Alternating Current[1]

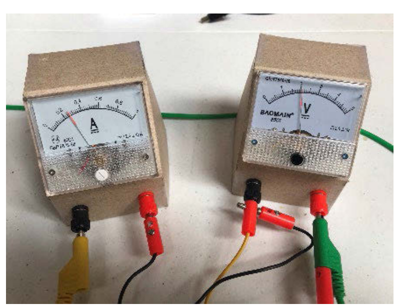
Howard J. Fisher

Although the topic of my talk is alternating current, I'd like us to begin with a few experiments using direct current. Direct current is what batteries produce, but we shall obtain it from one of our venerable "EFB" power supplies.

These preliminary experiments will exhibit Ohm's Law, the law that asserts proportionality between the voltage applied to a conductor and the current that flows in it. Nothing in the ideas of voltage or current themselves suggests that they should be proportional, nor does Ohm's Law in any way explain the proportionality when it does exist. We shall seek an explanation; but let us first observe that proportionality in practice.

The "lead" in a lead pencil[2] is a moderately good conductor of electricity. I have sharpened this pencil at both ends and connected it to the variable supply. I have a

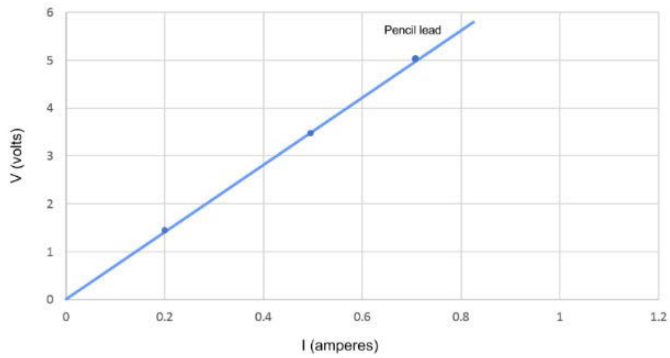
voltmeter to measure the voltage and an ammeter to measure the current. Watch as I vary the applied voltage**...**



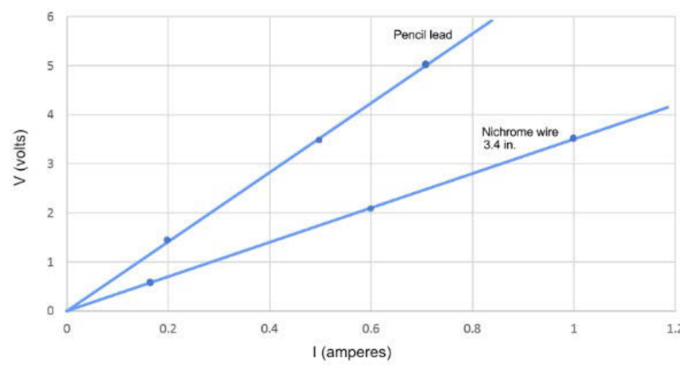
[1] Lecture given at St. John's College, Santa Fe, on 26 April 2023

[2] Pencil lead consists primarily of carbon in the form of graphite.

And here is a graph of the measured values. The straight line shows that the voltage is indeed proportional to the current.



Another conductive material is nichrome, an alloy of nickel and chromium. If we perform the same measurements on 3.4 inches of nichrome wire, we again get a straight line—but the slope of the line is smaller, meaning that less voltage was required to

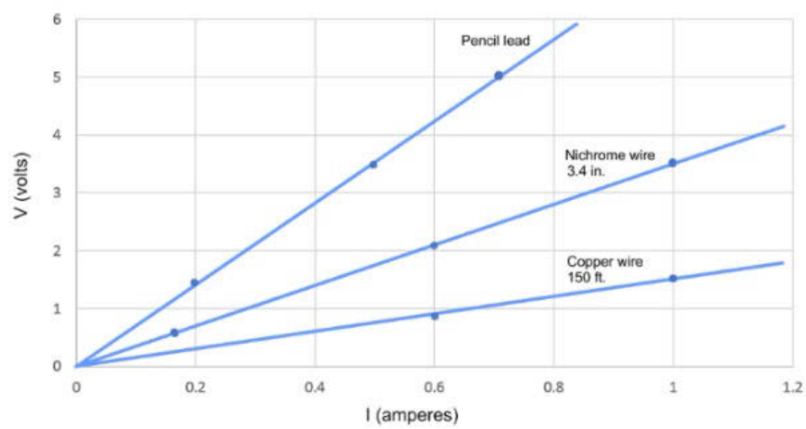


maintain the same current. Our nichrome sample is evidently a better conductor than the pencil; but both exhibit the proportionality that constitutes Ohm's Law.

