can be found (provided the ratios of these magnitudes can be expressed as ratios of straight lines). Hence let A:B::K:L and C:D::L:M, where K.L,M are straight lines. Then (A:B comp C:D)::(K:L comp L:M)::K:M, and so K:M::E:F. By the preceding proposition (K:M:(L:M)::K:L and (K:M):(K:L)::L:M. Therefore (E:F):(C:D)::A:B and (E:F):(A:B)::C:D. Again let (P:Q):(R:S)::T:U then I say that P:Q::(R:S comp T:U). Obviously from the above if (R:S comp T:U)::V:W then (V:W):(R:S)::T:U, hence (V:W):(R:S)::(P:Q):(R:S) and V:W::P:Q. Therefore (R:S comp T:U)::P:Q.

Q.E.D.

Proposition 3

The sum of two ratios having the same consequent is the same as the ratio of the sum of the antecedents to the consequent. That is, given any three magnitudes (of the same kind) A,B,C then (A:B + C:B)::(A+ C):B.

Since (A:B):(C:B)::A:C then by Euclid's Proposition 18, Book V (which it is easy to prove for ratios as magnitudes) (A:B + C:B):(C:B)::(A + C):C and by the preceding proposition (A:B + C:B):: [(A + C):C comp C:B] ::(A+C):B Q.E.D.

These three propositions in effect define ratio of ratios and addition of ratios and also redefine compound ratio as the inverse of ratio of ratios. That compounding also corresponds in some way to multiplication of numbers is evident from the next proposition.

Proposition 4

If four ratios are proportional, the ratio compounded of the extreme ratios is the same as the ratio compounded of the mean two ratios.

That is: given four ratios A:B, C:D, E:F, G:H such that (A:B):(C:D)::(E:F):(G:H) I say that (A:B comp G:H)::(C:D comp E:F). Since it is possible to find ratios the same as the above with either the antecedent or consequent prescribed, it suffices to prove the following:

if (A:B):(K:B)::(B:L):(B:M) then (A:B comp. B:M)::(K:B comp B:L)

But this is immediately apparent since (A:B):(K:B)::A:K and (B:L):(B:M)::M:L, hence A:K::M:L

Alternating according to Euclid's Proposition 16, A:M::K:L. Therefore since (A:B comp B:M)::A:M and (K:B comp B:L)::K:L (A:B comp B:M)::(K:B compB:L) Q.E.D.

Ratios and "Real Numbers"

The fourth and last of these propositions closely resembles Proposition 19 from Book VII of the Elements. Someone might well ask, how are ratio and number related? And the startling answer is that ratio is number and number is ratio, although in a somewhat larger sense of "number" than Euclid was familiar with. That is, ratios are the same as the "positive real numbers" of modern mathematical jargon.

All numbers are "relations in respect of quantuplicity, as are ratios. "Three" means nothing other than the relation with respect to quantuplicity of three things and one thing,* it cannot exist without a unit. (Even for Euclid "a number is a multitude composed of units.") The distinguishing characteristic of numbers is that they are relations with respect to a

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* Used as an adjective, of course, the number attributes this relation to the noun it modifies.

single and non-arbitary unit.

"An unit is that by virtue of which each of the things that exist is called one." Things are called one, moreover, by virtue of a relation, namely the relation in respect of quantuplicity between a thing and itself. Something is called one virtually because it is the same as itself.

Therefore the unit is in fact the ratio in which antecedents and consequents are the same. Euclidean numbers, i.e., integers, are multiples* of this unique ratio, or equivalently by our previous hypothesis, the ratios of multiples of any magnitude to the magnitude itself.

Besides the integers and the unit, the real numbers include the so-called "rational" and "irrational" numbers. The former, as their name implies, are ratios, distinguished from the latter by the commensurability of antecedent with consequent. The "irrationals", (in the modern sense of the term) are also ratios, but ratios with incommensurable antecedents and consequents. Although neither rational nor irrational numbers are multiples of the unit, they do relate to it quantuplicatively since the ratio of any real number (i.e., ratio) to the unit is the same as itself.

For example, (A:B):(C:C)::(A:B):(B:B) since obviously C:C::B:B. And (A:B):(B:B)::A:B. Therefore, (A:B):(C:C)::A:B. Furthermore it can be proved that ratios as real numbers, in toto show an isomorphism with the points of a line, that they are continuous, and that they are distributed copiously among the integers.

If Euclid had not chosen the word "greater" to describe one class of ratios not the same as a given ratio, and implicitly the word "less" to describe the other class, if instead of greater and less he had chosen to call them light and heavy, masculine and feminine, hot and cold ratios, these conclusions would be inconsistent with Euclid's thought. But he did choose that word. Therefore it is consistent. Q.E.D.

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* something can be multiplied without reference to number simply by adding to itself.

FOR HOUGHMAGANDIE

Charles G. Bell

Even the Romans in the decadence
That heaped the fallen world with monuments
Of their gloom, and when the bitterness of sex surprised them,
Loved it, sought it, prized it.

And from fabulous shade of the Dark Ages,
Out of crypts and cloisters, rise nameless voices --
Avowed monks on their knees inclining
To the dear socket of a girl's inturning.

Our own forbears, sanctimonious Puritans,
Warmed beds with bundling; staid Victorian
Dames, under bustles of silence, veiled the same fesses,
Smooth, indented, thighs spread for caressing.

Body, naked, cloven, supple, swaying --
Here while the great cloud waits in abeyance
At the sun's horizon, shall we not mate and slumber,
Our foldings wreathed on the leaves of Indian Summer?

THE NEW FALL

Charles G. Bell

"Triste, triste is the fall, the sad fall,
With wind and mist and leaves that fall."
(Victor Hugo, who was, alas,
The principal poet produced in France.)

While we, in a culminance of gold --
Tulip and oak, sassafras, ash,
With wine and scarlet of dogwood and gum,
And the filligree of creeper and thorn --

We, stripped to the skin, in rivers of sun,
On the hilltop stand, and cry the love
Of these twined selves and earth that moves
To the waste of winter by a turning road.

We swear the rapture of this new world
Is the sun's seizure on flesh of the fall.

ON THE SHORE OF BIRTH

Charles G. Bell

I wake from a cruel dream:
Odysseus, brandishing the blade
At the threshold of sleep,
Drives back the spirits
From the pool of blood,
Seeking the prophet.

I, clawing through dark
To the spilled sacrament
And wine of force,
Tear sleep with my cry:
"Odysseus, stern guard,
Put back the sword;
I am he; let my tongue
Blossom in speech."

I wake, my mouth filled
With the salt reek of blood.

ΓΝΩΜΩΝ

THE GNOMON: A NOTE ON THE "KNOWER"

Eva Brann

The gnomon, rival in interest to the canon (Collegian, Suppl., November 1962), appears thrice in the mathematical texts of the freshman year: in the fall in Euclid (II, Def. 2) as a geometric shape, in the winter in Nicomachus (I, 9) as a numerical scheme and in the spring in Ptolemy (II, 5) as an astronomical device. What do its forms have in common and why is it called "the knower" (ὁ γνώμων, from γιγνώσκειν, to know)?

I. Shadows as Images: the Astronomical Gnomon

> "Proper people usually take their shadow along when they go into the sun".
> Chamisso, Peter Schlemihl

It is among the wonderful facets of the visible world that every solid body under the sun casts a shadow. To be exact, there are two sorts of bodily shadow, which Leonardo da Vinci, who was fascinated by this appearance, called "primary" and "derivative".[1] The first belongs to the body itself and is a consequence of the curious fact that bodies, in their opacity, stand in their own light, just as they hide the larger part of themselves from sight (whence the eye and the sun are often compared; in optics this leads to the notion of visual rays -- cf. Euclid, Optics, Defs.). Therefore their surface, or as the Greeks said, their "apparency" (ἐπιφάνεια)[2] always has a bright and a dark side. In the ancient world, which still lived under one primary source of natural light, this was an overwhelming fact. Greek pot-painting, for instance, -- a craft humble and popular enough to permit us (as no classical text ever does) to speak of the predilections of "the Greeks" -- testifies to this. Athenian pottery, which was produced in tremendous volume and sent to the ends of the known earth, used two basic techniques, the one belonging roughly to the "archaic" sixth, the other to the "classical" fifth century B.C. Archaeologists speak of them as "black-figure"[3] and "red-figure" respectively. In the former technique, the figures of men, boys, horses, armor, girls (listed in descending order of interest) and other paraphanalia are put in rich black glaze onto the bright orange ground of the Attic clay. Therefore they appear to be seen against the light, as schemata, as flat, sharp, black silhouettes, slimmed, as they would be in nature, by the encroachment of the light around them. In the later technique the reverse is done -- the figures are "reserved" out of the black glaze covering of the pot. Therefore they appear as if the sun were full on them, and, being nourished as it were by light, are generally larger than the black figures, while the addition of black interior contours gives them a more substantial look: Leonardo aptly called such body shadows "the expression of body". Their use was elaborated by Greek painters into a technique called "shadow-painting" (σκιαγραφία), a species of trompe-l'oeil which makes the image look more natural than nature; it is often cited by Plato as the very paradigm of the art of <u>illusion</u>. (e.g., <u>Republic</u>, 365C, 523B, 583B, 586B, 602D).

But the two grandest and most instructive examples of all such shadows were, as Empedocles taught, the darkness of night, for

> "Earth makes night by standing under the rays of the sun"[4]
> (νύκτα δὲ γαῖα τίθησιν ὑφισταμένη φαέεσσι <ἠελίους>),

and those amazing phenomena after which the sun's own circuit was named the "circle of light failure" (ἐκλειπτικὸς κύκλος) -- the eclipses.

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However, the sort of shadow relevant to the gnomon is really only the second, the cast shadow, which the painter, considering it as darkening another body, calls "derivative", but which is, taken in itself, a very peculiar kind of <u>image</u>.

As an image, it is first of all natural, like a reflection (<u>Republic</u> 510E), and inseparable from its body, being its "doppelgaenger": σκιὰ ἀντίστοιχος -- as Euripides says, (<u>Andromache</u>, 745). Indeed, the ability to produce a shadow is the very sign that a thing is a proper body among other bodies; it is at once the warrant and the effect of its substantiality.[5] Secondly, shadows are negative images, images by deficiency, in which that which is visible in surfaces is almost entirely absent, "for what is seen is color" (τὸ γὰρ ὁρατόν ἐστι χρῶμα - Aristotle, <u>De Anima</u> 418a29). To this kind of shadow too, the first accounts of the cosmos are indebted, for as soon as failures of light in the heavens were recognized as shadow phenomena, the outline of the earth could be seen upon the moon, and from its phases could be inferred its character as

> "a night-shining borrowed light, wandering about the earth"
> (νυκτιφαὲς περὶ γαῖαν ἀλώμενον ἀλλότριον φῶς).
>
> Parmenides, Fr. 14.

But the most honorable function the cast shadow has ever served was to be admitted to the place shown in this diagram of the metaphor for the road to being in the <u>Republic</u> (507ff.):

eye	⟶	shadow	:	body	⟵	sun
≀		≀		≀		≀
thought	⟶	mathematical object	:	being	⟵	good.

The properties of shadows which fit them to be the sensible representatives of mathematical objects are just those described above and developed below: shadows are schematic consequences of the shapeliness of bodies, preserving as natural images a certain truth even under deformation; they are the lightless witnesses of all visibility, more easily consulted about its ways than is its dazzling source itself -- and so they are to the eye what mathematical objects are to thought: the nether aspect of being.

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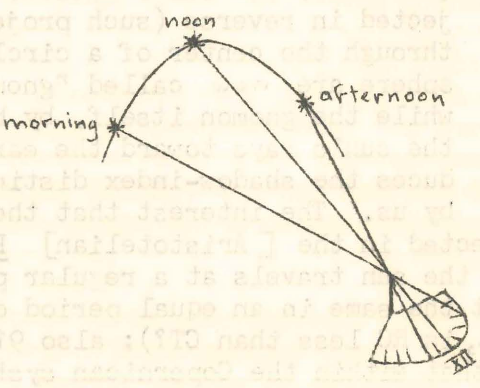
The third characteristic of the cast shadow, namely that it changes in length and direction independently of the body which produces it, is that which the astronomical gnomon particularly exploits. These changes are so fixed and familiar and so much more easily observed than the course of the sun in the sky (the sky itself is named after the shadow [σκιά] its clouds cast -- Partridge, Origins) that the reversal of their progress becomes the very signal of God's special concourse: "Behold, I will bring again the shadow of the degrees, which is gone down in the sun dial of Ahaz, ten degrees backward. So the sun returned ten degrees, by which degrees it was gone down" (Isaiah 38: 8, II Kings 20: 11). The gnomon, by which shadows are observed, is a thin upright, set up so as to cast a slender, neat shadow, whose changes with the hours and seasons can be closely watched. A gnomon may be no more than a stick in the ground, but it can also be the pointer of a sundial with carefully calibrated base, or even a tall obelisk-like official monument.[6] Such instruments were often called "shadow-viewers" (σκιόθηρα).

