# Looking in Freshman Lab II -
# How measuring, weighing, and counterbalancing can inspire us to see things in new ways

This is the second in a series of lectures[1] for freshman on ways of looking encountered in the first year of laboratory. In the second segment of the class, the students are invited to consider various ways of mathematizing the world.[2] They may come to see certain phenomena as *μαθήματα*, that is, as objects of study that can be used to learn through insight. While these phenomena are also investigated empirically, the lecture will explore whether they can be viewed in ways that lead to insight.[3]

The lecture will be limited to the ways in which the characteristics of length and heaviness and different sorts of equilibrium can lead the students to new ways of looking and seeing with the mind's eye.

## PART ONE

The first exercise performed in the second segment of the lab is measuring the quantity *length*. We measure the length of a table by laying out, end to end, a strip of cardboard—our *standard unit length*—as many times as it will fit on the table, say, seven. Then, unless we are very lucky, so that it happens to fit *exactly* seven times, we claim that the table is between seven and eight units long.

In order to measure the table more exactly, we make a new unit by dividing our first unit in half. Then we may say, for example, that the table is between 15 and 16 new units long. Continuing the process of bisecting, at some point—due to the thickness of the pencil mark or to the limitations of our vision—we find ourselves, perhaps, after four bisections, unable to perform a further one. At that point we might determine that the table was longer than 123 and shorter than 124 of the last standard unit. We'd go on to say that the ratio of two lengths, that of the table to that of the last standard unit, is somewhat greater than the ratio of the numbers[4] *123* to *1* and is somewhat less than the ratio of the numbers *124* to *1*. We claim we have measured the length of our table on "a *ratio scale*," in that we have determined "the ratio between a measured magnitude and the standard magnitude as … equal or nearly equal [to the] *ratio between two numbers*" (E&M, p. 5).[5]

We are asked (E&M, p. 5) think about whether any of the "practical limitations to achieving … exactitude … in the exercise … might be overcome." Let's take this question as an invitation to imagine, in a way proposed by the philosopher Edmund Husserl,[6] a series of practical refinements to our standard and the corresponding improvements in our ability to measure length exactly.

We begin with our last cardboard standard, sharpen the pencil we used to make it, and make an improved standard. Next if we think we can do better, we may sharpen the pencil more. Or we may come across a different tool for making even finer marks that can be made closer together. And so on. We imagine that as "technology progresses" we are able to distinguish separate lines that are ever closer together. Perhaps a machine is invented that can draw lines *.1* mm apart. We might be able to view them under a microscope and see that the edge of the table falls between two of them. As we imagine "the ideal of perfection …pushed further and further," we never get to the point of thinking that no further improvement could ever be conceived.

We can imagine repeated experiences of moving, in the "again and again" (*im Immer wieder*), toward a more perfect standard.[7] If we think of this perfecting process as carried to its ideal limit, we may think of an *ideally exact limit-standard* of length. That standard would be "like [an] invariant and never attainable pole…, which" we'd approach, but never reach, as ever more perfect standards were produced.[8]

Our imagining of this progress and our thinking of this standard appear to have led us to what Socrates, in conversation with Simmias, called *recollecting*. He spoke of recollection as a potential accompaniment to our recognition of a thing we are sensing—an accompaniment that would take place whenever someone "not only recognized (*γνῷ*) that thing but also thought (*ἐννοήσῃ*) of another thing, the knowledge of which isn't the same" as the recognition of that thing. When we recognize a set of marks on the cardboard, we *think* – short, *parallel*, *straight-line* segments, *equidistant* from each other. As Socrates might have said: from these (apparently) equal distances and this (apparently) straight edge, we think of the Equidistant Itself and the Straight Itself. From our perception of the equidistant marks along the edge of the cardboard standard, we think that they "both are reaching after the [Equidistant and the Straight] … and are in a condition of falling short of [them]." Socrates would say that "that of which [we] grasped the thought (*ἔννοιαν*) was recollected." It was not found *in* our sensing of the marks on the cardboard. Rather it felt as if we knew it from "sometime before" (*πρότερόν ποτε*; as in *a priori*). This kind of experience is what we call "learning through insight" (*Phdo* 72e-75b), as opposed to merely making empirical observations, interpreting them, and learning through experience.

Recollection is a *two-in-one* experience—like visually recognizing something in a crowded room as a face—or as a lyre—and, at the same time, thinking of it as Simmias's face—or thinking of the boy who is always playing that lyre. When, for the first time, someone recollected an ideal limit to the perfecting process described above, it might have been experienced as a thought that *suddenly* fell into her or his mind (in German: *ein Einfall*). The experience of recollecting may have shared certain features with Wittgenstein's description of the experience of suddenly "noticing an aspect": "I'm considering a face, suddenly I notice its likeness to another. I *see* that it has not altered; and yet I see it differently."[9] As Socrates pointed out, however, an additional feature characterizes the recollection-experience. For a key part of the way I see the "face" "differently" in recollecting is that I see it as wanting to be of the same sort as the ideal-exact limit-standard—like Socrates' pair of equal sticks, wanting to be of the same sort as the Equal itself—but as falling short of it.

We are also asked (E&M, p. 5) to consider a second question: whether, given any table, we could *always* subdivide the standard enough times, so that the length of that table would "turn out to be an *exact* multiple of the subdivided standard." Whereas the first question led us to recollection, involving a a two-in-one experience—*recognizing* a piece of cardboard with pencil marks on it and *simultaneously, together with* that, *thinking* of an ideally sharp standard for length—this second question leads us to a *single* seeing, or, perhaps better, a fantasizing. For it raises the possibility that the things in the world of our experience might themselves actually *be*—rather than reach after being—ideally exact. If that were the case, it would then be theoretically possible to express the length of every particular rectangular table exactly as a ratio of whole numbers. We might then be thought to have accomplished a *ratio-nalization of the world*.

At the end of Ch. I, we turn from length to the characteristic of heaviness, or weight. The first two questions posed are: how would it be possible to determine that two weights are "virtually indistinguishable" and how could we arrange different instances of heavy bodies "in a linear series," from lightest to heaviest. If we were able to do both, then we'd claim that we could measure heaviness on an *ordinal scale* (M&E, p. 4).

In order to experience what it means concretely to measure heaviness on an ordinal scale, we use a device called a "substitution balance." Here is a picture of one:

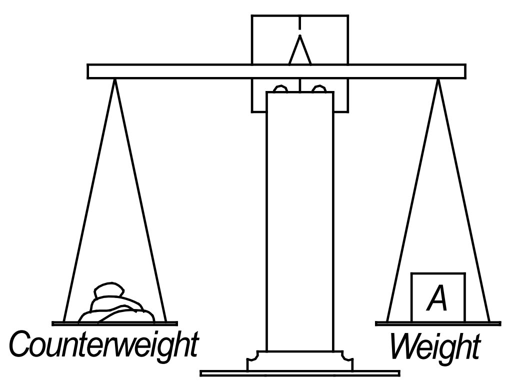


Figure 1

> This instrument consists of a straight wooden beam that rests on a knife-edge called the *fulcrum*; from each end of the beam a plastic pan is suspended…. the arms of the beam are not equal…. The beam is free to swing within the limits established by the two bumpers (M&E 8).

We place an object *A* in the weight pan on the right and enough clay in the counterweight pan on the left, so that the beam comes to rest somewhere in between the two bumpers. Then we mark the pointer's position.

In order to determine *experimentally* that two bodies, *A* and *B*, are equally heavy, [we] carefully remove *A* from the "weight" pan, "without disturbing the counterweight, and substitute *B* in its place…. If *B* causes the pointer to rest exactly where *A* did, then *A* and *B* are equal in their … weight; and if not, not" (M&E 9).

Students find this balance and this experimental definition of equality of weight to be "counter-intuitive." It is especially puzzling to them to read: "No inferences whatever may be drawn, by means of the balance, as to the comparison of the 'weight' and the 'counterweight.'" Why can't we say that the sinking of the counterweight would show that it is heavier than the weight?