In this way we can rank any number of different conductors as better or poorer: better conductors yield a gentler slope, poorer conductors yield a steeper one. The slope of the line, expressed as the quotient of voltage by current—V/I—is a measure of a conductor's imperfection. Modern terminology calls it the *resistance*; and if we represent resistance by the constant R, the equation $R = V/I$ is a mathematical expression of Ohm's Law.

We do not always prefer better conductors to worse ones. When we wish to regulate current, for example, imperfect conductors are often exactly what we need; and since they then serve a positive purpose, they enjoy a positive name: instead of calling them "imperfect conductors" we call them "resistors." The slide shows resistors from the 1920's, the 1960's, and today.

Most purposes, however, do call for good conductors; and among the best of these is copper. Copper is such a good conductor that 150 *feet* of number 20 copper wire has less resistance than 3 *inches* of our nichrome wire. Here is the graph. Once again we have a straight line; showing that copper wire, too, exhibits the proportionality that



constitutes Ohm's Law. As before, the slope measures the resistance of the sample; and note that even an excellent conductor like copper has some resistance—no conductor is perfect. "Conductors" and "resistors" are not different species; they are all conductors, they differ only in degree.

Measurement exercises like these are what nearly all instructional laboratories rely on to introduce Ohm's Law; but they contain a logical flaw, which hardly anyone ever bothers to acknowledge. The problem is that ordinary voltmeters include an internal resistor, and the scale markings of the meter are calculated by the manufacturer on the basis of that resistor's value. Thus the very design of the voltmeter already *assumes* Ohm's Law; so we commit circular reasoning if we rely on that instrument to establish Ohm's law. Ohm's Law actually rests on a sounder basis: it is the consequence of relations of energy.

Maxwell defined voltage (his term was difference of potential) as the energy required to transport one unit of electricity from one point to another, that is,

$$V = \frac{E}{q},$$

where E is the energy and q is the quantity of electricity transported. But current is the rate of electricity transfer per unit time, or

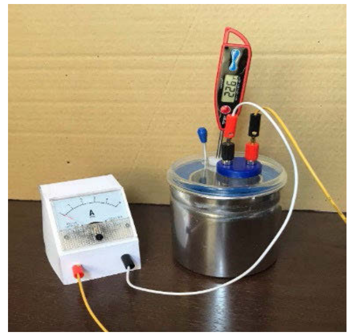
$$I = \frac{q}{t}.$$

Then the product of voltage and current will be

$$V \cdot I = \frac{E}{q} \cdot \frac{q}{t} = \frac{E}{t}.$$

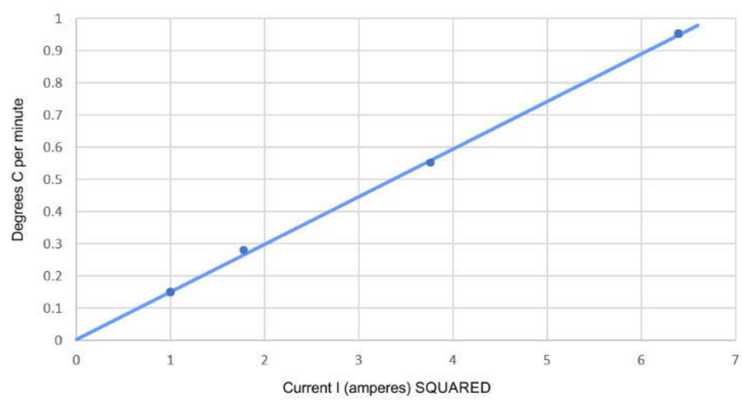
Thus E is the (electrical) energy expended to maintain current I for time t.

But whenever an electric current flows through a resistance, it generates heat. We can measure the amount of heat produced by immersing the resistive element in a water calorimeter, and noting the temperature rise just as freshmen do in the first semester laboratory. We shall use nichrome wire as the resistance element, shown here on the left;



and on the right is the electrical setup. Notice that it does not use a voltmeter at all, so no question of circularity arises.