The first skiothera are said to have been set up in Sparta by Anaximander in the first half of the sixth century.[7] One of them seems to have been a sun clock (ὡροσκοπεῖον, ὡρολογεῖον) to tell the hours of the day.



It seems fitting enough that the Spartans, then in the early days of their discipline with its many public occasions, should have been the first to want a public clock. Later on most public places must have had one; the Athenian market place, for instance, had a very large public water clock--sometimes also called "gnomon" (Athenaeus II, 42b, but usually κλεψύδρα -- "water stealer").[8] The name gnomonics came to be applied by the Greeks and Romans to the flourishing practical art of dialling generally, and particularly to the construction of a great variety of sun clocks (see Vitruvius IX, 8), among them some with inclined rather than upright pointers, such as the one which may have occasioned the early study of conic sections, described in IV. below. (Indeed Kepler thought it a fact that gnomonics "begot for us the geometrical doctrine of conic sections -- Epitome, p. 205) But the first and most ubiquitous gnomon was always a natural one -- man, the upright creature (Aristotle, Parts of Animals 653a31), talking away the morning as his shadows waned:

> These three houres that we have spent,
> Walking here, Two shadowes went
> Along with us, which we our selves produc'd;
> But, now the Sunne is just above our head,
> We doe those shadowes tread;
> And to brave clearnesse all things are reduc'd.

The theme of the human gnomon will recur.

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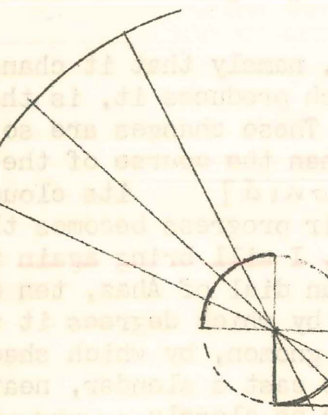
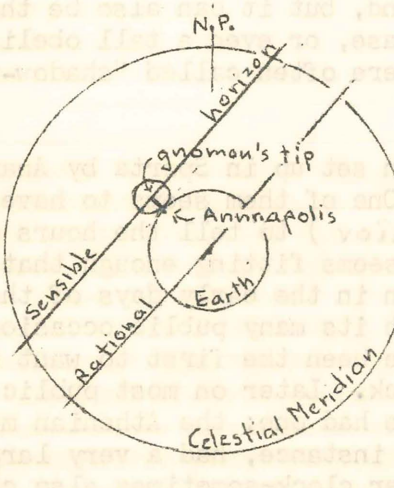
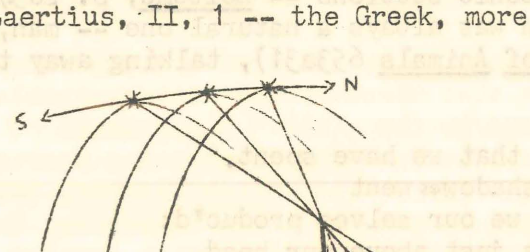
The gnomon in its properly astronomical function depends only on the shadow at high noon.[9] Its construction is, of course, identical with that of the sun clock and one fundamental diagram serves both, as in V. below. The principle on which it works is that of a projective image. The gnomon's vital part is therefore its shadow-casting tip, called by the Romans its "navel" (umbilicus). Because the earth is to the heavens as a point (Almagest I, 6), this tip can be taken as the center both of the rational and of the sensible horizon. Thereupon the upper half of the small circle drawn about it in the north-south plane serves as a miniature meridian of the heavens, while the part toward earth may become a sensible quadrant, measuring the elevation of the sun. Hence in the gnomon's tip coincide the center of the world and a kind of cosmic eye on whose retina (the dial) the sun's motions are projected in reverse (such projections through the center of a circle or sphere are now called "gnomonic"), while the gnomon itself, by blocking the sun's rays toward the earth, produces the shadow-index distinguished by us. The interest that the ways of such travelling markers exited is reflected in the [Aristotelian] <u>Problems</u> (611a14ff.): "Why is it that although the sun travels at a regular pace the increase and decline of shadows are not the same in an equal period of the time?" (i.e., why, in the figure of V., is RC less than CT?); also 912a34ff.-- It is a curious and significant thought that within the Copernican system the gnomon projects the earth's own motion onto itself.

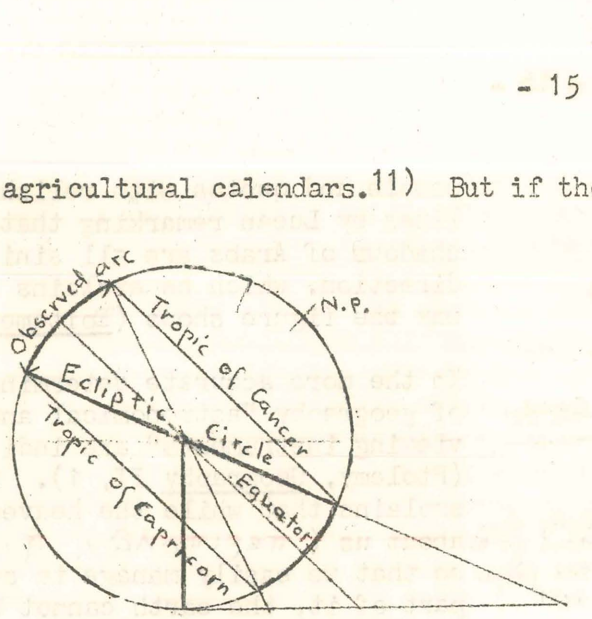




a. What motion of the sun does the noon shadow given off by the gnomon follow? Anaximander, when he furnished the Spartans with a sun clock, seems also to have installed a gnomon "to signal solstices and equinoxes" (Diogenes Laertius, II, 1 -- the Greek, more appropriately, says ισημερίας -- "equal days").[10] In our latitudes the shadows of the culminating sun get shorter toward summer, come to a standstill (solstice) and then grow longer again, being of the same length both at spring and autumn equinox. These seasonal changes in the shadows of the culminating sun are perfectly familiar even to primitive peoples, who use gnomons as

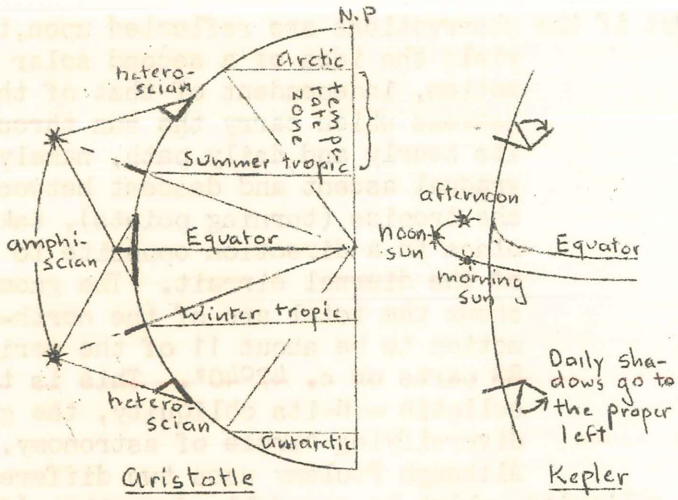


agricultural calendars.[11] But if the observations are reflected upon, they yield the idea of a second solar motion, independent of that of the heavens which carry the sun through its hourly and daily path, namely a gradual ascent and descent between the tropics (turning points), taking place in a direction opposite to that of the diurnal circuit. The gnomon shows the total arc of the north-south motion to be about 11 of the meridian's 83 parts or c. 47°40'. This is the ecliptic with its obliquity, the great diversifying circle of astronomy. Although Ptolemy uses two different instruments for obtaining the obliquity which he carefully describes (Almagest I, 12), the various men who were credited with this great discovery, Thales, Anaximander and Pythagoras in the sixth and Oenopides in the fifth century,[12] were all associated with the gnomon also (see II. below), and the gnomon was certainly instrumental to it.



b. But there is yet something more contained in the gnomon's shadows. If the dates at which they reach their various places are carefully kept, it is found that the time from spring equinox to summer solstice is $94\frac{1}{2}$ days, but from summer solstice to autumn equinox $92\frac{1}{2}$ days (Almagest, III, 4). This discovery was made by Meton and Euktemon in the latter part of the fifth century B.C.[13] Such an irregularity of the sun's motion would scarcely ever, I imagine, have been noticed, had it not been for the gnomon. For no eye can directly follow the path of "the heaven's glorious sun that will not be deep-search'd with saucy looks" -- as it may that of the planets. Thus the gnomon brings down the image of an anomaly which, once known, becomes a prime stimulant to the formulation of those rationalizing hypotheses which were the chief preoccupation of technical astronomy in antiquity (cf. Almagest III, 3).

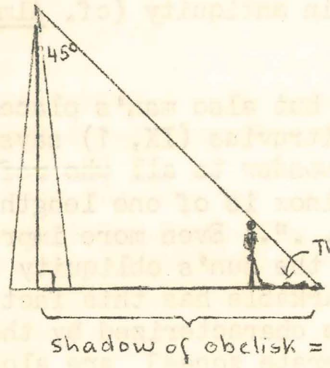
c. Not only the sun's motion in the heavens but also man's place under the sun is indicated by the gnomon's shadow. Vitruvius (IX, 1) says: "It is due to divine intelligence and a very great wonder to all who reflect upon it, that the shadow of a gnomon at the equinox is of one length in Athens, of another in Alexandria, of another in Rome . . .". Even more impressive as a direct index of the diversifying effect of the sun's obliquity are the varying directions of the noon shadows. So remarkable has this fact always seemed that the habitable zones of the earth are characterized by the directions of their shadows: "These [the temperate zones] are alone habitable while the regions beyond the tropics are not, for the shadow would not fall to the north, while it is known that places become uninhabitable before the shadows cease altogether or change to the south [i.e., south of the summer tropic], and the regions beneath the Bear [above the arctic circle] are uninhabitable because of the cold" (Aristotle, Meteorologica, 362b6ff.). So that according to Aristotle, men can live only in the temperate, "heteroscian" (ἑτερόσκιος - "other-shadowed") latitudes, while Ptolemy admits also the "amphiscian" parallels down to the equator (II, 6; see also Copernicus, II, 6); Kepler also was fascinated by this way of identifying



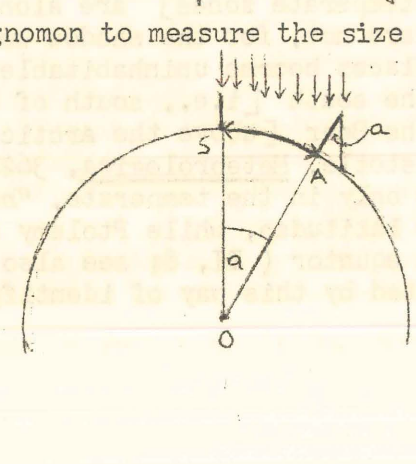
people and quotes with relish some lines by Lucan remarking that the shadows of Arabs are all sinister in direction, which he explains in the way the figure shows (Epitome, p. 205).