In response, after having tentatively considered *A* and *B* to be equally heavy, whenever *B* causes the pointer to rest exactly where *A* did, the students formulated the following insight: It makes sense because "whether *A* or *B* is in the weight pan seems to make no difference to anything at all, since everything is the same either way. *B* leaves the pointer in the same position as *A* did." As one student said: Weighing using the substitution procedure "challenged my presuppositions" about what equality of weight really means.

It was as though working with the method of substitution played an analogous role, for the students, to the one Socrates plays for the slave boy in the *Meno*. The students, too, at the beginning, without knowing, "suppose [they] know" how to determine equal weights; then they become aware that they "don't know" and are "perplexed" by the procedure proposed; and at the end they arrive at "true opinions" about it (*Meno* 82e, 84ab, 85c).

After that each lab team is given an identical metal cylinder, called a *baros*, to use as unit standard weight for making two pieces of clay, each equal in heaviness to one *baros*. By repeated halving they go on to construct a ratio scale for measuring weight and use it to assign weight-numbers to various bodies, in a way very similar to their assignment of length-numbers to the edge of the table.

A student asked: “What’s the point of spending even more time with such primitive equipment, like the substitution balance?” A possible response arises out of the fact that in performing the *baros*-exercise many of the weighing teams ended up *not* using the method of substitution. Instead, they reverted to their initial “knowledge” that the two weights at opposite ends of *a* horizontal beam are equal. Perhaps, then, the student’s earlier realization of having had his presuppositions challenged may have been like the slave boy’s true opinions, which had merely been “stirred up in him, like a dream” (*Meno* 85c). It may be that his realization didn’t “stay put” but rather ran away (97d-98a), or, through the lens of a different metaphor, flew up to mix in with and fly around with his previous opinions (cp. *Theat* 197cff) about what weighing is.

Socrates says to Meno that in order for true opinions to “stay put” they must be tied up “with a giving-an-account of what is responsible” (*αἰτίας λογισμῷ*)—a tying-up process that they’ve also agreed to call “recollection” (*Meno* 97d-98a). Perhaps it’s a recollection-process that makes use of recollected things, like a Square, in this case. Socrates had said earlier that in order for one to go beyond having true opinions “stirred up … like a dream” and to come “himself to draw up [knowledge] again out of himself (*ἀναλαβὼν αὐτὸς ἐξ αὑτοῦ*),” one needs to be asked “these same questions many times in different ways” (*Meno* 85cd). Perhaps by thinking out afresh, each time, our responses to those questions about the relations among the parts of a recollected Square, we become able to recollect them, too, and to articulate them, eventually, with assurance.

After the *baros*-exercise, in discussion the students realized the mistake they’d made. Did their questioning and the rethinking of the exercise lead them, at this point, genuinely to draw up again out of themselves and to formulate an account of what is truly responsible for showing equality of weights? Or not? They may have only been remembering that they were *supposed* to follow the *recipe* for the substitution procedure and had merely forgotten to do so. This latter suggestion is reinforced by the fact that in an exercise three weeks later that again required a weighing using the substitution balance, no one in the class actually used the counterweight.

## PART TWO

Chapter II, dealing with equilibrium, revisits the balance from a different point of view. A suitable balance allows us to go beyond measuring the heaviness of a single body to determining a certain relationship among two bodies and two distances.

The students perform simple practical exercises in the lab and study a treatise on equilibrium by Archimedes. How are we to think about the exercises in relation to the treatise? Does the latter provide hypotheses, which the students test with the aim of learning, through their experience of the former, what *does* happen? Or could Archimedes, be, as it were, the Euclid of weights-in-equilibrium? If so he'd be offering us a way to learn through insight into what *must* happen when bodies are counterbalanced on a true Balance.

At the beginning of the new chapter, we perform five simple exercises, in order to gain a sense of how heavy bodies behave when suspended from a point. Then, in our imagination, we take a second look at those experiments, in order to seek out the essence of the bodies' behavior.

We, first, make a number of pinholes around the edge of a piece of cardboard as shown in Figure 2, and suspend it from each in turn.  After the cardboard comes to rest, we draw a vertical line through each pinhole.

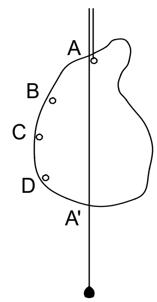


Figure 2

The lines drawn come very close to passing through a single point, which we call the cardboard's *center of weight* (M&E, p. 13). The name is apt in the sense that the cardboard shape "behaves as if all of its weight were concentrated at or near" this point.

Second, we confirm this claim by, sticking "the head of a pin through this center" and then balancing "the cardboard, in a horizontal position, on this pinhead."

Third, when we "insert the pin through the center of weight and hold the pin horizontal," the cardboard remains at rest in any orientation in its vertical plane. This rest position is *neutral*, in the sense that the cardboard has no preferred orientation; it does not rotate out of any position in which it is placed. In comparison, the rest positions observed in our first set of experiments, where the center of weight rotated until it came to rest vertically below the point of suspension, are *stable*. For even a small displacement leads to oscillation and, ultimately, to a return to the initial position.

Fourth, if we were very careful, we could bring the cardboard to rest—at least for a short time—with its center of weight vertically above a pinhole through which the horizontal pin was inserted. It would then be in an *unstable* rest position. For if our hand trembled ever so slightly, the cardboard would swing around in its vertical plane and come to rest in a stable rest position.

Finally, what is even more puzzling, when the center of weight is not even on the piece of cardboard, as in the case of the odd shape depicted in Figure 3, the lines drawn intersect at a point that is not located on the cardboard.

Generalizing from these empirical observations, we claim that all heavy bodies have a center of weight in the above sense and have the three kinds of rest position, which depend upon the relation of their points of suspension to their center of weight.

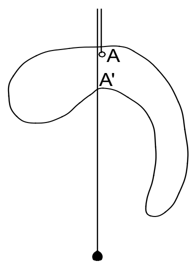

Figure 3

Could we say that, beyond such a generalization, these few exercises have led us, whenever we recognize a heavy body, to think, in addition, of a point that behaves as that body's center of weight? Would our experiences support the claim that the center of weight did not come *from* our empirical observations of bodies in the lab? Rather it occurred to us *in* our observation of them in the lab. If so our simple exercises with the pieces of cardboard would, perhaps, have allowed us to *recollect* the center of weight, so that it felt as if we knew it from "sometime before" (*Phdo* 72e-73d).

In thinking of the center of weight, we seem to be looking "through" the body with a sort of *X-ray vision*. Only the body's center of weight and the point of suspension, its "bones," show up as white on the X-ray. The other parts of the body, its "soft tissues", appear black. If the two bones were *given*, then we could see in the mind's eye—without our having to do further experiments[10]—that the actual amount of the weight and its disposition throughout the body would be irrelevant to the way the body behaves when suspended. X-ray vision, thus, would allow us to separate something essential in the situation of a suspended body from what is merely accidental.

We turn now to Archimedes' treatise "The Centers of Weight of Planes" (M&E, pp. 14ff). Figure 4 is the second figure in the treatise. It depicts two magnitudes, **A** and **B**,[11] of equal weight and

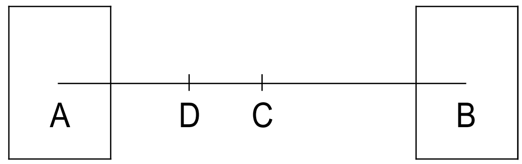


Figure 4

having centers of weight,[12] *A* and *B*, respectively. The previous figures, of the substitution balance and of the cardboard cutouts, were schematic depictions of physical objects. How are we to interpret Archimedes' figures? In what new ways do they invite us to look at material things?