We measure the temperature rise produced during the time of current flow; and look what happens when we graph the *rate* of temperature rise against the *square* of the current: we obtain a *straight line*, showing that the heat produced per unit time is proportional to



the square of the current. Expressing these results algebraically, if H is the heat produced in time t,

$$\frac{H}{t} \propto I^2, \quad \text{that is,} \quad \frac{H}{t} \propto \frac{q^2}{t^2}\ ;$$

or, expressed as an equation,

$$\frac{H}{t} = K\frac{q^2}{t^2}$$

where K is a constant of proportionality. Rearranging,

$$\frac{H}{q} = K\frac{qt}{t^2} = K\frac{q}{t}.$$

But recall Maxwell's definition of potential difference, which we saw earlier:

$$V = \frac{E}{q},$$

where E was electrical energy. Where does this electrical energy go? If it is converted to heat, then E will be equal to H; and if that is so, we can combine the last two equations to obtain

$$V = \frac{E}{q} = \frac{H}{q} = K\frac{q}{t} = KI \qquad \text{or} \qquad K = \frac{V}{I}.$$

But this is the very proportionality that constitutes Ohm's Law, so the constant K must actually be the resistance R. Thus Ohm's Law—that is, the proportionality between voltage V and current I—arises insofar as *electrical energy is converted to heat*. This conversion of energy is the explanation that Ohm's Law stood in need of; and it is the real foundation of that law.

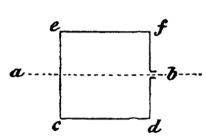
Then since Ohm's law can be independently demonstrated for the resistor which ordinary voltmeters contain, our previous use of the voltmeter was perfectly legitimate; and we need have no hesitation in employing the voltmeter as a measuring device. But remember that whenever we do so we are *assuming* Ohm's Law. Consequently, whenever we employ the voltmeter in new circumstances we must be sure to ask whether Ohm's Law has been shown to apply in those circumstances.

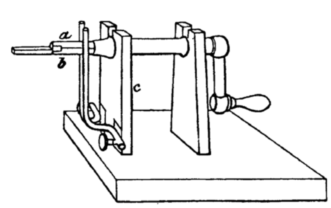
{II}

Now as I mentioned at the outset, the current used in our previous exercises was what we now call "direct current" or DC. Direct current flows consistently in one direction. But as most of you probably know, the kind of electric current we most often make use of in ordinary life, the current that energizes our household appliances and which supplies most of our indoor and outdoor lighting, is *alternating current*. Its prevalence alone would make alternating current worthy of study from a societal point of view; but our interest will be different. As we shall see, alternating current reveals several electrical phenomena whose existence we might never have suspected had we been restricted to direct current alone. Phenomena related to these prompted Maxwell to pursue some highly consequential analogies, which will occupy our attention, too.

Alternating current, AC, is so called because instead of flowing in a single direction like direct current, it continually reverses itself: first reaching a peak rate of flow in one direction, then, a fraction of a second later, reversing to reach an equal peak in the opposite direction. The rate at which these reversals take place is the frequency of the current.

The simplest way to produce alternating current is by rotating a loop of wire in a magnetic field, as Faraday did in his Twenty-ninth Series; his drawing is shown on the left, where ab is the axis of rotation. Recall that when a wire moves across magnetic

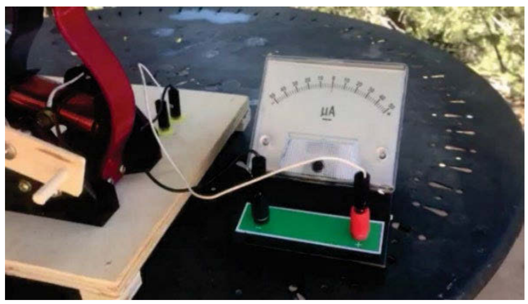




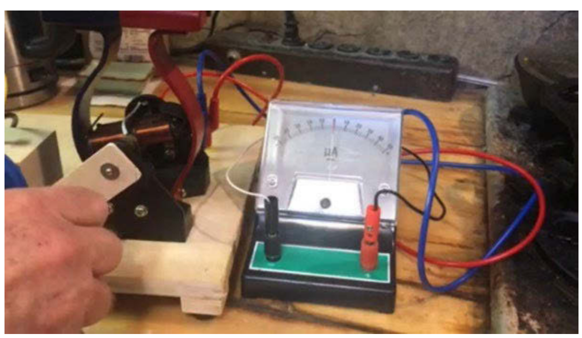
lines of force, a current is produced. But in the course of one revolution of the rectangular loop, a leg like *ef* or *cd* reverses its direction of travel across those lines, so that the resulting current likewise changes direction every half-revolution. Those repeated changes of direction were undesirable for Faraday's purposes; and he employed the commutator, pictured on the right, to reverse the loop connections every half turn and so negate the current's own alternation. But what for Faraday was a *problem to be overcome* will be, for us, the chief object of study.