In the more accurate determinations of geography "astronomical and shadow-viewing instruments" are indispensable (Ptolemy, Geography II, 1). Ptolemy explains that while the heavens turn about us (περιπολῶν ἡμᾶς), so that we easily manage to see a great part of it, the earth cannot be travelled over and surveyed by one or even several men; one of the obvious consequences is that to locate themselves on earth then must, strange to say, take their bearings on the more accessible heavens. In fact, the grid of our terrestrial maps is still a projection of celestial circles. (The figures in V. will show graphically how the length of the equinoctial noon shadow immediately gives the latitude above the equator.) It follows that each parallel is associated with a ratio, that of the gnomon's height to the equinoctial noon shadow (Almagest II, 6). In order to calculate this ratio the Pythagorean theorem is necessary, and it is perfectly possible that the gnomon problem invited its solution -- as we will see in III., the gnomon is first and foremost a Pythagorean affair.

d. And finally the gnomon can, once geometric similarity is understood, be used to make large-scale measurements of size. Diogenes Laertius (I, 27) reports that Thales measured the pyramids by using himself as a gnomon, "having observed, when it [the human shadow] is equal in size to us" (παρατηρήσαντα ὅτε ἡμῖν ἰσομεγέθης).14) This story is a good one on several counts, first because the thought of Thales outdoing the Egyptians, who are supposed to have been the Greeks' masters in practical geometry, is a nice one, secondly because it seems so characteristic that a Greek, coming from a country where buildings were built to human scale,15) should apply this scale to these Egyptian monstrosities, and last, because it is a literal exemplification of man as measure. On a larger scale, Eratosthenes (third-second century B.C.) used the gnomon to measure the size of the earth itself. The method is as follows:



S is Syene, a city lying under the summer tropic;
A is Alexandria, under the same meridian but north of Syene,
O is the center of the earth.



Since the sun is very far away, its rays strike parallel to each other. At noon of the summer solstice they strike the gnomon at A at angle a, equal to 70°30' or one-fiftieth of four right angles. Arc SA is known to be equal to 5,000 stades, angle AOS to angle a. The whole circumference is therefore equal to 50 x 5,000 or 25,000 stades = 24,662 miles.

* * * * * * *

We have looked at the gnomon as producing reflections of daily time, of celestial motion and of terrestrial extent. Of these works, time-keeping is the first and greatest. At least Anaximander, the supposed discoverer[16] of the gnomon, must have ascribed to the measurement of time the greatest office possible. In his one extant fragment[17] he makes time the assessor of the penalty of destruction and the recompense of return which things owe to each other for the outrage they have committed by abandoning the common pool of the boundless to come into being. But if the world's changes occur "according to the command of time" (κατὰ τὴν τοῦ χρόνου τάξιν), then in keeping time men discover that order.

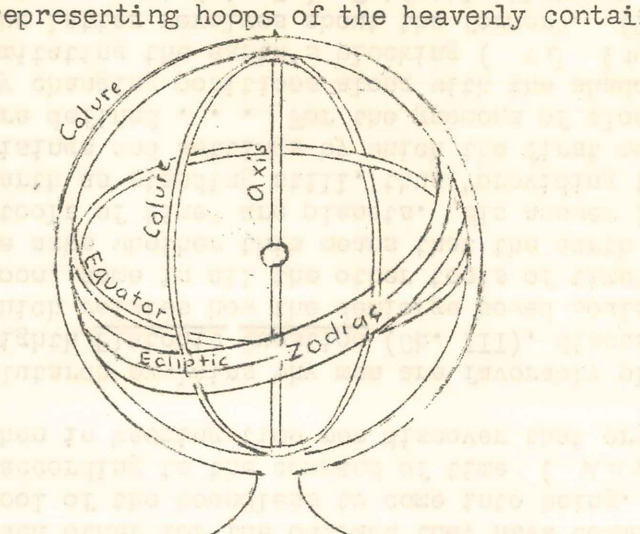
Plutarch explains why men are favorably placed for time-keeping. In the eighth <u>Platonic Question</u> (Ch. III), discussing the passage in the <u>Timaeus</u> which relates how the demiurge sowed souls, "some in the Earth, some in the moon, some in all the other tools of time" (ὄργανα χρόνου -42D), he asks whether this means that the earth too revolves, for all the other "tools of time" are planets. His answer is that it is best to think of the earth as standing still, thus "providing those bodies which revolve with risings and settings by which the first measures of time, night and day, are defined . . . For the gnomons of clocks also act as tools of time not by changing positions along with the shadows but by standing still, thereby imitating the earth's blocking (τὸ ἐπιπροσθοῦν) of the sun as the latter revolves about the former". (There follows the Empedocles passage quoted in I.) Evidently Plutarch interprets the earth's role as a tool of time to be complementary to that of those parts of the heavenly clock which "have the epithet 'wanderers'" and which were made for the enunciation (διορισμόν) and keeping (φυλακήν) of the numbers of time" (38C). For the earth, by "opposing itself to the sun", brings about the first sensible effect, the first reflection, the first <u>measured</u> measure of time. Thus because it is the home of shadows, those moving images of time -- just as time itself is a "moving image of the motionless"[18] (37D5) -- earth becomes a measure-taker, while the planets are measure-makers. Hence it is here on earth that images of the heavenly clockwork and instruments for fetching its motions down to earth are made.

II. <u>Images as Tools: The Geometrical Gnomon</u>

> "truste wel that alle conclusions that han be founde, or ellys possibly might be founde in so noble an instrument as is an Astrelabie ben unknowe parfitly to eny mortal man in this regioun, as I suppose."
>
> Chaucer, <u>A Treatise on the Astrolabe</u>

This same Anaximander, significantly the first writer of a book of the kind entitled in later references "On Nature" (Περὶ φύσεως), not only "invented the gnomon . . . and was the first to draw an outline of land and sea, but also to build a sphere" (εὗρεν δὲ καὶ γνώμονα πρῶτος ... καὶ γῆς καὶ θαλάσσης περίμετρον πρῶτος ἔγραψε, ἀλλὰ καὶ σφαῖραν κατεσκεύασε -- Diog. Laert., II, 1-2). These are the basic tools of the sciences of heaven and earth. They all share an apparently simple, but on reflection very questionable character -- they are in some way models. By a model I do not here mean something that may be talked about like this: "This is only [!] a model, the real atom contains nothing of the sort" (Eddington, The Nature of the Physical World, Ch. IX, speaking of Bohr's model of the hydrogen atom). These instrumental models are rather to be regarded as true images, which, before they are put to use as instruments of observation, can serve as teaching models, as "visual aids" for acquainting the student with the rational structure of the world.

All ancient astronomical instruments that I know of embody some such part of the universe, but the descendent of Anaximander's sphere, the astrolabe, whose construction Ptolemy describes in the Almagest (V, 1), is the most complete in this respect.[19] It coincides basically with the Pythagorean model presented in the Timaeus (and here it is of interest to note that Pythagoras may have been Anaximander's pupil[20]). It consists of fixed "colures" representing hoops of the heavenly container acting as reference meridians and the band of the "other", the zodiacal circle in whose center lies the sun's path, the ecliptic.[21] The astrolabe, however, while it omits the axis and the earth "wound around it" of the Timaean model, adds calibrations, sliding pointers and movable meridian and horizon circles. (Such index circles are called "gnomonic", Kepler, Epitome, p. 134). The whole is therefore a skeleton of the cosmos, that aspect of the visible world which it has in the reflective view, in "theory" (θεωρία , from θεωρός -- a "spectator").



Here arises an apparently simple-minded question. What need is there of observation if the best parts of the world are already before us? This question might elicit from the astronomer a most important fact. The model which he possesses has, it will turn out, a different origin and a different standing from the astronomical theory he is working on. He has taken it over from the physical philosophers, who themselves used only the simplest tools to obtain it -- particularly tools like the gnomon, which produce natural images. The artificial model they have constructed does indeed incorporate all that is knowable of the world -- its center which is coincident with the center of reflection, its roundness which is the only perfection possible to a moving body, and its orthagonal and oblique principles which act as sources of regularity and irregularity. However, this latter principle gives rise to appearances not construable in terms of a fixed and timeless structure, but only by complex moving diagrams. The astronomer's enterprise, therefore, consists in finding such diagrams as will "save the appearances", that is, as will show their irregularities to be merely apparent by producing a geometric diagram which will account for them. But since more than one such diagram can be produced, and since none are entirely satisfactory, they remain mere hypotheses, quite different in standing from the model, which simply represents what is and which may therefore serve as a guiding frame for fixing the aberrant appearances.

Yet even once the usefulness of the instrument is accepted, its very nature remains a problem. The model, as has just been emphasized, is not a model of the total world -- it seems to overlook some appearances, in fact most of them, while it contains all sorts of parts no one has ever seen. It is a model only of the world's rational structure, of its geometry — but it is also a sensible artifact, made of "circles accurately turned with four perpendicular surfaces each", and existing in time. Is it not, then, a copy of some pure model, a timeless paradigm which has been espied beyond the appearances? Is it not rather the embodiment of the model for the world than an image of the world?

The Timaeus presents a fantasy answering the perplexities of a model-making science such as that of the Pythagoreans. It shows that such a science cuts across, overlooks the world as a visible, appearing world and by going always to the geometry behind it, avoids the only road which can lead to being, that which confronts the contradictions of appearances as appearances. It shows further that its principles of account giving lead to a hopeless confounding of "before" and "after". For wherever the human account is to be embodied in a mathematical, i.e., rational image, the thing to be accounted for must itself be an image incorporating a rational paradigm. But this paradigm arouses the suspicion of having been construed into or placed under the world by the human inquirer himself -- he may have taken his account of the world for its being. Hence arises the following circle, in which every image turns into a model and vice versa:

noetic		natural		man-made
pure model	⟶	sensible image	—	rational model
↑		↓		↑
pure image	⟵	sensible model	⟶	rational image

The Timaeus grandly stops this circle by decree: ". . . to which of the two models (παραδείγματα) did the builder make the copy alike, to that which is always the same with itself and alike, or to that which has become? Certainly, if the world is a masterpiece (καλός), and the artisan good, it is clear that he looked to the eternal (29 A) . . . And if this is so, there is, again, every necessity for the world to be an image of something. The great thing is for everything to begin according to its natural beginning. (ἀρχήν). Thus concerning the image and its model, we must bring out distinctly that accounts are like in kind with those very things which they expound. Accounts of what is stable and strong and belongs to thought will clearly be stable and incontrovertable . . . while accounts of that which images that model, since it is itself an image (εἰκόνος), will be but likely-likenesses (εἰκότας), being like the former only by similitude" (29 B-C). What we must not forget is that Timaeus has introduced the very account which is to follow as a "likely myth" (εἰκὼς μῦθος - 29 D 2), so that it is, by the criterion of this passage, an account of that world which is made in the <u>image</u> of a <u>sensible model</u> -- and the <u>Timaeus</u> itself turns out to be the likeness of a cosmologist's quandary.

One more reflection on the principles underlying the use of instrumental models: as instruments of observation these ancient tools are of course deeply and significantly different from those which most captured people's imagination at the beginning of modern times[22], namely the telescope and the microscope. For the latter are extensions of sight beyond and below the range given by nature and take the observer <u>to</u> objects beyond his scope -- for instance, in his <u>Starry Message</u>, which announces discoveries made through the "spy-glass", all tending to assimilate the nature of the heavens to that of the sublunar regions, Galileo says: "It is a very beautiful thing, and most gratifying to the sight, to behold the body of the moon, distant from us almost sixty earthly radii, as if it were no farther away than two such measures"[23]; the very name of the treatise, <u>Siderius</u> <u>Nuncius</u>, which has the alternative interpretation, countenanced by Galileo, of <u>Starry</u> <u>Messenger</u>, implies that the observer has been to the stars. The astrolabe, on the other hand, fetches the star to the observer, as the name ἀστρολάβον ὄργανον -- the "star-fetching tool" (Ptolemy, <u>Geogr</u>. II, 1) implies. Thus, for instance, in giving instructions on sighting with the instrument, Ptolemy says: "the star, as if stuck to both surfaces, is sighted on the opposite side. . ." (V, 1). The rationale of this might be said to be that the star to be sighted actually becomes part of the astrolabe, being itself, as an appearance, a mere fore-sight, serving to line up the instrument. What is really observed is not so much an appearance, as a reading on the instrument, the numerical stuff of a geometric diagram (at least in theoretical astronomy). The readings will be accurate because, except in the case of the moon, the sensible and the rational horizon coincide and thereby make the center of the sphere a portable little epi-center of the world, which the observer may at once be at and look at. Kepler, who was deeply interested in the "sphaera materialis" (see Note 21), particularly in the reason why it could be used even in a heliocentric system, explains its character in just this way. The material sphere, he says, is an "effigy of the world, such as our sight imagines for itself; it is made in such a way that the theory (ratio) of the prime movement and of movements dependent on it can be demonstrated to the eye as with an instrument", and the earth is the "home of eyes" (domicilium oculorum -- <u>Epitome</u>, pp. 101-102).