He clearly shares two ways of looking with Euclid. First, his vision is *abs-tractive* in Aristotle's sense. He, too, sees the "things that result from drawing [*trahere*] [something] away [*abs*-]" (*τὰ ἐξ ἀφαιρέσεως*), namely, things that our mental removal of other sensible characteristics—like "hardness and its opposite, and also hotness and coldness, and other pairs of contrary perceptible attributes," "leaves behind."[13]

If we look for bodies that have typical shapes, like oval, roundish, or oblong, our looking ignores, or has "stripped off" all characteristics—including heaviness—except shape. If, instead, we were preparing to look at bodies with Archimedes' vision, we'd perform almost the same stripping-off. However, in addition to shape we'd leave behind weight—a characteristic that does not, like shape, manifest itself fully to the eye.

Second, Archimedes shares with Euclid—and with our previous consideration of the limit-standard of length—a vision of ideally exact limit-formations. For instance, in the Archimedean Figure 5 below, as we do with Euclid's figures, we take it as imaging an ideally exact straight line, on which there are three ideally exact points,[14] as indicated. But, in addition, we shall take the respective *sizes* of the two rectangles as *indicating* an *exact* relationship between the weight of **A** and that of **B**.

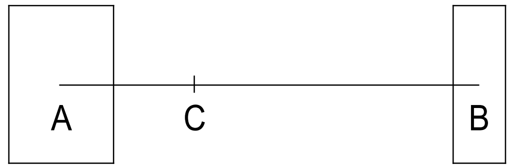


Figure 5

In particular, if we take the straight line *AB* in Figure 5 as representing a beam and *C* as representing the tip of a fulcrum, then we'll say that the figure images an *ideally exact limit-balance*. The treatise will show us what *must* happen when we weigh heavy bodies on such an ideally exact limit-balance.

Furthermore, while Archimedes' rectangular figures are, like Euclid's, intended to be *images-and-likenesses* (*εἰκόνες*) of exact *shapes*, in this lecture we shall focus on them only as *indices* of relative *amounts of heaviness*. That is, we shall neglect the shapes of the two rectangles **A** and **B** *as* shapes and attend only to their sizes, which will be taken to indicate the relative weights of the two represented bodies.

We shall interpret not only Archimedes' figures of weights but also bodies in the laboratory as indices of relative amounts of weight. Many of the relevant features of indices are borrowed, with some modification, from C. S. Peirce's treatment of the *index*.[15] The connection of likeness between the recognized object and the thought of the Thing itself, in the experience of recollection, is replaced, in the case of an index, by a direct, factual, non-arbitrary space-time connection between the index and the indicated thing. Here are some examples—many of which are experienced by animals—of indices and what each points to: rock – hardness; frost – coldness; dark clouds - impending rain; (for a fish in the sea) direction of greater light - warmer water; limping gait – physically impaired animal; scowling human facial expression - displeasure or concern; particular way of pronouncing a word - particular geographic place or social group.[16]

As can be seen from those examples, the "naturalness" of the connection between pointer and pointed-to may involve *innate knowing* in some cases, in others, *learning through experience*[17] (neither through teaching nor through insight)—including the experience of "artificial" correlations, such as, between a beep from the oven and the cookies' readiness to be taken out or between a red traffic light and the urgency of stopping the car. Rather than "seeing" what is being pointed to, we may feel an urge to, "hear" an invitation to, or tend to make an inference to it—for instance, at the sights of smoke or footprints, at the sounds of thunder or of the doorbell ringing, or at the tastes of flavors. Thus, the correlation need not be perfect, in order for a thing to act as an index.

Our earlier experiences both with weighing on the substitution balance and with suspending pieces of cardboard from pins through holes, as well as our everyday experiences of lifting bodies and our childhood play on seesaws, all contributed to our having learned through experience, or associatively, the connections between, on the one hand, Archimedes' rectangles or the bodies we work with in the laboratory and, on the other hand, the weights that they indicate. Contrary to mathematical symbols, these rectangles and those bodies indicate the relative amounts of heaviness in an *inherent* way; *bigger* means *heavier*, in some rough sense. But it is a way that is in between being an image-and-likeness—it does not resemble its signified object, in this case, heaviness—and being arbitrarily assigned to signify heaviness. Whereas recollection was a two-in-one experience, indication is an experience of duality, but of a "naturally" connected duality.

There is a meaningful connection between magnitude of size and magnitude of weight, but the relationship between the size of the rectangle and the magnitude of the weight has to be stipulated if there is to be an exact account. If we *sup-pose* that inherent connection, then Archimedes may lead us to recollect Equilibrium Itself. So, for instance, the ratio of the sizes of the rectangles **A** and **B** in Figure 5 is taken to indicate the exact ratio of the weight of body <u>**A**</u> to that of body <u>**B**</u>.[18] Our X-ray vision then shows us that the weight of any body at all could be indicated by the same rectangle **A**, provided its center of weight were at *A* and its weight were equal to that of **<u>A</u>**.

For the first time in the measurement segment, we are using things that we have recollected in order to arrive at further insights into relationships within what has been recollected. This is a third way in which Archimedes is following the Euclid's way of seeing; he shows how a sequence of conclusions can be generated from the ideal-exact limit-formations. For as the arithmeticians do based on "the odd and the even" and as the geometers do based on "the geometrical shapes, the three kinds of angles," etc., Archimedes "is forced to seek on the basis of sup-positions (*ἐξ ὑποθέσεων*), proceeding not to a beginning but to an end." The conclusions about the equilibrium of weights are "in agreement with that from which [he] set [his] inquiry in motion" (*Rep*. 510bd; Sachs trl., modified). In that way Archimedes engages us in learning through insight.

The first part of Archimedes' title is "On the Equilibrium of Planes." In some of the postulates and propositions,[19] we read: *to be in equilibrium,* or, equivalently, *to sink,* or *incline, equally,* and ii) *to sink*, or *incline, toward*[20]—which latter seems to be a specification of *not to be in equilibrium*. Archimedes does not follow Euclid's procedure by stating precisely what it is for weights to be "in equilibrium" or, for that matter, for a point to be "held fixed" or to be "the center of weight."[21] Perhaps for the first two of these expressions, he relies on the reader's everyday experiences with things like seesaws; for the third he may be counting on the reader's having played around with suspending objects from strings, just as we did in the earlier exercises. Or one might adopt a view introduced in the last century[22] and claim that Archimedes defines these phrases *implicitly*. Then the statements in which he uses them would aim to specify, without ambiguity, how they are to be interpreted in terms of other, familiar terms.

He also often[23] mentions equal or unequal weights *at* certain or equal or unequal distances, before making explicit, in Prop. 4, that each weight is taken to be "at" its center of weight and that each distance is from a body's center of weight to what the manual calls "a pivot"—what was referred to earlier as the "point of suspension" of the cardboard or the "fulcrum" of the substitution balance.

Whereas *length* and *weight* are characteristics of individual bodies, *being-in-equilibrium* specifies a relationship among five entities. For the setting-out of the above figure, involving bodies **A** and **B**, they would be—the two weights, **A** and **B**, their respective centers of weight *A* and *B*, and the pivot point *C,*[24] on the rigid, straight, weightless beam *ACB.*

The first postulate of the treatise states "that equal weights at equal distances are in equilibrium, and that equal weights at unequal distances are not in equilibrium but incline toward the weight which is at the greater distance." The insight expressed here is of a kind familiar from Euclid. Recalling what the students had said in relation to the substitution balance, we see that if we exchanged the two equal weights with each other, it could make no difference to anything at all. Everything would be the same either way.

In addition to the X-ray vision involved in seeing centers of weight, Archimedes introduces a few other ways of looking. Prop. 4 mentions "*the magnitude composed from both magnitudes.*"[25] When we first glance at Figure 6 below, we see two separate bodies, **A** and **B**, of weights **A** and **B**, respectively, and their respective centers of weight, *A* and *B*.