Here is a small mechanical generator, which I shall use as an alternating current source. It is basically little more than Faraday's rotating loop—without his commutator—but this loop consists not of a single winding but many; and in addition it has been fitted with an iron core. The generator also contains its own permanent magnet instead of relying on the much weaker terrestrial magnetic field. All these enhancements greatly increase the amount of current the generator can produce. Let us first observe the periodic reversals that give alternating current its name.

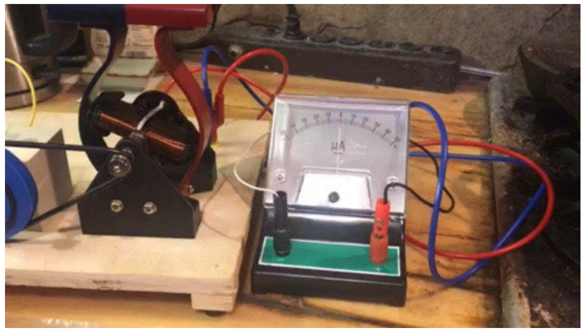
I have connected the generator to a galvanometer whose needle rests at center scale. Since the galvanometer can withstand only very small currents, I have also included a



poor conductor (a resistor) to limit the current. (In Faraday's Third Series he used a *wet string* to achieve a similar purpose.) I turn the crank and—of course, the galvanometer swings from side to side, every half-revolution. But this raises the question, what kind

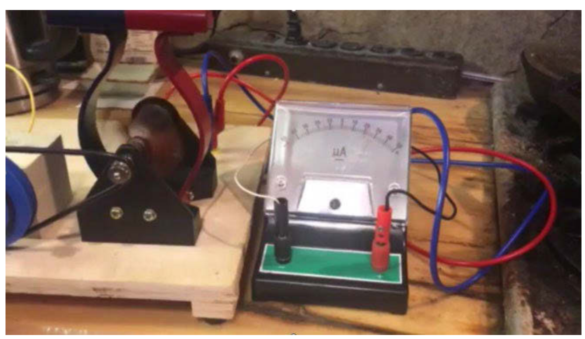


of instrument can possibly measure alternating current, since the ammeter and voltmeter that we have been using are constructed exactly like this galvanometer and would respond the same way: at low frequencies swinging back and forth, as we just saw; while at high frequencies hovering uselessly about the zero line, like this:



There are a number of ways to make DC ammeters and voltmeters measure alternating current. The simplest method uses the *rectifying diode*, pictured here. This

device conducts in only one direction; so the meter, instead of being exposed to successively reversing pulses of current, receives only those pulses having the chosen direction, and accordingly registers in one direction only, like this:



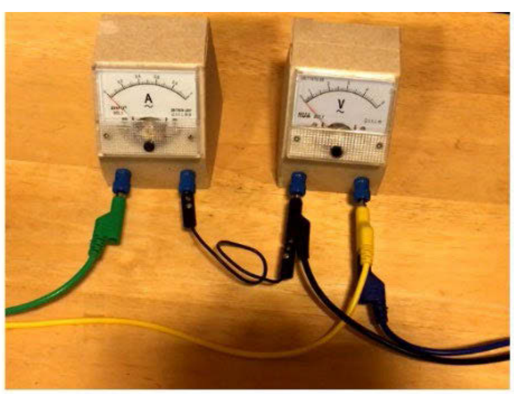
But what does this relatively steady reading represent? At these high speeds, the meter's needle cannot keep up with the current's rapid excursions between zero and peak value; the meter indicates only the *average value* of the pulsating current. Calculating that average is not difficult; you'll find it in the handouts here on the table; and you are welcome to take a copy later. But let me state at once that the average current turns out to be a *fixed fraction* of the peak current. For our present purposes it doesn't matter what that fraction is; whatever its value, our meter readings will be proportional to the *peak* values of current or voltage; and it is these peak values we shall be interested in. So remember that even though the meter is steady, the alternating current (or voltage) in the circuit will be continuously oscillating between those positive and negative peaks.