It is my guess that the discoverer of the gnomon conceived the very earth itself in the image on an instrument. Anaximander held, and was probably the first to hold, that the earth was in equipoise at the center of the world. Scholars are therefore somewhat embarrassed by his notion that the earth is cylindrical, looking something like a column drum of which we inhabit the upper flat surface[24] -- it seems so incongruous with his geocentric spherical model. But could it not be that this cylinder is conceived precisely in the image of a gnomonic column, an "analemma" (see V. below), the pedestal of a gnomon? Could it not be that Anaximander saw the earth's shape in the image of that very instrument of observation which had shown him something of the sun's circuit?

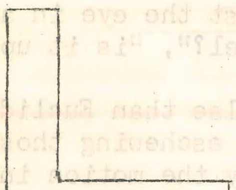
* * * * * * * *

To say, then, that instruments are rational copies of the world must first of all mean that they incorporate geometric principles. But it means above all that they are constructed. Now the model-making activity is itself in want of tools; I mean tools whose function lies inbetween the "theoric" tools discussed above and the mechanical tools which literally do work (ἔργα) and therefore give their name to the whole class (ὄργανα). These tools must be a kind of embodied geometry -- we might call them "demiurgic tools". Of course the Timaean demiurge himself, as a noetic artisan, has no need of them, since their function is to guide the body and to aid the senses: they "rule" the human hand so that it is not distracted by the fits and starts of the body, and they assist the eye in answering the questions "is it straight?", "is it on the level?", "is it upright?".

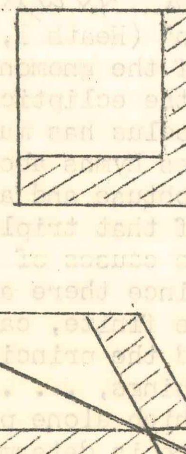
The geometry incarnate in such tools is nothing else than Euclidean ruler-and-compass geometry, a. because it is timeless, eschewing those shapes whose production is mechanical, i.e., which are produced by the motion in time of interconnected rigid parts (for instance, it is reported that "Plato himself blamed those . . . who reduced the doubling of the cube to a construction by means of instruments and machines" -- ὀργανικὰς καὶ μηχανικὰς κατασκευάς - Diels I, 429, 9; this is of course precisely the kind of construction so fascinating to the seventeenth century, whose cultivation issued in Newton's theory of fluxions; cf., for instance, Descartes Geometry, Bk. II); b. because it is "preferential", always working from the special and more "perfect" elements, such as the right line, the right angle and the circle.

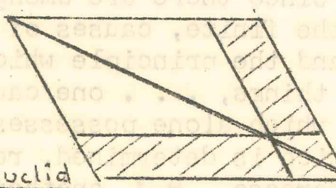
Now Proclus comments on Euclid I, 12, the proposition on the construction of the perpendicular, and in particular, on its name — ἡ κάθετος γραμμή, "the line let fall (as a plumb line)", as follows: "This problem was first investigated by Oenopides [5 cent. B.C.] , who thought it useful for astronomy; he, however, calls the perpendicular, in the archaic manner, [a line drawn] gnomon-wise (κατὰ γνώμονα), because the gnomon is at right angles to the horizon" (Heath I, p. 271). This Oenopides, who studied the geometric construction of the gnomon, is the same man to whom the discovery of the obliquity of the ecliptic is most authentically attributed (see I.a., above). But Proclus has much more to say about the perpendicular; in fact he almost writes hymns about it. Commenting on Euclid's Definitions X-XII of right, obtuse and acute angles he says: "But the Pythagoreans take the solution of that triple distribution back to principles and do not hesitate to define the causes of those differences among rectilinear angles. In effect, since there are among the principles on the one hand those which reside in the finite, causes of limit, of sameness, of equality . . . and there is on the other hand the principle which resides in the infinite . . . giving diversity to things, . . . one cause, arising from the finite, produced the right angle which alone possesses equality and similarity with every right angle, which is determined, remains always the same and admits neither increase nor decrease. But another cause, arising from the infinite, which is inferior and of double nature, produces two angles . . . which have an unlimited tendency to more or less . . . . This is apparently also the reason why the Pythagoreans refer the angles back up to the pure causes of the divine arrangements . . ., for that which is right . . . belongs to the gods." (Proclus, Commentaires sur le Premier Livre des Eléments d'Euclide, ed. Ver Eecke, p. 120.) And again: "The right angle is in fact the symbol of unyielding power which is united to equality, limit, or boundary. It is for this reason that Timaeus calls the circle of the other, which possesses for the divine soul the ratios of sensible things, also right; for in our souls the circle is broken into fragments of every sort and undergoes the various deformations which result from generation, while in whole souls it aintains itself intact and stable in the presence of sensible things" (p. 249).

It is clear that the demiurge of the sensible model must work with a carpenter's square, in fact, with the prototype of all carpenter's squares. This tool is the very symbol of the builder's craft, providing by art those prime shapes which our sublunar habitat cannot produce by nature. The Greek word for the carpenter's square was gnomon . Theognes, in the mid-sixth century B.C., mentions it along with two other such tools; he says that the envoy sent to Delphi to consult the oracle should be "straighter than the τόρνος [compass] , the στάθμη [plumb-line] and the γνώμων [carpenter's square]" (Heath I, p. 371). It is, however, not known which application, that to the shadow-casting upright or to the carpenter's square, was the earlier; the Theognis passage above appears to be among the earliest uses of the word in a text. I would guess that the workman's gnomon came first -- the builder wants to know that posts are perpendicular and corners square and it is the plumb-line and the "gnomon" that can tell him.

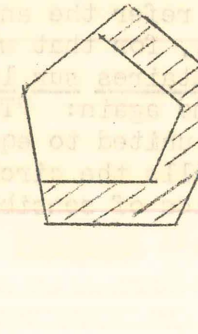


However that may be, the name of the tool was next transferred to the corresponding geometrical shape. For instance, Aristotle, when he wants to illustrate that alteration is different from growth, says that "the square, when the gnomon is placed around it, grows but does not alter" (Categories 15a31).[25] Euclid seems to have been the first to extend the definition to all parallelograms (I, Def. 3): ". . . let any one whatsoever of the parallelograms about its diameter with the two complements be called a gnomon". And last, Heron of Alexandria (Definitions, 58) defines the geometrical gnomon as any figure which when added to any figure whatever makes the whole new figure similar to that to which it is added; and he goes even further: "In general, a gnomon is every addition which makes the whole well-proportioned (εὔρυθμον) or a figure (σχῆμα)".





The scholiast, in commenting on Euclid's definition, explains the existence and the name of the gnomon thus: "It is to be noted that the gnomon was discovered by the geometers for the sake of brevity, but the name arises from an incidental property. For from it the whole is known



(γνωρίζεται) -- either that of the whole area or of the remainder, when the gnomon is either placed about it (περιτιθῆται) or taken away. And in sundials it only serves to make known the actual time" (Euclid, ed. Heiberg, V. pp. 225-226). Presumably he means that from the geometric gnomon one may calculate the number of an area, while from the upright one may tell the time. Does this do justice to the origin, the name and the functions of the "knower"?

III. <u>Tools as "Knowers": the Arithmological Gnomon</u>

Gnomon: know-man

Florio

The Elizabethan lexicographer's jeu d'esprit which is quoted at the head of this section will prove to be more apt than the scholiast's exegesis, for the gnomon's name is no accident. (The once popular use of "gnomon" for "nose"[26]) evidently alludes to a similarly pregnant homonym -- as everyone knows, it is often the nose that knows.) But how can the gnomon, or any tool, be said to know?

a. Obviously the gnomon as throwing off a shadow pointer might be said to know what it points out, while as a carpenter's square it knows the "right" angle in the same way that a plumb-line knows how to find the perpendicular and a ruler can command the hand. So also rules of right conduct are called the "plumb-line of life" (σταθμὴ βίου) or the "gnomon of morals" (γνώμων ἠθῶν - Diog. Laert. IX, 12).[27])

b. There is in Greek, as in English, a use of the word in which it is largely synonymous with the more popular word "canon". Κανών means simply a rod (cf. "cane") used for keeping things in shape; for instance, the staves bracing a shield are called κανόνες in the Illiad (XIII, 407). Later it comes to mean a guide line or straight edge such as is used by masons. Its assumption into the theoretical realm is best exemplified in the monochord, a calibrated sounding board displaying the geometrical realization of a system of intervals worked out by compounding the simplest number ratios, namely those which as the Pythagoreans had discovered, sounded as consonances when expressed in string lengths. The monochord could be used theoretically for experimenting and practically for tuning other instruments. The theory of proportions according to which the canon was "cut" (<u>Sectio Canonis</u>), or any such system of proportions, came to be itself called a "canon", as for instance, the Canon of Polycleitus, which was both the title of that sculptor's book on proportions and the name of the statue which embodied them: "Chrysippus holds beauty to consist in the proportion not of the elements . . . but of all the parts to each other, as they are set forth in the Canon of Polycleitus."[28]) Thus the word "canon" came to designate the <u>codified didactic exposition of any theory</u>, much as we speak of "a theory" or a "body of knowledge" as if they were objects. For instance, in the corpus of Democritus' works a logical canon (περὶ λογικῶν κανῶν - Diels II, 91, 10) is listed, just as later Aristotle's logical writings were called the <u>Organon</u>; "The Tool". Therein lies, of course, the whole secret of this usage -- theory is here regarded as an instrument which, when mastered, makes a man expert. This is the use which Vitruvius has in mind when, reciting the names of the great Greek theoreticians, he says that "they left to posterity many things concerning machines and observational instruments (organicas et gnomicas)".[29] And this is the use most agreeable to the moderns -- in the initial book of modern science, called the Novum Organum, theory is explicitly treated as a mental tool: ". . . It is by instruments and helps that the work is done, which are as much wanted for the understanding as for the hand. And as the instruments of the hand either give motion or guide it, so the instruments of the mind supply either suggestions for the understanding or cautions" (Bk. I, Aphorism II -- such books of aphorisms are precisely what the ancients called a "gnomology").

c. The Tables containing the records of readings are usually called canons, for instance the various catalogues and tables in the Almagest, whence instruments of observation in general, insofar as they yield lists of readings, are called gnomonic or canonic. In this sense a canon is a handbook of information.

In all these three applications -as index, as theory, or as information, the gnomon might be said to act as a tool of knowledge. But how can there be such tools and what is the manner of their action?

* * * * * * *

The possibility of the thing depends, it seems to me, on the possibility of conceiving of reason itself as an instrument or of thought as being at work. The notion that our bodies are tools, either our own or someone else's, seems natural enough since we can indeed feel ourselves "pushing ourselves" or being "handled". And since the "handiest", the most skillful part of our body, is the hand, inasmuch as it can itself wield tools and also because it is our hand that feeds us, all men (except perhaps young men) do think of it easily as the most instrumental part of their bodies. But the body possesses also other tools whose very name means "instrument", namely the "organs" (ὄργανα) of sense. These seem, usually, to work effortlessly and to produce nothing new but only to make us aware -- as we suppose in practice, whatever our theory may be -- of what already is outside us. Among them the eye seems to bring us the most variegated awareness and from farthest off. (All these considerations can be found in Aristotle -- proof that they are natural; see Note 30).

But anyone who has ever thought or "used his mind", particularly if -though the effort was great- he has found it blunted and undiscerning, finds it natural to think of it too as an "organ". Usually one of the two chief bodily organs, the eye or the hand, is chosen as the model in the image of which the reason or thought is conceived. The Stoics are the great proponents of the latter metaphor[31] which is always taken more or less literally by those who choose it. Consequently all things tend to be reduced to the tangibly material, so that even sight may become a species of touch,[32] and knowing itself is understood under the type of "grasping" -- Zeno, the founder of the Stoic school, is said to have been in the habit of demonstrating the meaning of "knowing" by showing his open hand and saying, " 'A visual appearance is of this sort;' next he closed his fingers a little and said, 'assent is of this sort', and then he pressed his fingers closely together and said that that was comprehension -- and from this analogy he gave the name, which it had not had before, to the thing itself: catalepsis [κατάληψις - concept] . . ." (Cicero, Academica II, 47).