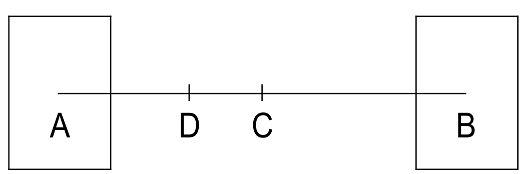


Figure 6

Then, in a second, *compositional looking*, we see "the magnitude composed from both magnitudes," that is, we imagine **A** and **B** as composing two parts of a single body, say, **A/B**, in which they are joined by a rigid, straight, weightless beam, *AB.*

In this way we are thinking of **A** and **B** as forming a shape reminiscent of the odd cardboard

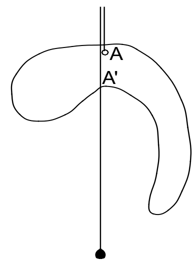


Figure 7

shape from the beginning of Ch. II, shown again in Figure 7. We then imagine molding that cardboard (illustrated on the left side of Figure 8)—simultaneously shrinking and straightening its central portion into a straight line and shaping the two end portions into congruent rectangles—all the while still viewing it as one object, so that we end up with the shape on the right side of Figure 8.

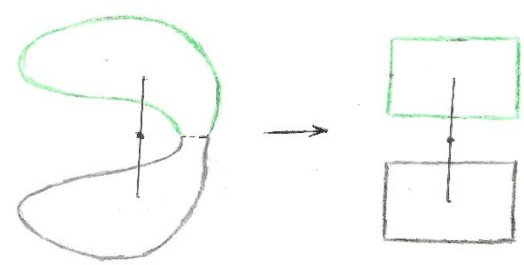

Figure 8

Another way of looking is the reverse of the above, namely, *decompositional looking*. Just as we can compose, so we can decompose. We can view a single body as two separate ones.

Archimedes sometimes accompanies compositional or decompositional viewing with a third mode of looking. In the proof of Prop. 4, he shifts from viewing a point *as* the center of weight of a composite body **A/B** to seeing that same point *as* a pivot about which the two bodies **A** and **B**, into which **A/B** is considered to be decomposed, are in equilibrium.[26]

In these ways Archimedes is inviting us to shift back and forth, at will, between ways of looking. First, like *seeing* Figure 9 now *as* a duck and now *as* a rabbit, we can *consider* a figure *as* depicting *either*

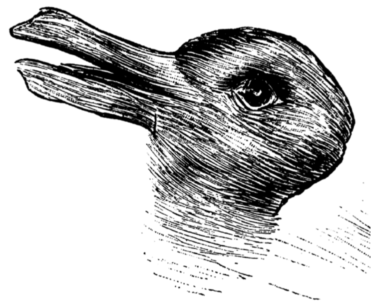

Figure 9

two separate bodies **A** and **B** *or* one composite body **A/B**. Second, like *seeing* Figure 10 now *as* an old

Figure 10

woman and now *as* a young one, we can *consider* a particular point *as either* a center of weight *or* a pivot point for equilibrium. Whereas, for a viewer who actually does see them, the shifts back and forth between seeing the duck or the rabbit or between seeing the young or the old woman—each of which is a genuine *seeing*—are involuntary, for us the shifts Archimedes prescribes are voluntary; and each of them might be more a *viewing-as* or even only a *thinking-of*.[27]

In Prop. 6, in a virtuoso display of all of his ways of looking, Archimedes shows us that what the students had unthinkingly assumed in working with the substitution balance, namely, the so-called "Law of the Lever"—that "bodies will be in equilibrium at distances from the pivot which are reciprocally proportional to the weights of those bodies" (M&E 20)—*must* be true of an ideally exact limit-balance.

Figure 11 shows two bodies **A** and **B** that are given as having commensurable weights, **A** and **B**,

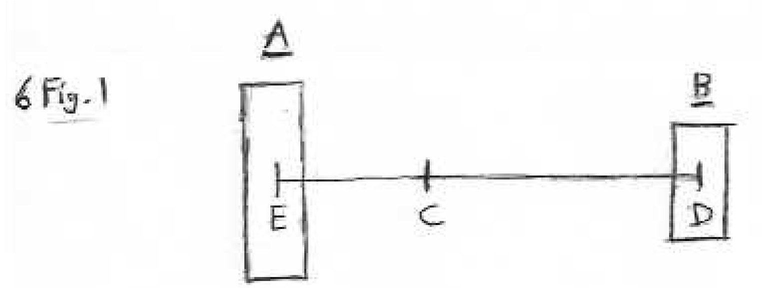


Figure 11

and as having their centers of weight on a straight-line beam at points *E* and *D*, respectively. Their weights are also given as having the following relationship to their respective distances from a point *C*, located on *ED* between *E* and *D* – $\overline{CD} : \overline{EC} :: \mathbf{A} : \mathbf{B}$. What is to be shown is that **<u>A</u>** and **<u>B</u>** are in equilibrium about point *C*.

The givens permit us to carry out the proof by assigning particular numbers to the distances and weights. In Figure 12 $\overline{CD}$ is shown as 5 units long and $\overline{EC}$ as 3 units, where each unit is labeled $\overline{N}$.

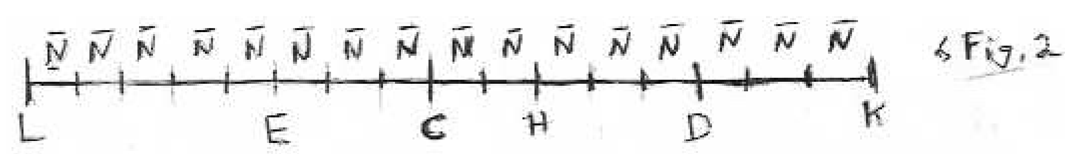


Figure 12

Moreover, in the construction $\overline{EC}$ has been extended to the left by 5 $\overline{N}$ and $\overline{CD}$ to the right by 3 $\overline{N}$, as pictured. In Figure 13 **<u>A</u>** is shown divided into 10 equal unit weights **<u>Z</u>**, and **<u>B</u>** into 6, respectively.

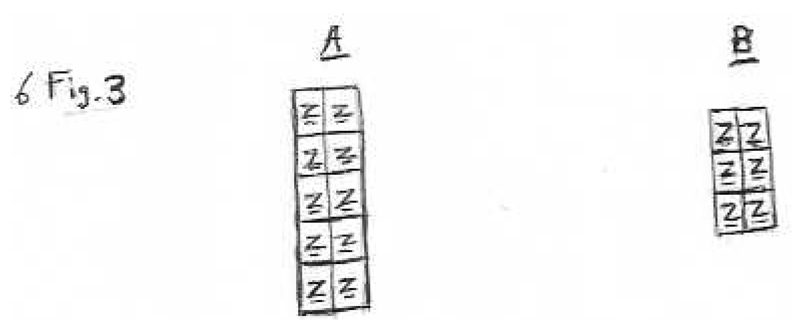


Figure 13

Then in Figure 14 the centers of weight of the 10 **Z** are shown as having been placed at the midpoints of the 10 $\overline{\mathrm{N}}$ on the left end of $\overline{\mathrm{LK}}$ and those of the 6 **Z** at the midpoints of the remaining $\overline{\mathrm{N}}$ on the right end of $\overline{\mathrm{LK}}$.