The AC ammeter and voltmeter, then, are just adaptations of the DC ammeter and voltmeter; and therefore the AC voltmeter, like its DC counterpart, assumes in its very design that Ohm's Law applies. But exposure to alternating current would seem to be one of those "new circumstances" in which we have to ask whether Ohm's Law *does* still apply. So it is important to know that the calorimeter method I described earlier gives the same results for alternating current as it does for direct current. Therefore Ohm's Law will hold for the resistor in an AC voltmeter, too; and on that basis I will make free use of ordinary commercial AC meters in the work that follows.[3]

Now, as we saw a moment ago, when the speed of the mechanical generator increases, both the frequency and the current increase together. With the simple mechanical generator, therefore, we cannot control the strength of the current independently of the frequency. This is a serious disadvantage for experimenting. For that reason I will instead use this electronic generator to produce alternating currents at varying frequencies but with independently adjustable strengths.

Our earlier experiments revealed the ratio of DC voltage to DC current as definitive of the property called resistance. Does the ratio of AC voltage to AC current share that same significance? We would certainly expect so, since, as I mentioned already, calorimeter measurements on nichrome wire give the same results for AC as for DC. If the same is true for conductors generally, then measurements on copper wire should give the same straight line graph for AC that we earlier obtained using DC. Let us test that expectation with our 150-foot length of copper wire. And let me note that for these measurements I have strung it like a clothesline—75-feet out and back again, and therefore nearly straight except where it reverses at the middle. I have connected the ends of the wire to the electronic generator; and once again I will subject the wire to a varying voltage and we will measure the resulting values of current. But remember that we are now using alternating current, not direct current as we did before; and we are using AC meters, which have been adapted from DC meters in the way I described. Watch as I gradually increase the voltage...

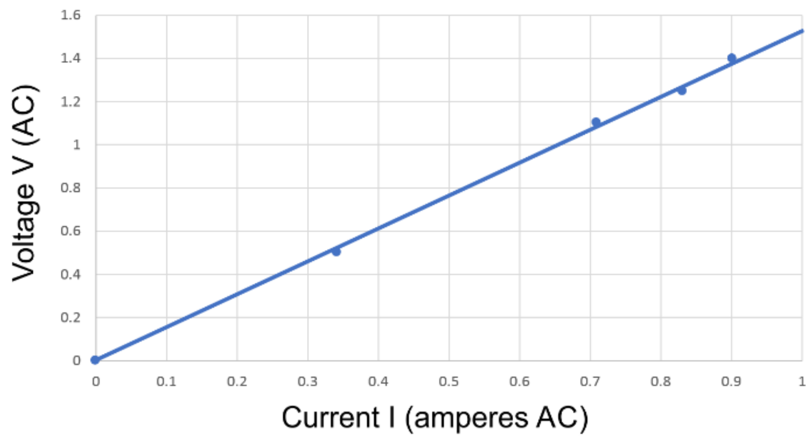
To AC generator



To 150 ft. copper wire

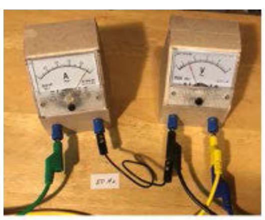
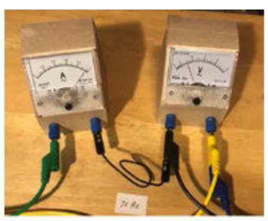
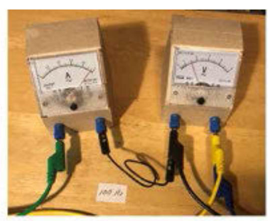
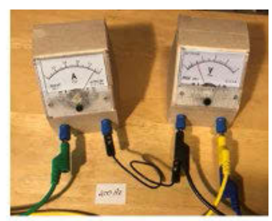
[3] I will pass over an additional complication which the use of commercial instruments introduces. For various practical reasons, standard AC ammeters and voltmeters are calibrated to read not the average value per cycle but rather about 90% of that average. But since those readings are still proportional to the peak values, that fact will not affect our reasoning about the peak values.