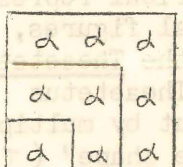
Now insofar as gnomons are said to make figures hold their shape (σχῆμα) by acting as confining straight edges (κανόνες) and directrices (εὐθυντῆτες) -- this is in fact a second account of the name given by the Euclid scholiast (p. 226) -- they may indeed be said to "comprehend" or "grasp" their figure. And by reason of this grasping function they may, in turn, be used as the tools by which reason itself can grasp figures. Some such commonplace of exegesis has presented itself to several of those who have been struck by the name.[33] And yet it misses the point -- for "gnomon" does not mean "instrument for knowing" but "knower", and first and foremost the human knower;[34] (so, for instance, certain Athenian officials, whose duty it was to inspect the countryside to see if the law against cutting down olive trees had been violated, were called γνώμονες, "men who are strict because they know" - οἱ ἀκριβεῖς ἀπὸ τοῦ γνῶναι -- Etymologicum Magnum, p. 236).

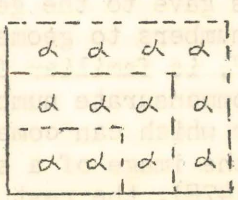
* * * * * * *

The Pythagoreans, however, understood their gnomon, the numerical gnomon, properly -- according to its name. In a passage of the Physics (203a10-16) Aristotle characterizes it, albeit incidentally:

"Again, they [the Pythagoreans] think that the limitless (τὸ ἄπειρον) is the even (τὸ ἄρτιον). For it, even when it has been enclosed (ἐναπολαμβανόμενον) and limited by the odd (ὑπὸ τοῦ περιττοῦ περαινόμενον), still brings to the things that are their indefiniteness (τὴν ἀπειρίαν). A sign of this is what happens in numbers. For when gnomons are placed around (περιτιθεμένων) the one, the form (τὸ εἶδος) is one, and without the one (καὶ χωρίς)[35], it becomes always other (ἄλλο ἀεὶ γίγνεσθαι)."

The main point of the passage is that the principles of "limit" and "limitless" reveal themselves in an arithmetical form for the Pythagoreans. For the successive odd numbers can be added gnomon-wise to the original single unit, that is to say, they can be placed around it so as to preserve the square. On the other hand, if successive even numbers are placed about the original two, this cannot be done gnomon-wise (see Note 35), and the proportions of the resulting rectangles change with every addition (although they approach squareness).

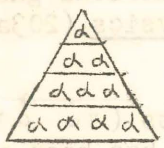




Three particular observations must be made about the passage:

a. Aristotle does not explicitly say that the gnomon meant is the square gnomon, but this is sufficiently clear 1. from the analogy with the geometrical gnomon which _is_ primarily square (see II. above); 2. from Simplicius' commentary, who says: "The Pythagoreans call the odd numbers gnomons because when they are placed around square [numbers] they preserve the same shape, just as do the gnomons in geometry" (_Comm._ _in_ _Arist._ _Graec._ IX, p. 457, 1-3) and from Theon's presentation, who generates the square number no less than four times; 3. and most important, from the "table of opposites", the list of contrary principles distinguished by the Pythagoreans, which ends with "square" (τετράγωνον) and the "oblong" (ἑτερόμηκες - _Metaphysics_ 986a25).

b. Although the square gnomon is clearly the first and most important (see c.), the arithmological, like the geometrical gnomon was later given an extended meaning. For it is possible to reproduce "triangular" numbers as well, namely by the addition of rows of numbers increasing by one, a reproductive addition which is properly speaking a canon, a straight-edge. The triangular gnomon may even be said to have a certain priority (Nicomachus, II, 7; Theon. p. 37, 7ff.) because it is the image of the series of natural numbers, because the triangular number is the first possible kind of plane number, and because the triangle of four rows, consisting of the beginning one and the first three gnomons, constitutes the "tetractys", the perfect ten (which contains all the numbers whose ratios are the consonances[36]), which is the very "nature of number", and by which the Pythagoreans swore[37]). Analogously, pentagonal,[38] hexagonal and all the succeeding numbers can be generated by gnomons, provided only that the beginning is the unit; if it is two, nothing can be done to preserve the shape.



c. Aristotle speaks of the _eidos_, the figure or form of numbers, and we have mentioned triangular, tetragonal and pentagonal numbers. What are these shaped or "figurate" numbers? "It is the way (ἔθος) of the Pythagoreans to draw shapes (σχηματογραφεῖν)" says Simplicius (p. 457, 16). _Schematographia_ was the name later writers gave to the geometrical representation of numbers. This assimilation of numbers to geometrical figures, still evident in our word "square numbers", is familiar from the _Theaetetus._ In order to find one appellation for all commensurate numbers Theaetetus "called square and equilateral that number which can come about by multiplying a number by its equal, considering it in the image of a square shape" (τὸ σχῆμα ἀπεικάσαντες τετράγωνον - 147E), the rest were to be called oblong (προμήκες σχῆμα)[39], and the sides or roots which could square these oblongs are incommensurable with those belonging to the square numbers as lengths but commensurable as areas, i.e., the root of the oblong with the side numbers 2 and 1, namely $\sqrt{2}$, is incommensurable with other numbers as a side - but the area upon it, namely 2, is commensurable with them. The crux of this correspondence is that it points to a class of geometric magnitudes for which there are no numerical counterparts, namely the sides which square the oblongs, the irrationals, which can be made again commensurable only by the "power" of the square.

The Pythagoreans, however, understood their figurate numbers -- although they ranged them under the same two heads of square and oblong -- in a way basically different from that of Theaetetus. Pythagorean numbers are first of all <u>shapely</u>. This appears in the fact that the extended class of the square includes in principle all regular geometric figures, while the Thaetetan scheme, relying primarily on the <u>areal</u> nature of figures, is confined to rectangles of varying shape. The square number is the paramount number-shape only because it is always "gnomonized" by an odd number,[40] and is therefore the very "sign that odd number is figure-producing (εἰδοποιόν) and limitative (περατωτικόν)" -- Simplicius, p. 456, 16. For in the square the leading principle of the Pythagoreans, limit, assumes the character of the odd[41], while the odd, in turn, appears in the square both as its principle -- as the beginning one, and again in the consequences -- as the series of odd gnomonic numbers. Where it is present, a number retains its <u>eidos</u> through all changes of size[42]; where the unlimited is at work, rectangles of infinitely many proportions arise.[43]

But in what sense can a number be said to have an <u>eidos</u>, a form or figure? It should, first of all, be emphasized that these numbers are intended to be truly prior to their figure: "from numbers came points; from points, lines; from lines, plane figures; from plane figures, solid figures; from solid figures, sensible bodies."[44] It is to this priority of being that the Euclid scholiast refers when he says that the geometric gnomon has its name "by analogy" (ἀπὸ μεταφορᾶς), although he has already, just as correctly observed that it was discovered by the geometers (Heiberg, p. 227, 3).

The figure arises when the units composing the numbers are disposed non-linearly as points in a plane. This means that there must be at least three units in the number and so "the triangle . . . is the most original and elementary form of plane number" (Nicom. II, 7). One of these units is always regarded as the beginning of the number and is "potentially a triangle" (Nicom. II, 8) or a square,[45] etc.; all the following numbers, "which generate (ἀπογεννῶντες)[46] the triangular, square and polygonal numbers", etc. are the gnomons. The Pythagorean mode seems convincing enough if one considers that all geometrical figures seem to include in their determination some reference to number -- as is shown in the name "tri-angle" or "tetra-gon" -- yet the language of the commentators always tends to give figure the priority. This happens in the very expression "tetragonal number", which, if the intended priority were to be observed, should rather be "triadic figure". A particularly blatant case is that of Philolaus (see below) who on the one hand speaks of the decad as the "source and leader" of all things, and on the other, calls geometry the "source and metropolis" of all of mathematics. (Diels, I 399, 26; 411, 11). Thus the besetting Pythagorean difficulty concerning priorities (see II. above) turns up once more in the thesis of the generation of figure by number.

* * * * * * * *

It is the gnomon's nature, then, to preserve the <u>eidos</u>. What kind of a thing is it that can have such an office? The fragments of Philolaus the Pythagorean,[47] the man with whom Cebes had studied (<u>Phaedo</u>, 61E), can throw light on this question. The chief of these runs as follows (Diels, I. 411, 14ff.)

"The nature (φύσις,) of number is to make things <u>knowable</u> (γνωμικά) and to guide and to teach anyone who is at a loss (ἀπορουμένῳ) about anything and does not know something. For not one among things would be clear to anyone, either by itself (ποθ' αὐτό) or as it is towards others (πρὸς ἄλλο), if number were not, or its being. But as it is, in fitting (ἁρμόζων) them to accord with the soul (καττὰν ψυχάν) it makes all of them <u>knowable</u> (γνωστά) to sense - awareness (αἰσθήσει) and conversant (ποτάγορα) with one another <u>according to the nature of a 'knower'</u> (κατὰ γνώμονος φύσιν),[48] making them into bodies, and distinguishing (σχίζων) the ratios (λόγους) of things both unlimited and limiting -- each apart".

Interpretation: number makes things knowable by its very nature. For entering into those things which have the form of first principles, the unlimited and the limiting things, it makes them bodies so that they become perceptible to sense, and it also gives them their distinctive ratios so that they can stand in numerical relation with each other. Thus it disposes things according to the nature of a knower, for the knower has a soul composed of ratios.

The passage is full of allusions to Pythagorean doctrines: 1. limit and the limitless are the principles of things and are to be regarded as their stuff (<u>Metaph.</u> 987a13); 2. therefore things are made of numbers, the immediate consequences of these principles (987a19); 3. hence numbers constitute sensible bodies (1080b16). 4. The soul is a harmony, i.e., a compound of number ratios (<u>De Anima</u> 407b27; <u>Timaeus</u> 35ff.); 5. like is known by like (<u>Timaeus</u> 37ff.). The other fragments all propose one or the other of these doctrines.

Concerning the last of these the following passage from Sextus Empiricus (<u>Against the Mathematicians</u>, VII, 92) is of interest: "The Pythagoreans say that the logos distinguishes [is a κριτήριον] , but not generally, since the logos that arises in mathematics is, as Philolaus says, contemplative (θεωρητικόν) and, seeing the nature of the whole, has a certain kinship with her, if it natural for like to be known by like."

Logos, it appears, had for Philolaus a double aspect -- it distinguishes and it relates. This is precisely what the fragment plays on. For number, in distinguishing the ratios (logoi) of things, makes it possible for the soul to discern them and to give accounts (logoi) of them. This is why the Pythagoreans "who simply join number with the soul" call it the "discerning tool of the world-working god" (κριτικὸν κοσμουργοῦ θεοῦ ὄργανον - Diels I 109, 29). But by making things into bodies it makes them sensible and therefore available to sense and to the soul. And since the soul is a system of number ratios, so must the bodies be "conversant" (προσαγορά) with or related to each other, i.e., they must have ratios (logoi) among each other. Then they will form a system of ratios or a harmony and so they will become known to the soul as like to like.

It follows: as number makes things <u>to accord with</u> the soul, so it makes things <u>according to</u> the gnomon, which Philolaus, quick to see a likeness, interprets as the soul's representative in the <u>visible</u> world. For as the soul gives an account of nature so does the gnomon give proportion (ἀναλογία) to a shape; as the soul is the reason for the harmony of nature; so is the gnomon the source of similarity in figures; as the soul takes to its like in world, so does the gnomon fit its figure; as the soul embraces and binds body, so the gnomon delimits and clasps number;-and so both the soul and the gnomon are the guardians of εἴδη , the looks of the world.[49)]

continued
Overleaf

IV. THE SUNDIAL AND THE ORIGIN OF CONIC SECTIONS

O. Neugebauer (*Proceedings of the American Philosophical Society*, XCII, 1948, pp. 136-138) shows that the peculiarly restricted way of obtaining sections in the pre-Apollonian theory of conics (attributed to Menaechmus, c. 350 B.C., a pupil of Eudoxus) is accounted for if the study of conics was first suggested through observation of the shifting shadows cast in the course of a day by the gnomon of a certain type of sundial.

In the early theory (1) only right circular cones, i.e., cones with axes perpendicular to their (circular) bases are used, and (2) the cutting plane is always perpendicular to a generating line. The different sections are then obtained by varying the vertical angle (see opposite page; cf. *Collegian*, October 1963, pp. 36ff.). Both conditions are present in the geometry of the -- mathematically very convenient -- sundial set out on the opposite page:

- G: center of rational horizon but also shadow-casting tip of gnomon GS_n;
- GSO: axial triangle of a right cone with vertex G and the circle of the sun's daily path as base;
- $\measuredangle\delta$: angle of the sun's declination, i.e., latitude above the equator;
- $\measuredangle\alpha$: verticle angles of opposite cones; since the maximum for $\measuredangle\delta = 23^\circ 51'$, $\measuredangle\alpha = 180^\circ - 2\delta$ is always obtuse;
- GS_n: the gnomon, a generating line of the opposite cone;
- DIAL: a cutting plane perpendicular to the gnomon;
- $S_f S_n S_a$: a conic section traced out by the tip of the gnomon's shadow as the sun progresses; the gnomon's base itself forms the vertex.