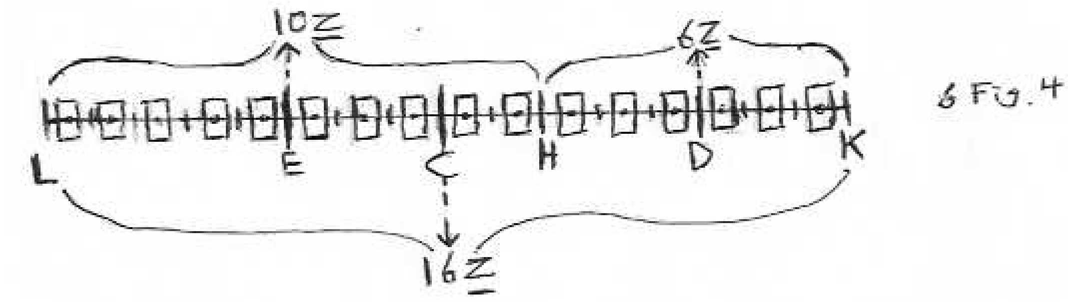


Figure 14

Now Archimedes simply has us look at these 16 **Z**, successively, in different ways and in relation to the given weights **A** and **B**:

As we look at the 16 equal weights **Z**, each at the midpoint of an equal length $\overline{\mathrm{N}}$, we see

i) on the left end of $\overline{\mathrm{LK}}$, five pairs of **Z**s situated symmetrically about point *E*, respectively;

ii) on the right end of $\overline{\mathrm{LK}}$, three pairs of **Z**s situated symmetrically about point *D*, respectively;

iii) on the whole of $\overline{\mathrm{LK}}$, eight pairs of **Z**s situated symmetrically about point *C*, respectively.

Hence, we see, *by the first postulate*, that each pair must be in equilibrium about the point about which it is symmetrically situated.

*By shifting from duck to rabbit*, we consider each of these pairs of **Z**s (ducks) as a composite weight (rabbits). *By simultaneously shifting from old woman to young woman*, we see that for each composite of two weights, the point of equilibrium (old woman) of the pairs in question—*E* for i), *D* for ii), and *C* for iii)—is its center of weight (young woman).

*By again shifting from duck to rabbit*, we consider each of the three sets of pairs of composite weights (ducks)— i) the five pairs on the left (from *L* to *H*), ii) the three pairs on the right (from *H* to *K*), and iii) the eight pairs on the whole (from *L* to *K*)—as itself a composite weight (rabbits). So—by an analogue to Euclid's first common notion: Composites of weights having the same center of weight also have that center of weight—we see that *E* is the center of weight of i) the composite of the 10 **Z**s, *D* of ii) the composite of the 6 **Z**s, and *C* of iii) the composite of the 16 **Z**s.

*By X-ray vision* we then see that

i) the composite of the 10 **Z**s is the same as **A** as originally given, since both have the same center of weight *E* and same total weight.

ii) the composite of the 6 **Z**s is the same as **B** as given, since both have the same center of weight *D* and same total weight.

So [by a common notion: Composites of pairs of weights (the first pair is the composite of the 10 **Z**s and the composite of the 6 **Z**s; the second pair is **A** and **B**) having the same respective centers of weight (*E* for the first member of each pair and *D* for the second) and pairwise equal in weight are equal and have the same center of weight.], we see that the center of weight of the composite weight **A/B** is also *C*, the center of weight of the composite of the 16 **Z**s, that is, of the composite of the 10 **Z**s and of the 6 **Z**s

Finally, *by shifting back from rabbit to duck*, we consider the composite weight **<u>A</u>/<u>B</u>** (rabbit) as two separate weights **<u>A</u>** and **<u>B</u>** (duck). And simultaneously, *by shifting back from young woman to old woman*, we consider *C*, the center of weight (young woman) of the composite weight **<u>A</u>/<u>B</u>**, as a pivot point for equilibrium (old woman). So, we see that: "therefore if **<u>A</u>** be situated at *E* and **<u>B</u>** be situated at *D*, they will be in equilibrium about the point *C*." *QED*

Given two bodies on a beam that counterbalance each other in the lab, we consider them, first, as counterbalancing on an Archimedean true Balance. That is, as we perceive the lab situation, there occurs to us, in recollection, the thought of two ideal-exact centers of weight and an ideal-exact pivot point on the true Balance. On the basis of our sup-position based on the experienced "natural" connection to ideal-exact weights and of his suppositions about the *μαθήματα*, Archimedes' Prop. 6 may have enabled us to learn through insight (*μαθεῖν*), precisely which relationships among those five entities associated with the two given weights are *necessary*, in order for them to be seen as a complex image-and-likeness (*εἰκών*) of what Socrates could call *Equilibrium Itself*—that at which our actual counterbalancing in the lab can be thought of as aiming, while still falling short. If so then we now expect that the closer our laboratory equipment comes to imaging an ideally exact limit-balance, the more exactly our experiments will manifest the Law of the Lever.

## PART THREE

At the beginning of Chapter III of the manual, we again take up the Archimedean approach to looking at situations of counterbalancing. But now, first of all, Archimedes has stretched the meaning of *being-in-equilibrium* to refer to two portions of a body of water pressing down upon certain "parts" of the water below them, instead of referring to two solid bodies on a beam about a pivot.  His treatise *On Floating Bodies* shows us how to consider this pressing-down as analogous to weights' tending to push downward on a beam and, thereby, to incline it to sink.

Second, he offers us a way of looking different from the X-ray vison and the compositional or decompositional looking familiar from the earlier treatise. We shall use this way of looking, in order to see, or imagine, certain things *into* given laboratory situations, almost as if we were looking at the latter through a PowerPoint transparency, placed over the actual things in the lab. For instance, as we look at a piece of wood floating in a bucket of water, we'll be able to use our imagination and *see* that a portion of it must be below the surface of the water and why it must be so. It might be as if the "giving-an-account of what is responsible" (*αἰτίας λογισμῷ*), which Socrates and Meno agreed to call "recollection" (*Meno* 97d-98a), were transformed into *an imagining* of what is responsible.

In the previous treatise Archimedes had us look at his figures with a voluntary considering-as, analogous to the *in*voluntary seeing of a figure as a duck or as a rabbit. As a result of that treatise, we were able to consider both drawings of his figures and certain beam and pivot arrangements in the lab as imaging the true Balance. However, the multiple shifts in looking required to see the truth of Prop. 6 are so elaborate that we are not in a position to execute them while we look at a given beam and pivot arrangement in the lab. That is, we can only "apply" Prop. 6, as when we refer to the Law of the Lever; we cannot "see" it immediately *in* the given beam and pivot arrangement.

In his second treatise Archimedes offers a recipe for looking that shifts what we can immediately see as true of a given object in a pail of water in the laboratory—without relying on the mediation of the statement of a proposition, which we remember having proven.

This new way of looking is introduced in Prop. 2, where Archimedes shows that "the surface of any liquid[28] which remains motionless," such as the water in a bucket, "will have the form of a sphere which has the same center as the earth." In this and the next few propositions, the treatise depicts the given liquid as a portion of a sphere, which is to represent an earth-size ball of liquid. If the given liquid were water in a bucket, we'd be imagining a ball of water the size of the earth and a tiny portion of it, which coincided with the water in our bucket.

Archimedes justifies this way of imagining by showing that if we supposed that the surface of the water in the bucket did *not* coincide with the surface of a sphere, then we'd be led to contradict ourselves. Figures 15 and 16 are two versions of a schematic picture of what we'd see under this