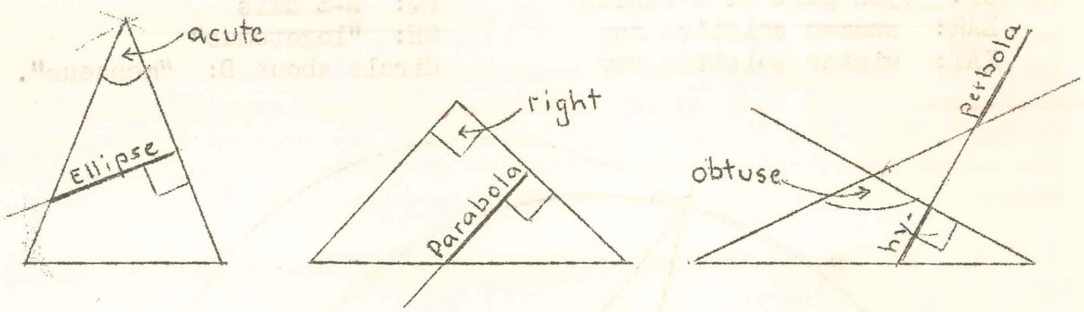
Since the cone in this construction is always obtuse, the section will always be a hyperbola, but the other sections are easily interpolated.

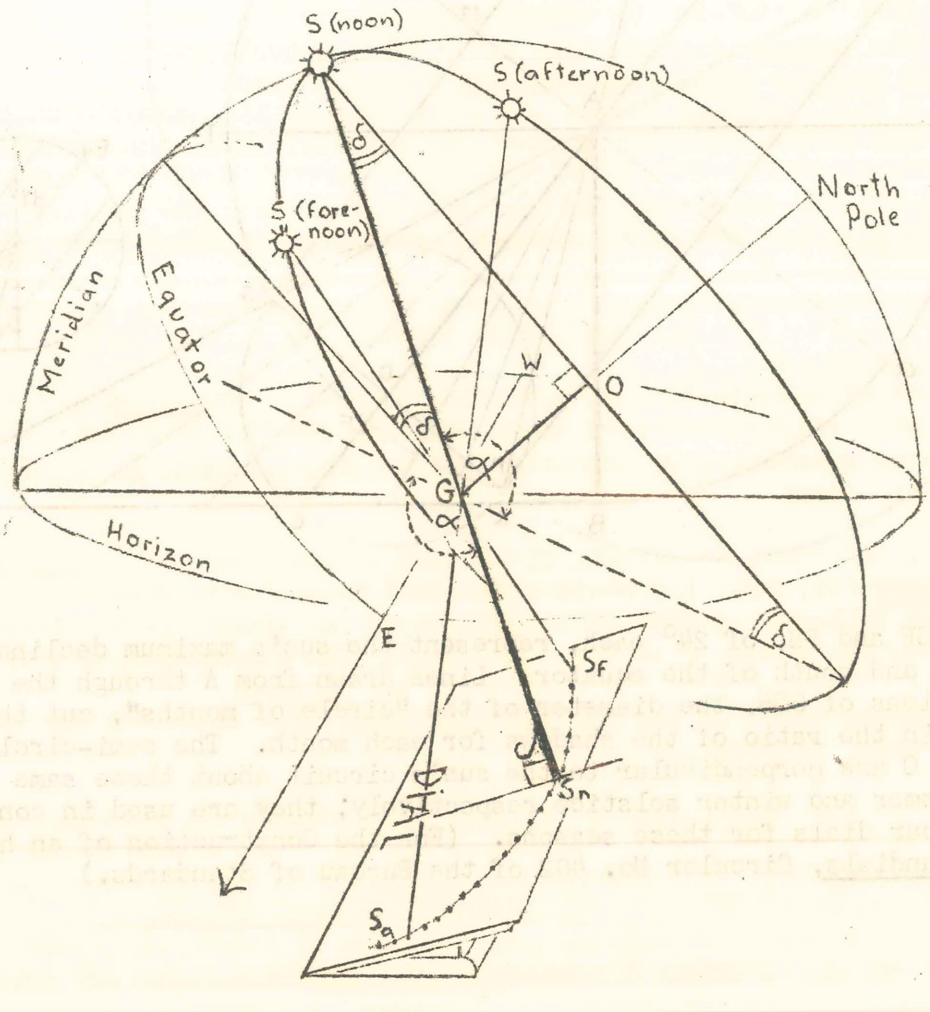
V. VITRUVIUS' SUNDIAL

A mathematical construction of a sundial is called an analemma, literally, a "pedestal". (Ptolemy wrote a whole treatise "On the Analemma", his construction is summarized by Neugebauer, *The Exact Sciences in Antiquity*, sec. 87 with note.)

Vitruvius, the Roman architect, gives such an analemma in his *Ten Books on Architecture* (IX, 7). The basic (observed) ratio, from which the rest of the figure follows, is that of the height of the gnomon to the length of the equinoctial shadow (5:4 for Annapolis). The numerical latitude can then be calculated (39°N; for the method see *Almagest* II, 5). Vitruvius' dial is marked only for the sun's annual path. (For construction see opposite page, overleaf.)

Early Conics
and
Sundial

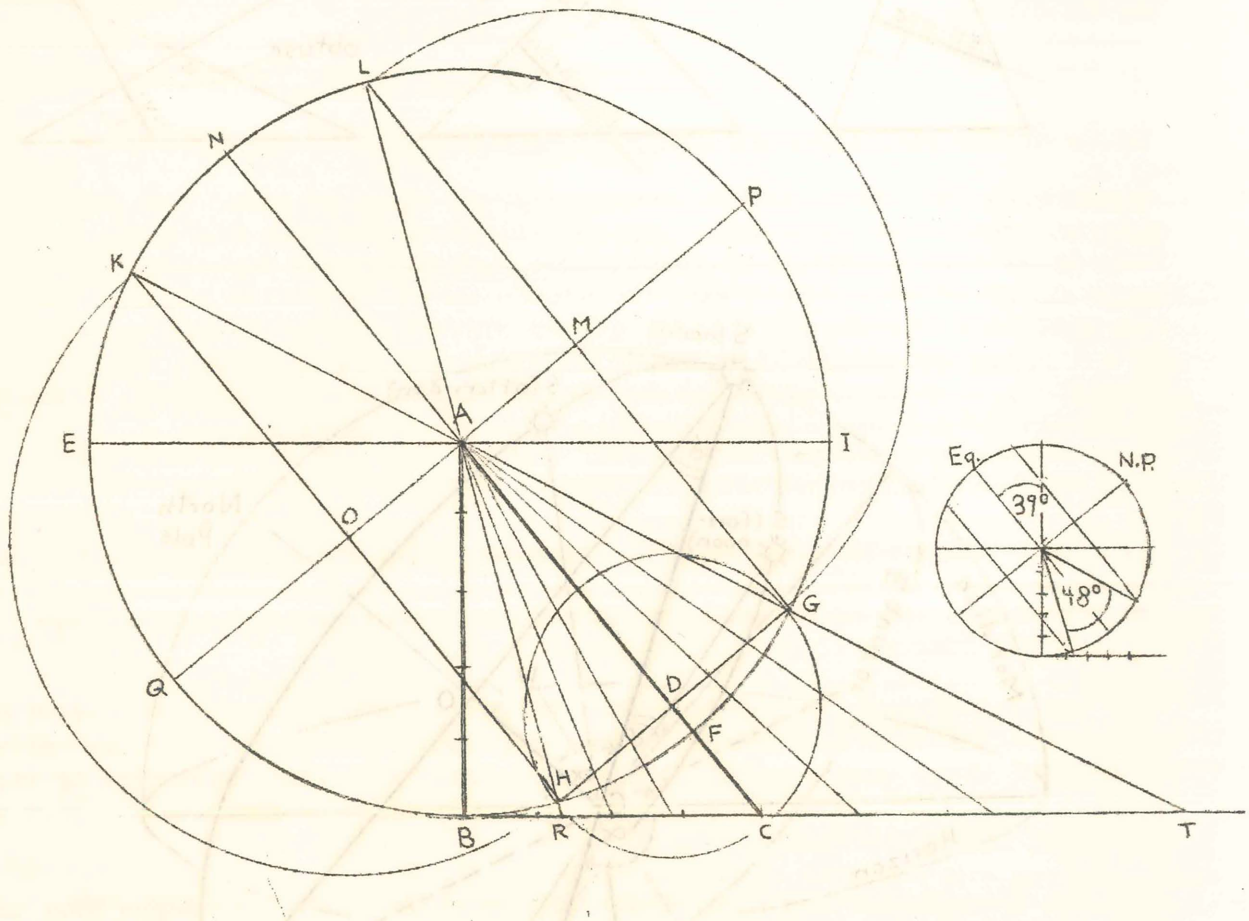




Vitruvius' Sundial

The parts of the analemma, in the order of their construction, are:

AB: gnomon
EBIL: meridian in N-S plane
BC: equinoctial shadow
NAC: equinoctial noon ray
GF: 15th part of meridian
LAR: summer solstice ray
KAT: winter solstice ray
FN: equator
GL: summer tropic
HK: winter tropic
N,K,L: culminating sun
PQ: N-S axis
GH: "logotomus"
Circle about D: "menaeus".



Arcs GF and FH, of 24° each, represent the sun's maximum declination north and south of the equator. Lines drawn from A through the even divisions of GDH, the diameter of the "circle of months", cut the base line in the ratio of the shadows for each month. The semi-circles about M and O are perpendicular to the sun's circuit about those same centers at summer and winter solstice respectively; they are used in constructing the hour dials for these seasons. (For the Construction of an hour dial see Sundials, Circular No. 402 of the Bureau of Standards.)

VI. Notes

General references for the gnomon: *The Thirteen Books of Euclid's Elements*, ed. T. L. Heath, Dover, 1925; W. K. C. Guthrie, *A History of Greek Philosophy*, Cambridge, 1962; G. S. Kirk and J. E. Raven, *The Presocratic Philosophers*, Cambridge, 1963; see indices.

1. *The Notebooks of Leonardo da Vinci*, ed. MacCurdy, pp. 947-985; also R. Arnheim, *Art and Visual Perception*, pp. 256ff.

2. the contrary of which is the transparent (τὸ διαφανές), according to Aristotle (*De Anima*, 418b1ff) not itself a body but that *through which* bodies appear.

3. See J. D. Beazley, *The Development of Attic Black-figure*, Berkeley, 1951.

4. H. Diels, *Fragmente der Vorsokratiker*, 7th ed., I, pp. 331, 20.

5. This is the theme of Chamisso's wonderful story of Peter Schlemihl who sold his shadow to the devil. The general principle that the image makes the man is held in universal respect in our world, where a person is not considered to be "identified", which means to say, to be the same with himself, until he can produce a certain paper with his own picture on it. I have noticed a more innocent version of the same thing among the modern Greeks, who will sometimes, at a first meeting, show you photographs of themselves, intending to prove thereby that they are people of some consequence.

6. See the picture in Ptolemy, *Canon of Kings*, ed. Halma, p. xliv.

7. The texts on this matter offer certain difficulties (discussed by Kirk and Raven, pp. 99-103) the solution to which may be that Anaximander set up two sciathera, one calibrated for astronomical purposes and another for telling the hours.

8. See *The Athenian Agora, a Guide*, 1962, pp. 108-109. Besides this large piece of architecture, there has been found also an example of those portable klepsydrai which were used in Athenian law courts to limit speeches to six minutes (*ibid.*, p. 164). These pots have been known to inspire the envy and admiration of visiting American senators.

9. The noon shadow seems to be distinguished by primitive people too, who stay indoors then, unable to bear the uncanny feeling of going about without a shadow, cf. Arnheim, p. 257.

10. For sun caves, an early form of solstice marker, see Kirk and Raven, pp. 52-54.

11. For instance in Borneo, see *A History of Technology*, Oxford, 1954, I, p. 117, Fig. 46.

12. See Heath, *Aristarchus*, Oxford 1959, pp. 21, 130ff.; Kirk and Raven, pp. 81, 101, 103. There is some question whether the earlier men actually discovered the oblique ecliptic or some partial feature like the tropics and the equinoctial circle.