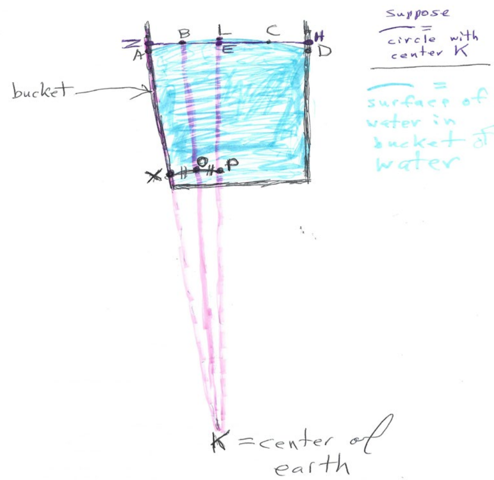


Figure 15

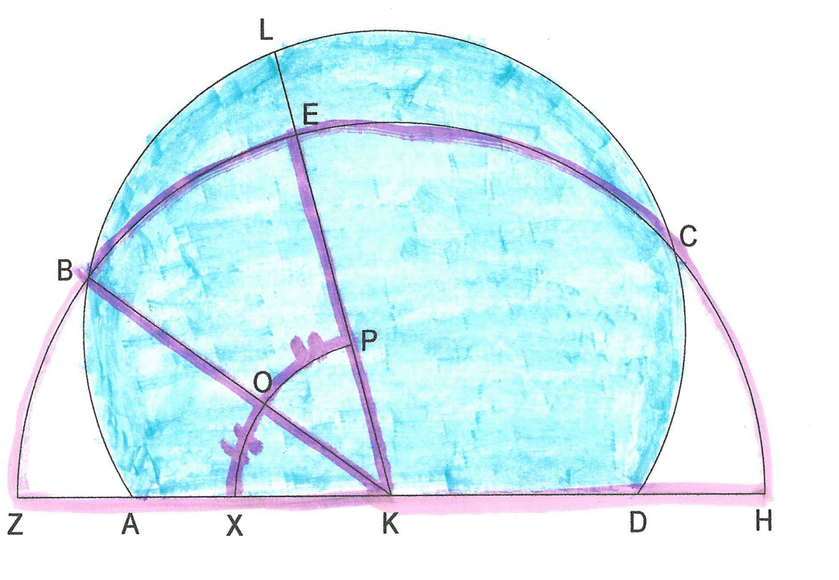


Figure 16

supposition.

If the surface *ABLCD*[29] of the water (colored blue) in the bucket, did not coincide with the surface of a sphere centered at the center *K* of the earth, then we could describe some *other* sphere, an outline of which is partially pictured in pink, as *ZBECH*, centered at the center *K* of the earth, passing through point *B* on the surface *ABLCD* of the water in the bucket. As shown, the sphere *ZBECH* would also pass through two other points, one (*Z*) above the water in the bucket and one (*E*) in the water. In this way the supposed spherical surface would be above the surface of the water in the bucket from *Z* to *B* and below the surface of the water from *B* to *E*.

In order to allow us to see the contradiction that would be lurking here, Archimedes has us look at two things, which we have imagined into the figures above: first, the two thin pyramids of water *ABK* and *BLK*; second, an imagined smaller sphere—part of which is shown as *XOP*–whose center is also *K*, and which cuts through the water in the bucket and intersects the pyramids in *XOP*. Then he allows the small spherical slice *KXOP* to play a role analogous to that of the beam-and-pivot in his previous treatise.

He has asked for (postulated) only one thing of us in this treatise—that we assume that a liquid, like water, is such that

> if its parts lie evenly and are continuous, that part which is pressed the less is thrust out by that which is pressed the more; and each of its parts is pressed by the liquid which is vertically above it, if the liquid is not enclosed in anything and is not pressed by anything else (M&E, p. 25).

In the above two figures, the pyramidal slice of water represented by *ABK*, which is inside the corresponding portion of the sphere (*ZBK*), is smaller than the slice *BLK*, which encompasses the corresponding portion of the sphere (*BLK*), congruent to *ZBK*. Hence, we can see that the part above *OP* is being pressed more by the water above it than is the part above *XO* by the water above it. Therefore, when applied to this situation, the postulate requires that the part *XO* be thrust out by the part *OP*, which means that the water has to be in motion—a contradiction of the given motionlessness of the water. Therefore, the supposition that the surface of the water in the bucket did *not* coincide with the surface of a sphere is necessarily false. *QED*

Archimedes goes on to show how imagining an object and a bucket as placed within and near the surface of an earth-size ball of water could lead to other discoveries. In the next proposition he supplies the fourth ingredient in his recipe for looking, namely, a shape imagined as being mentally outlined within one of the two pyramids in the liquid and as equal and similar to the shape of (the immersed part of) a given object.

Say, we look at a heavy piece of wood—which happens to be exactly as heavy as the same volume of water—being slowly lowered into the bucket, until it remains at rest exactly where it happens to be when released. *Suppose* that this remaining-at-rest happened *before* the wood had been completely immersed in the water, as depicted in Figure 17, and that at the instant when the lowering stopped all motion ceased in the bucket.

Now imagine looking at this piece of wood through a transparency of an earth-sized globe of water, positioned so as to intersect with a plane through the center of the earth in the blue hemi-sphere *KALBCMRYND* and having "the form of a sphere which has the same center as the earth" (Prop. 2, enunciation). In what follows what is pictured is always the intersection with that same plane

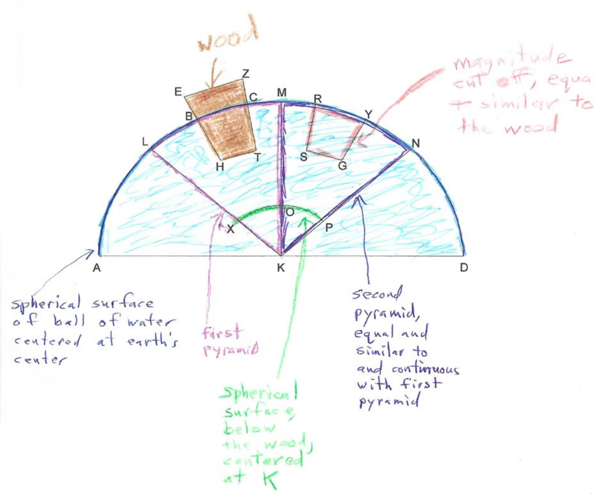


Figure 17

through the center of the earth. The brown figures *BEZC* and *BHTC* represent, respectively, the parts of the wood above and below the surface of the water.

Then imagine over the transparency of the globe of water *another* transparency with two adjacent pyramids, “equal and similar to” and “continuous with” each other and each “having for its base a parallelogram [lying] on the surface of the liquid, and its vertex the center of the earth” (Prop. 3, proof). The left pyramid, *KXLBCMO*, is shown containing the submerged part, *BCTH*, of the wood.

In the right pyramid, *KOMRYNP*, *we* imagine a *third* transparency, showing “a magnitude [*RYSG*] … cut [out] from the liquid, equal and similar to the [submerged] part of the” wood, outlined in pink and in the same relative position, in relation to the right pyramid, as is the submerged part of the wood, in relation to the left pyramid.

Finally, we imagine, over the other three transparencies, a *fourth* one (outlined in green in Figure 17) containing the surface, *XOP*, of a portion of a smaller sphere cutting through the bucket, below both the wood and the cutout, and also centered at the center *K* of the earth.

Having superimposed such a detailed set of transparencies, we look at the one having the two pyramids. It is clear that each would contain two pairs of portions of things pressing-down equally—namely, the submerged wood *HBCT* and, seen through the third transparency, the congruent cut-out *SRYG*, respectively; and the remaining pyramidal portions *XLBHTCMO* and *OMRSGYNP* pressing equally—on the left and right “parts,” *XO* and *OP*, respectively, of the surface of the smaller sphere, shown on the fourth transparency. But it is also clear that the portion of the wood protruding above the water, *BEZC*, would be pressing on *XO* without being paired with anything additional pressing on *OP*. So, we *see* that *OP* would be “that part which is pressed the less.” Thus, by the postulate *OP* would be thrust out by *XO*, so that there would be motion in the water—a contradiction of the supposition that the water remain motionless. Therefore, the piece of wood *could not* have come to rest before it was completely immersed in the water. *QED*

Here Archimedes has employed all the ingredients in his recipe for looking, namely, the four transparencies through which we look at the actual object in the bucket. They show, respectively:

> a) *a spherical earth-sized globe of water*, centered *at the center of the earth*, a small portion of the surface of which *coincides with the surface of the water in the bucket*,
>
> b) *two equal and similar, adjacent pyramids of water*, cut from the globe, continuous with each other, and having vertices at the center of the earth, in the left of which is the immersed (part of) the object,
>
> c) in the right pyramid, *a cutout of water equal and similar and similarly placed* to the immersed part of the object, and
>
> d) *a spherical surface*, with center also at the center of the earth, and which cuts through the water in the bucket, near its bottom.

In contrast to the components of the figures in the first treatise—beam, pivot, and weights—these ingredients are *mental additions* to the things actually visible in the laboratory and to what a representation of them would show. The objects on the transparencies are neither images-and-likenesses nor indices of the bodies in the lab.

As we are looking we add them mentally to any given object-in-bucket situation in the lab that we wish to understand, such as those to which the next few propositions in the treatise might refer—for instance, a piece of wood taped to a piece of metal that stays wherever it is released in the water, some wood floating partly above the surface of the water, or a metal block that comes to rest at the bottom of the bucket.

Imagining the first transparency containing the globe of water and imagining the other transparencies laid over it will allow us to *see* clearly, in various situations in the lab, whether or not, with the specified givens, the left or right "part" would be being pressed more. For all we need do is look at matching pairs of congruent shapes in the left and right pyramids and look for anything else that might be pressing down on either of the two arcs that does not have a match on the other side. This is the second step in cooking with the recipe. We *think about* what we see through the transparencies, in the bucket of water. We can see in the mind's eye whether or not there is equal pressing right and left. In the final step we may consult the postulate and see with insight whether or not, based on it, either "part" must be "thrust out."

When we first looked at the wood and bucket of water, we did not have anything like an earth-sized globe of water or pyramids in mind. In the first step Archimedes' recipe leads us to look at the bucket *while imagining* the transparencies overlaid onto the actual bucket. So, in using this treatise, in addition to recognizing the wood and bucket of water, we are thinking and imagining all that is in the four transparencies.

We might say that one who is following Archimedes' recipe "sees a thing according to an interpretation,"[30] or, rather, several interpretations, as instructed by Archimedes. Each transparency might be like a textbook illustration, which would always be accompanied by an interpretation. We can *imagine* each transparency as superimposed over the sight of the bucket. That will enable us to *think* of the transparency's figure as being "in" the bucket. We may then *view* that figure *as* ingredient in the bucket. Perhaps, also, we might *see* it in this way.

In the second step we *think about* what we have thought and imagined onto the wood and bucket of water. By viewing the latter through the transparency, we can see in the mind's eye whether or not there is equal pressing right and left, and if not, we likewise see, in the third step, that the postulate requires there to be motion and that the supposition of motionlessness has been contradicted.

Is it the case that familiarity with Archimedes' recipe for looking is a recipe for essential seeing, that is, for seeing what is essential in certain situations? Does it enable an imagining-and-thinking that, in turn, allows something essential to be *seen* and a truth to be *recollected*?

As noted near the beginning of the lecture, the experience of recollection is a *two-in-one* experience, like that of visually recognizing something as a face and simultaneously having a thought of it as Simmias's face. It is seeing a straight line on the blackboard as an image-and-likeness (*εἰκών*) of the Straight Itself. When we follow Archimedes' recipe, our experience seems rather to be a *dual* one of recognizing the water, the bucket, the objects, and, separately, of imagining-and-thinking. However, it is a dual one in which the correlation is due entirely to Archimedes' instructions; we have had no experiences of pyramids of water, cut from the globe, or of cutouts of water from which we could have learned, associatively, to connect them to an object in a bucket of water. As opposed to the "naturally," or inherently, connected duality of index and indicated, this duality is externally superimposed.[31] We "lay" the figures from the transparencies over the figure of the bucket of water on paper or over our sight of it in the lab.[32]

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[1] A Friday Night Lecture, delivered at St. John's College, Annapolis, on February 25th and 26th, 2022; revised, April 17, 2022.

[2] This kind of mathematization is independent of the "new mathematics" of Viète's algebra and Descartes's analytic geometry. So, no equations are involved.

[3] All page references to the manual are to a revised version, entitled *MEASUREMENT AND EQUILIBRIUM: Archimedes' and Pascal's Ways of Looking at Weight, Water, and Air*, edited especially for a Graduate Institute Preceptorial offered in Santa Fe in the summer of 2021.

I would like to thank all of the members of my Freshman Laboratory classes in the last several years and, especially, the participants in the Graduate Institute Preceptorial in Santa Fe in the summer of 2021: Isabel Ballan, Bill Blais, Patricia Burk-Travis, Nicolae Federspiel-Otelea, Hugh Himwich, and Susan Olmsted,.

[4] In calling *1* a number, we are diverging from the strict sense of "number" as an assemblage of units.

[5] The way our table stands (σχέσις; Euclid, *Elements*, Bk. V, Df. 3) relative to the length of our new smallest unit is nearly the same as the way the number 123 units or 124 units stands relative to one unit.

[6] This approach was proposed by the phenomenological philosopher Edmund Husserl in "Die Frage nach dem Ursprung der Geometrie als intentionalhistorisches Problem." See *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy*, tr. D. Carr (Northwestern: 1970), trl. modified/*Die Krisis der europäischen Wissenschaften und die transzendentale Phänomenologie* (Haag: Martinus Nijhoff, 1962), pp. 353-78/365-86. The quotations in the following two paragraphs are from this work.

[7] "From the praxis of perfecting, of freely pressing toward the horizons of conceivable perfecting in the 'again and again' [*im 'Immer wieder'*], limit-shapes become sketched out, toward which, as invariant and never attainable poles, the particular series of perfectings is progressing" (Husserl, *ib*., 26/23).

[8] Our imagining of this progress and our thinking of this standard leads us to view the standard in the way we may have looked at a chalk line on the blackboard in the Freshman Mathematics Tutorial. We'd have seen it as imaging a line that matches Euclid's definition of straight line—perfectly sharp, without thickness or waviness or fuzziness. All the shapes referred to by Euclid were ideally sharp in this sense. Whenever we look at a drawing of one of them, what we are invited to think of is characterized by ideal sharpness. (We could perhaps arrive at an "ideal visual acuity" in the same way.)

[9] The example is borrowed from Ludwig Wittgenstein, *Philosophical Investigations* (NY: Macmillan, 1953), tr. G. E. M. Anscombe, p. 193.

[10] Nevertheless, the manual later (M&E, p. 19) proposes an exercise in which, simultaneously, the cardboard is replaced with a meter stick and an axis through a hole drilled at the 50 cm. mark substitutes for a pin through a potential center of weight of the cardboard. By adding or removing small amounts of clay as needed, the students are able to bring the meter stick into the three different kinds of rest position. In what way, if any, does this exercise deepen our insight into the center of weight or of the possible rest positions?

[11] Here and throughout the treatment of Archimedes, the manual's symbols are changed.

[12] "Center of weight" is mentioned in some of the postulates (4, 5, 7) and in several propositions (e.g., 4 and 5) and proofs (e.g., of 6 and 7).

[13] The translations of Aristotle generally follow those of Joe Sachs, in *Aristotle' Metaphysics* (Green Lion Press, 1999), K.3, 1061a29-b4. See also Sachs's note on p. xlix.

[14] The points *A* and *B* represent, of course, the two centers of weight, at which the two bodies **A** and **B** are thought of as suspended.

[15] Cf. C. S. Peirce, *Collected Papers of Charles Sanders Peirce* (8 Vols.). (1931–58). Ed. Charles Hartshorne, Paul Weiss & Arthur W Burks. (Cambridge, MA: Harvard University Press), 4.447: "The kind of representamen termed an index. … is a real thing or fact which is a sign of its object by virtue of being connected with it as a matter of fact and by also forcibly intruding upon the mind, quite regardless of its being interpreted as a sign. It may simply serve to identify its object and assure us of its existence and presence. But very often the nature of the factual connexion of the index with its object is such as to excite in consciousness an image of some features of the object, and in that way affords evidence from which positive assurance as to truth of fact may be drawn. A photograph, for example, not only excites an image, has an appearance, but, owing to its optical connexion with the object, is evidence that that appearance corresponds to a reality. … An icon has such being as belongs to past experience. It exists only as an image in the mind. An index has the being of present experience."