13. See J. S. E. Dreyer, <u>A History of Astronomy</u>, Dover, pp. 93, 106.

14. On the story see Kirk and Raven, pp. 83-84. The fact that pyramids are not thin objects actually makes the geometry of the situation rather complicated; it is given in T. Dantzig, <u>The Bequest of the Greeks</u>, New York 1955, pp. 52ff.

15. The Parthenon, for instance, (built of course a century and a half after Thales) is a good example of a human scale applied even to a superhuman building. For a Greek temple rests on a platform of steps (crepidoma) whose risers will obviously be felt to bear a relation to the human step (cf. R. D. Martienssen, <u>The Idea of Space in Greek Architecture</u>, Johannesburg, 1958, p. 83). The Parthenon steps are just twice as high as those of a man, and for human convenience one intermediate step must be added for each rise. Its proportions are therefore double the human.

16. a. Herodotus, insisting for some purpose of his own on the practical nature and the foreign origin of both geometry and astronomy, says that the Egyptians developed geometry for the purpose of assessing the lands swept away by the Nile, "whence it came to Greece, while the Greeks learned both the gnomon and the polos from the Babylonians" (II, 109). This seems to conflict with the reports giving Anaximander as the discoverer of the gnomon (cf. Kirk and Raven, pp. 101-102; Kahn, p. 91, n.3). Furthermore, most of Anaximander's inventions have also been ascribed to Thales (Guthrie, p. 74). It seems scarcely possible to settle the matter except to point out that the gnomon, the most natural of all tools, was scarcely in need of discovery in an original sense in any case -- what the sources ought to be read to mean is that he was among the first to develop it as a theoretical tool, i.e., in a cosmological context.

b. It is not known what instrument in particular the polos was; some identify it with the gnomon (Guthrie, p. 33). If it had anything to do with the gnomon, it may have been just its tip, for that corresponds to the geometric understanding of "pole"; Proclus, for instance, says that if an upright gnomon is set at the center of a circle its tip will be a pole (Heath, I, p. 185 -- actually the gnomon itself is a locus of geometric poles, a pole being any point not in the plane of a circle which is equidistant from all the points on the circumference).

Now the common later meaning of the word is pivot, as in the [Aristotelian] treatise <u>De Mundo</u>: "The whole of the heavens,* as I have said; but there are necessarily two points which are unmoved, opposite one another, just as in the case of a ball being turned in the lathe (τόρνῳ); they remain solidly fixed, holding the sphere in its place (συνέχοντα), and the whole burden moves in a circle around them; these points are called poles. If we think of a straight line joining these two together, which some call the axis, it will be a diameter of the cosmos, having the earth at its center, and the poles at its extremities" (391b). However, this meaning misses the original sense of the word, which is "something going round", from πέλομαι , "go". Thus it is not surprising to find an earlier use in which polos means "heavens" because "it itself goes round and through it all things go" (Schol. to Aristophanes, <u>Birds</u>, 179). It is therefore equally likely that the polos was a spherical model, perhaps with movable parts.

(The pivotal poles become the crucial parts of the first planetary model to embody a hypothetical account of planetary motions, that of Eudoxus, which consisted of numerous homocentric spheres each with different poles (see Heath, <u>Aristarchus</u>, Ch. XVI).

* the whole cosmos, is spherical and moves continuously,

17. The text: Kirk and Raven, pp. 117-121. On Anaximander in general see C. H. Kahn, <u>Anaximander and the Origins of Greek Cosmology</u>, Columbia 1960.

18. Translating the pun on ἀ-ιών -- "un-going" in αἰών eternity" cf. 37D7: ἰοῦσαν αἰώνιον εἰκόνα. The pun expresses the character of time as the source of self-contradiction.

19. Ptolemy's astrolabe became what was later known as an "armillary sphere", used, at least by the fifteenth century, more for demonstration than for observation. A picture of it is given in Cornford, <u>Plato's Cosmology</u>, frontispiece. It is significant that the Neoplatonist Nicholas of Cusa was one of the best known designers of such spheres (see I. Hart, the <u>Mechanical Investigations of Leonardo da Vinci</u>, p. 21; this book contains a brief survey of simple astronomical instruments, pp. 19-31.)

The astrolabe of later times, on the other hand, is a plane instrument, as was Ptolemy's "planisphaerium". Neugebauer (<u>The Exact Sciences in Antiquity</u> pp. 185, 219) says that both were based on projections of the heavens upon a plane. Such plane instruments as a class are called "theoric" (Kepler, <u>Epitome</u>, p. 31).

20. See Kirk and Raven, p. 100, n. 1.

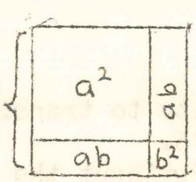
21. Archimedes says explicitly that " 'universe' is the name given by most astronomers to the sphere whose centre is the centre of the earth and whose radius is equal to the straight line between the centre of the sun and the centre of the earth" (<u>The Works of Archimedes</u>, ed. Heath, Dover, p. 221). This notion must have been suggested to technical astronomers by the cosmological models, for in these the ecliptic is one of the containing circles.

22. See M. Nicolson, <u>Science and Imagination</u>, Ch. I, "The Telescope and the Imagination"; Ch. VI, "The Microscope and the English Imagination".

23. <u>Discoveries and Opinions of Galileo</u>, ed. Drake, Anchor, pp. 27ff.

24. cf. Kirk and Raven, p. 134.

25.



$$\begin{array}{rl} & a+b \\ & \sqrt{a^2+2ab+b^2} \\ & \underline{-a^2} \\ a+b & 2ab+b^2 \\ \text{gnomon} \rightarrow & \underline{-(2ab+b^2)} \\ & 0 \end{array}$$

Hence in the arithmetical algorism for taking the square root the <u>second</u> subtrahend is still called the gnomon.

26. The N.O.E.D. can document this usage from the time of Ben Jonson:

> ". . . Her nose, the <u>gnomon</u> of Loues diall, that tells you how
> the clocke of your heart goes" --

to Cowper:

> "The emphatic speaker dearly loves to oppose,
> In contact inconvenient, nose to nose,
> As if the <u>gnomon</u> in his neighbour's phiz,
> Touched with the magnet had attracted his".

Like "canon" and "organ[um]", the word gnomon has become part of both technical and common English, used in the former mostly in connection with <u>gnomonics</u>, the art of dialling. In one curious obsolete use of the plural, taken from the Greek, gnomons designate those of a horse's teeth which tell its age. These are the equine equivalents of "wisdom teeth".

27. Thus Theognis: "to judge (δικάσσαι) [a suit] by the plumb line and gnomon" (v. 543). The Latin for carpenter's square was "quadra" or "<u>norma</u>" -- for the Freemasons it becomes a symbol of rectitude.
It says much about the way we live now that it is a journal of reviews of books on antiquity which presently bears the title <u>Gnomon</u>.

28. cf. G. Richter, <u>The Sculpture and Sculptors of the Greeks</u>, New Haven, 1957, pp. 248-249.

29. Vitruvius I, 1, 16. The preceding passage is pricelessly Roman: "As for men upon whom nature has bestowed so much ingenuity, acuteness, and memory that they are able to have a thorough knowledge of geometry, astronomy, music, and other arts, they go beyond the functions of architects and become pure mathematicians. Hence they can readily take up positions against those arts because many are the artistic weapons with which they are armed".

30. For example, the body as tool: <u>Parts of Animals</u> 642a11; a man's body as another's tool: <u>Politics</u> 1253b; the eye as organ of awareness: <u>Metaph.</u> 980a23.

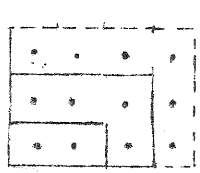
31. The analogy itself comes from Aristotle: "The soul, then, is like a hand, for the hand is a tool of tools and the intellect a form of forms and sense of sensible forms" (<u>De Anima</u> 432a1-4).

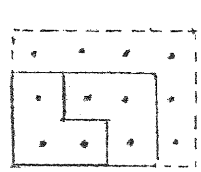
32. See for instance Descartes, <u>Dioptric</u>, Ch. VI for sight conceived in terms of touch.

33. A. Boeckh, <u>Philolaos des Pythagoreers Lehren</u>, Berlin 1819, p.144; Heath, I, 371.

34. See Liddell and Scott, γνώμων , nos. 1., 2.

35. καὶ χωρίς has given a great deal of difficulty to translators; see <u>Physics</u> (ed. Ross), pp. 542-545, commentary on 203a13-15.
The possibilities are: καὶ χωρίς means either 1. "without the gnomon" or 2. "without the one". In the first case the numbers may be given in an arithmetical series like that suggested by Themistius: 1, 1+2, 1+2+4 . . ., or they may be arranged in rectangular patterns. In the second case, which Ross adopts, a. the beginning number may be two, or b. there may be no beginning number, so that the gnomons would look as arranged in the figure. (It is not, however, likely that gnomons were ever placed around anything but a beginning number.)

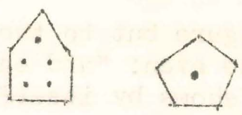

2a


2b

The question is whether a geometrical addition can be called a gnomon when it changes the proportion of the figure. In its latest and over-extended such an addition could be called a gnomon if it preserved, as do the gnomons of 2., the numerical relations of the figure, i.e., n(n+1). But this is surely not a possible function of the original Pythagorean gnomon which Aristotle and Philolaos mean; even Simplicius still shows some hesitation about using the word in this way: "And although even numbers are not called gnomons in the primary sense, because they do not preserve the same form, yet they may be added as gnomons to show the unlike in figurate numbers" (σχηματογραφία- 458, 5-7). On the other hand it is evidently possible to speak of numbers being "placed around" (περιτιθεμένον) as non-gnomons. Therefore καὶ χωρίς must mean that numbers which are not gnomons are placed around the two, which is the first even number, and that the resulting figure is even, and different with each addition.

36. cf. Nicomachus II, 26.

37. cf. Aetius, translated in Kirk and Raven, pp. 230-231.

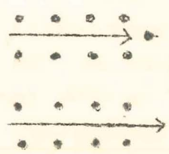
38. 
There are alternative ways of generating the pentagon, as shown; see Nicomachus, Introduction to Arithmetic (trans. D'OOge), II, 8-11, especially p. 244, n. 1.

39. See also Euclid VII, Def. 16, and Heath's Commentary (II, p. 287ff.).

40. Galileo, too, felt the fascination of the tetragonal gnomon, for he recognized in it the form of his rule for free fall. Having given its geometric proof, Galileo adds a corollary in which he formulates the rule numerically; "While therefore during equal intervals of time the velocities increase as the natural numbers, the increments . . . are to each other as the odd numbers beginning with unity" (Two New Sciences, Third Day, Th. II, Cor. 1).

41. which is, in turn, one of the two elements (στοιχεῖα) of number, as Aristotle says (Metaph. 986a18).

42. The Euclid scholiast says that "when the gnomon is added the σχῆμα grows, while the εἶδος does not change in quality (οὐκ ἀλλοιοῦται)", where the former must mean a particular figure of particular size. (Heiberg, p. 277, 16).

43. Aristotle's explanation of the way in which the odd represents the finite and the even the infinite has an alternative, given by Simplicius, p. 455, 20

(see Kirk and Raven, pp. 244-245): as the figure shows, the even can be divided as long as it remains even, while the one in the odd blocks division and binds the number into a finite atom.

44. Alexander as quoted by Cornford, Plato and Parmenides, p. 8, q.v. for a discussion of Pythagorean number. Passages relevant to this theme are collected in Kirk and Raven, pp. 242ff.

45. so the Euclid scholiast, speaking of square gnomons, calls the unit the

first square number (Heiberg, 226, 8).

46. Theon, p. 37, 12; it seems as if with ". . . ἀπογεννῶντες ... γνώμονες καλοῦνται" he were trying to comment on the gnomon's name.

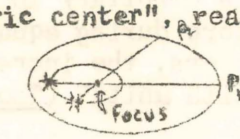
47. On Philolaus see Guthrie, pp. 329-333; Kirk and Raven, pp. 307-313. A large part of E. Frank, Plato und die sogenanten Pythagoreer is devoted to proving that all the fragments of Philolaus are spurious (esp. pp. 291ff.). Scholarly literature on the "Philolaic problem" exceed reflections on the content of the fragments by better than ten to one, roughly. There is, however, an article by Newbold in Arch. für Gesch. der Phil. (1906) which discusses the gnomon fragment (pp. 176ff.) and which I have not seen.