[16] Most of these examples, as well as some in the next paragraph, are taken from https://legacy.cs.indiana.edu/~port/teach/103/sign.symbol.short.html.

[17] A special case of learning through experience is getting the hang of seeing photos as photos, and not as mere patterns on paper. This kind of learning through experience is relevant to the image-and-likeness, rather than to the index.
[18] Using three different symbols involving the letter "A," we may say: "Body **A**, having center of weight at point *A*, and of weight indicated by rectangle **A**…" The same rectangle is taken both to represent a body and to indicate the amount of that body's weight.
[19] See Postulates 1-3 and 6 and Propositions 1-4.
[20] The manual points out (14, n. 1) that the Greek word for being-in-equilibrium *(ἰσορροπεῖν)* could also be translated inclining-downward-equally. The manual has already sometimes used "downward tendency" as equivalent to "weight" (M&E, p.9). Perhaps we are to imagine that two bodies, each tending or striving to move downward, are joined by a weightless beam-and-pivot arrangement and are each endeavoring, in a struggle against one another, to incline the beam downward on its side of the pivot, and, in the case of equilibrium, are doing so with equal effectiveness. When one of the two wins the contest, the beam will come to sink, or incline, toward (*ῥέπειν ἐπί*) the side on which the winner is located.
[21] For instance, Prop. 3 claims that "two weights [**A** and **B** in the above figure] will be in equilibrium" with each other. Prop. 4 takes *A* to "be the center of weight of magnitude **A**" and proposes that *D be* "held fixed."
[22] Padoa, A., "Essai d'une théorie algébrique des nombres entiers," *Bibliothèque du Congrès international de philosophie* (1900) 3.
[23] For example, Posts. 1, 2, and 6; Props. 1, 2, and 3.
[24] If, inspired by Archimedes, we look at two weights in the lab, with the question of their being in equilibrium in mind, we can consider them as if they were situated at the ends of an ideal-exact straight-line, imaged by a seesaw on a playground, and ask: About which pivot point on the seesaw will neither side sink down. This Archimedean looking could be called *seesaw vision*. We, as it were, see an ideal-exact seesaw into the given actual situation—imagining the weights' centers of weight at its opposite ends and indicating their amounts by the sizes of the respective rectangles.
[25] Or: "of both <u>taken together</u>." See T.L. Heath, *The Works of Archimedes* (NY: Dover Publications, Inc., 1912), p.191; underlining added.
[26] If *D* is "the center of weight of a magnitude [**A**/**B**] composed" of two magnitudes [**A** and **B**], then the composite magnitude "will be in equilibrium "if *D* is held fixed"—the manual's footnote adds: "that is, as a pivot."

When we first arrived at the notion of a center of weight as a point on the piece of cardboard, at the beginning of Ch. II, we stuck a pin horizontally through it and showed that it was then also in a neutral rest position after any rotation about that point. That is, from a point's being the center of weight of a body, it followed that the body would be in (neutral) equilibrium about that point. Archimedes appears to be in agreement with this suggestion that being a center of weight is the more fundamental notion and that being in equilibrium about a point follows from it. Thus, it makes sense for the manual to have introduced the former before the latter. It is easy to see the truth of the converse of Archimedes' assumption, namely, that if equal weights are in equilibrium about a point, then that point is the center of weight of the weight composed of the two equal weights.
[27] In one paragraph Wittgenstein asks the reader to consider, as an example of "the danger of wanting to make fine distinctions," the aspects of a triangle"—such as, seeing it "as a triangular hole, as a solid, as a geometrical drawing; as standing on its base, as hanging from its apex; as a mountain, as a wedge, as an arrow or pointer." In the next paragraph he gives the response of an interlocutor: "'You can in doing so <u>think</u> now <u>of</u> *this*, now of *that*, can <u>view</u> (*ansehen*) it now <u>as</u> *this*, now as *that*, and then you will <u>see</u> it now *in this way*, now *in that way.*'" Wittgenstein immediately points out that it would be an error for the interlocutor to think that there is some way to make the *this* or *that* more determinate, by spelling out further *what* you are thinking of or that as which you are viewing it or the *way* in which you are seeing it: "— *In which way*? There is as a matter of fact no further determination." See Wittgenstein, *op. cit.,* translation modified and underlining added, p. 200.
[28] "The Greek word is derived from the adjective ὑγρός, which means 'moist' or 'wet'" (M&E, p 25n).
[29] In what follows we often speak of three-dimensional shapes, like spheres or pyramids. But, for convenience, all of the labels are of the shapes' intersections with a plane passing through the sheet of paper, or the computer screen, as the case may be.
[30] This example is borrowed from Wittgenstein, *op. cit*., p. 200, but used in a different sense.
[31] In that respect it is like Peirce's "symbol." See Peirce, *op. cit*., quoted above.
[32] The above analysis of Archimedes' second treatise would also apply to Pascal's *Treatise on the Equilibrium of Liquids*. Pascal uses one transparency. The figure on it is one or another version of VI, VII, or VIII in Figure 18 below.

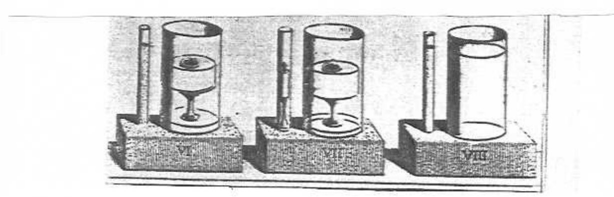


Figure 18

The box may be imagined to be filled with a weightless liquid (M & E 41: "without counting the weight of the water"). The large weights on the right are each of 100 pounds, resting on a piston, over the right-hand aperture; the small one in VII is 1 pound. The ratios of the right aperture to left aperture are 1 : 100. The heights of the liquids are all the same. The weight of the liquid in the large tube in VIII is 100 pounds; that of the liquid in the narrow tubes in VI and VIII is 1 pound. This Pascal box-and-tubes transparency may be laid over situations like those in IX, X, XI, and XII in Figure 19—representing vessels partially submerged in a river, and containing mercury or copper or cork, in part—in order to see what

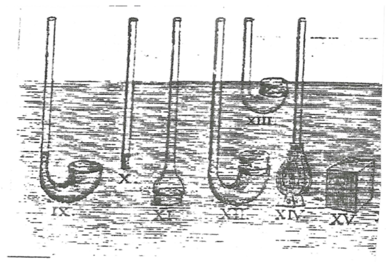


Figure 19

*must* happen. In laying it over, we must mold the narrow tubes of the vessel and of the Pascal-transparency, so that they are exactly superimposed; and, similarly, for the vessel's wide opening and the Pascal-transparency's wide tube, and so on, as suggested by Figure 20.

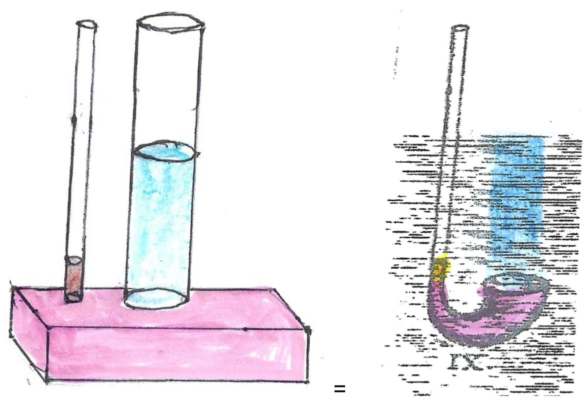


Figure 20

Figure 21 may suggest a way of relating the Pascal transparency to those of Archimedes.

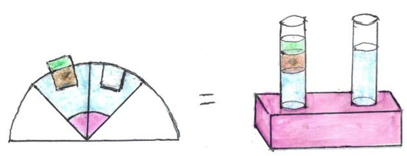


Figure 21