48. Diels (as rendered by K. Freeman, Ancilla to the Pre-Socratic Philosophers, p. 75) translates: "But in fact Number, fitting all things into the soul through sense-perception, makes them recognizable and comparable with one another as is provided by the nature of the Gnomon . . .". The differences are crucial to the interpretation, especially "according to" for "into" the soul, and "a" for "the" gnomon.

49. Philolaus applies the word eidos not to the figure but to the two "proper forms" (ἴδια εἴδη) of number, the odd and the even: "and each of these has many shapes (μορφαί) which each thing shows by itself" (Diels I, 408, 10).

VII. [Corrections to AN ANATOMY OF ORBITS, April Collegian, pp. 1-28.]

p. 8, ii, 1a add: In iv, 3 this is the center of gravity.
p. 9, iii, 2, l. 3: for "geometric center", read "focus".
p. 11, iv, 3, l. 4: for "geometric center", read "the focus of the orbits".
The figure for the last two should be →



p. 12, diagram: the sun is S
p. 12, vii, 2, l. 7: "eccentricity AB=AR"; so also in the diagram, p. 13.

FEAR AND WAR OR FEAR AND PEACE?

Andrea Jacobson

There is a paradox in Thucydides' History of the Peloponnesian War, and it concerns one of the strongest emotions that move men. It is the paradox of fear. Thucydides says that the truest cause of the war was "the growth of the Athenian power, which putting the Lacedaemonians into fear necessitated the war."1) And yet the Mytelinaeans say, "the equality of mutual fear is the only band of faith in leagues."2) Fear is presented both as a basis for war and a means for peace. How can such a seeming cotradiction exist?

We shall have two questions facing us in this discussion. The first of these is what fear is. In theology fear is sometimes a virtue, as in the case of fear of God. And yet fear is also often a derogatory epithet. One child taunts another with the challenge -- "What's the matter, are you chicken?" And men often hesitate to say honestly that they are afraid. What is the emotion that is subjected to such various interpretations?

And if we succeed in answering this question, we shall be ready to deal with the one that is behind the paradox discussed above. This second question is whether fear, which clearly seems to be a possible cause of war, can ever be the basis for a viable peace. What kind of fear would this be, and how is it to be distinguished from the fear which has such opposite effects?

Let us begin with the Lacedaemonians before the war as Thucydides describes them, and begin our attempt at clarifying fear with Hobbes' definition thereof -- "aversion with opinion of hurt from the object".3) The Lacedaemonian situation is certainly consonant with such a definition. They are averse to the growth of Athenian empire, as their actions and speeches show. And they do have an opinion of hurt from an object, from the Athenian empire, because it may grow large enough eventually to isolate Sparta and perhaps even subjugate her.

What does this fear involve? It is immediately clear that as a result the Lacedaemonians become aware of their separateness as a group or, with the confederates, as a group of nations. But it is one group as distinguished from everyone else. While at peace both men and states may forget that they are responsible for themselves and their own preservation, and that everyone else has a similar responsibility. And so they forget that each must be devoted to a different good. But fear, and an opinion of hurt, imply a hurting agent and thus a conflict between two distinct parties. And so fear reminds men that they stand alone and removes a possible false atmosphere of mutual love and self-sacrifice.

Fear is also concerned with the future, for it is the opinion of a hurt not yet actually present. This is shown in the speech of the Lacedaemonian confederates:4)

> "But now the Athenians are a match for us all at once, and one city after another too strong for us. Insomuch that

> unless we oppose them jointly and every nation and city set to it unanimously, they will overcome us without labor".

Consideration of Athenian action in the future leads to the conclusion that, if no preventive action is taken, there will result, in the future, conditions which are harmful.

At the time when this conclusion strikes someone, whether after thought or after the much shorter process of an unexpected agressive action by someone or something else, we can say that a person has been struck by φόβος, this being the word Thucydides uses in speaking of Spartan fear as the cause of the war. It is something that is outside the person involved. When he has been hit by it one of two things can happen, the person or city involved will follow one of two courses.

The first of these is blind panic. When, in the Peloponnesian exhortation before the battle at Naupactus, the soldiers are told that Athenian experience may be useless because "fear confoundeth the memory,"[5] the hope expressed is that the Athenians will feel so outnumbered and overwhelmed that they will consider their skill of little value. And so the important factor for success will become chance, which could equally favor them or the Peloponnesians.

But if chance is thus so much more powerful than skill, and there is no attempt made to use skill, the Athenians involved will not be making decisions. They will not be trying to understand what the purposes of the enemy are and deciding what their own strategy should be. In short, they will not be using their reason.

And if they feel that the injury they fear is completely unavoidable, they will hope only for some kind of miracle. And miracles by definition are of all things least subject to rationality. And so the opinion of hurt, together with an opinion, produced by the circumstances involved, that the hurt is not to be avoided primarily by one's own efforts, can lead men to panic or despair.

But there is another course that fear can take, and this is the one exhibited by the Lacedaemonians. For they are not easily subject to panic, since their case is far from a hopeless one. They are a powerful city, supreme on land, and possessing powerful confederates. "Wherefore, men of Lacedaemon, decree the war, as becometh the dignity of Sparta."[6] Whether or not they want this war, they are not panicked by it. This leaves the way open for rationality.

The first occasion for reasoning is the decision that Athens can be stopped only by war. Sthenelaidas may have been an irrational demagogue; but even wise and temperate Archidamnus thought a war might be necessary -- "and prepare withal for the war"[7] he tells the Lacedaemonians.

And the conduct of and preparation for a war are intensely involved with reason. The actions of the war are in the future, and the future can be known only by conjecture based on reasoning analysis of the past and its analogies to the present. The Spartans will be concerned with causal relationships -- what will result from an immediate invasion of Attica, is it better to wait and attack somewhere else, what should be done to train a navy?

The whole problem of confederates will also require much thinking. The Spartans must decide how best to arouse and hold their own, and also how to persuade Athenian confederates to revolt. The awareness of separation that is involved in φόβος will make them pay all the more attention to this aspect of the war.

And so fear can become rational, and as such is no longer something outside the person who fears. It is more what is meant by the Greek word δέος , and it is, as we have seen very much concerned with the future.

Thus we have arrived at our second question. Can such a thing as fear be the basis for a league or for peace? Clearly panic, with its heedless thoughtless action and its passivity, can not. But what of rational fear, what of δέος ?

The first difficulty is that anything that is to be the basis of peace must be very strong. Hobbes says, "In the nature of man we find three principle causes for quarrel -- competition, diffidence, and glory."[8] And these causes are powerful, for within their categories can be grouped all the passions, desires, and prejudices that move men to war upon each other. What makes us think any fear can control them, that it can override desire for gain and glory? Do not people often act despite their fears to obtain a desired end?

But there are reasons for believing that fear, if great enough, will be even more powerful than these desires. For while gain and glory are apparent goods and certainly sought after by men, they are goods that are to be enjoyed in the future. If a man has a decision to make, and one choice may lead him to gain or glory, these two goods will often come to him some time in the future. The profits on his business will begin to accumulate at the end of the week and by the next week he will be enjoying greater luxury; or perhaps, in the case of glory, he will be honored for his generalship when he returns to Athens the next year.

Fear, however, is usually more immediate and sure. Men tend not to have the foresight to fear things far in the future, precisely because fear is concerned with undesirable objects and it is natural not to want to think of such. The result is that when fear does arrive, the potential hurt may be very near in time. The choice between a fairly immediate evil and a good in the future will often be decided in favor of the former.

And, more importantly, gain can be enjoyed only by living men, and even glory seems better if one is alive at the same time that he is glorious. In other words, life is a pre-requisite to either of these goods, and a fear that contained an opinion of hurt to one's life would have necessary priority over the desire for some gain that, without life, would be worthless. Self-preservation is not only an old instinct, but it may also be of the first importance in our actions.

So fear of the right kind and in great enough quantities may be strong enough to overrule the other emotions. But what is the right kind of fear? The Mytelinaeans have spoken of mutual fear. Let us say that I am afraid of you because you are strong, but you are also afraid of me, for I too am strong. We agree not to hurt each other, because neither is sure of winning, neither wants the risk of losing, and neither wants to waste energy and life in wars. Also, if the two of us combine our strength, we will be that much more sure against the world. Therefore -- let us have peace.

But there is a great problem in all of this, one which makes this theory always eventually fail, and this is the factor of changes in power. My crops may fail one year, my slaves may revolt, or my city may suffer sedition. You, observing this, will judge me weaker and perhaps consider a war now worth the risk, because the odds have changed. Mutual fear is gone, because you no longer have a predominant opinion of hurt. And so too is the peace gone.

Mutual fear is not sufficient for the basis of a league. Neither is a fear that cannot be understood by all. For an example of this let us consider the Melian dialogue. We could say that we have the possibility here that the Melians are so struck by panic and so hopeless about their situation that they are no longer being really rational. This is probably the effect the Athenians hoped to have on the masses if they had been allowed to talk directly to them.

Let us assume, however, that this is not true of the Melian leaders, and that the fighting power of the Athenians has not completely panicked them. There is still fear involved here, and it is fear of the consequences of Athenian rule. Submission seems not only dishonorable, but also perilous, for they have no guarantee that Athens, once the master, would not rule with severity. And if the Athenian rule should become oppressive in the course of this long and expensive war, the Melian leaders would have difficulty in leading a revolt of the city that they had earlier voluntarily surrendered.

But is this a fear the Athenians can fully understand? Certainly they know that the Melians, like any other nation, would prefer to be free. But they do not realize how much the Melians would be afraid of their rule, not in terms of tribute that might be presently levied, but in terms of placing one's self in the power of people like the Athenians. The Athenian ambassadors say, "You shall take very absurd counsel in deciding to resist unless, excluding us, you make amongst yourselves some more discreet conclusion."9) Since the Athenians have not recently been a weaker, subjugated city, and since they don't see themselves as cruel or impetuous, they may not realize another nation could view them in this way.

And so perhaps throughout the dialogue the conversation is blocked by this Melian fear that the Athenians do not understand. And so it is clear that in any attempt at peace, if fear is used as a means, it must be understandable to all.

Thus our analysis suggests that fear is a powerful enough passion to control men in the actions that lead to war. As we have seen there is a problem of determining the proper object of such fear, for it must be fear of something enduring and must be a fear visible to all. There seem to me to be two possibilities. One of them is relevant to Thucydides' time and later is developed by Hobbes. This is fear of the state of war. If war places men in a state of nature -- "solitary, poor, nasty, brutish, and short"-- man should want to avoid the evils that such a state entails.

In the past, however, war has not involved a state of nature for many individuals. The men who govern countries are often not those who fight in foxholes (which might be a good modern equivalent of what Hobbes had in mind). Large parts of the population continue to live in civilized society, although the society may be changed in form by a war.

But now the possibility has appeared, through atomic weapons, of actually destroying all civilization and perhaps the earth as a suitable place for life. And this fear seems to fulfill the requirements for a fear that can lead to peace.

If it sounds too optimistic to place so much hope in fear -- and well it might -- we must remember that there is a large qualification underlying this thesis. This qualification is education and resultant applied rationality.

Even Hobbes says that the laws of nature are arrived at by reason. If man knew them without reason, there would never be any state of nature, or at least not one of the kind that Hobbes describes. So a man must be taught, either by experience or by other men, that war is the greatest of dangers.

But there is something else he must know, that may be even more difficult for him to learn. This is the fact that all men have some basic needs in common, and that all men desire many of the kinds of things he does. If he knows this -- and on the assumption that other men will be rational -- he can conclude that others have the same dread of war he does. And so he may be free to follow his aversion to war without the risk of domination by others that at first seemed apparent.

I will freely grant that this qualification is very large, and I do not know whether it is possible to educate men in such a way. But this theory may be a road -- a way applicable to both men and nations that avoids the cruelty of Machiavelli and the need for divine intervention.

* * * * * *

NOTES

(All footnotes are references to History of Pelopennssian War, unless otherwise noted.)

1) I: 23

2) III: 11

3) Hobbes' Leviathan, Part I, Chapter 6, p. 55

4) I: 22

5) II: 87

6) I: 86

7) I: 85

8) Leviathan, Part I, Chapter 13, p. 106

9) V: 111