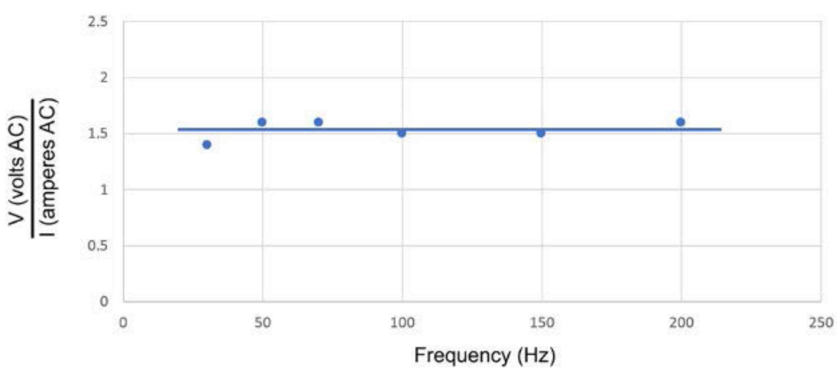
And here is a graph of the results.[4] The straight line shows that AC voltage and current are proportional in our copper wire, just as they were for DC. So it appears that



the ratio of voltage to current does indeed express the resistance of a conductor, whether the current is alternating or direct—just as we had anticipated.

But these measurements were made at a definite frequency of alternating current. Will the resistance be the same at other frequencies? Here are four of seven measurements I made over a frequency range between 30 and 200 cycles per second (modern terminology would say 30 to 200 Hz). When all seven measurements are

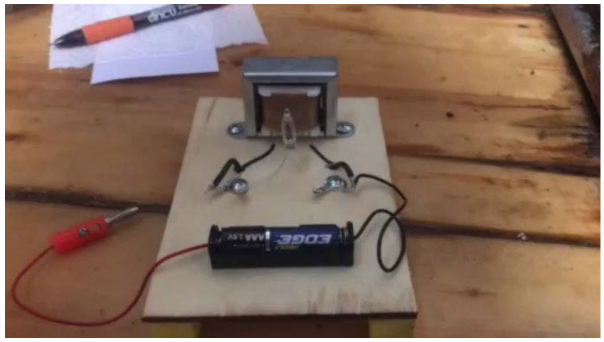


graphed, the horizontal line shows that the ratio of voltage to current is indeed the same throughout the range of frequencies. Evidently the resistance of a conductor to alternating current is *indifferent* to frequency.

[4] The oscillator signal was sent through an audio amplifier in order to obtain the large currents recorded here.

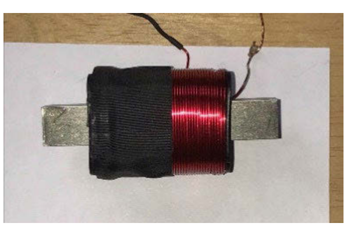
But what if we were to modify this conductor by winding it into a coil? You might not expect a mere change of configuration like that to make any difference; but in Faraday's First Series—and even more in the Ninth—he obtained unexpected effects from just such a coil, effects which were even more pronounced when iron was introduced.

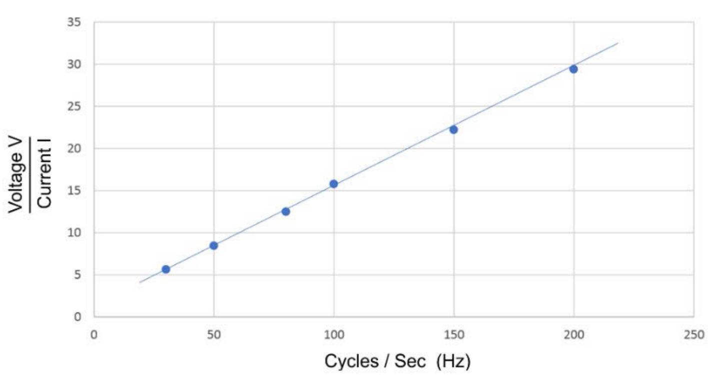
An especially dramatic example is this one, patterned after experiments in Faraday's Ninth Series. The device at the rear of the board consists of several hundred turns of



copper wire, wound about an iron framework. I have connected a small neon bulb across its terminals; the bulb requires about 90 volts to light up. But the battery supplies only one-and-a-half volts, not nearly enough to light the bulb. I make the connection, and of course the bulb doesn't light. Yet when I suddenly break the connection… *the bulb flashes!* Evidently the coil developed a sizable voltage, much higher than the voltage of the battery itself, and sufficient to light the bulb. But what is important to us is the fact that this occurred at *the moment the connection was broken*; that is, in conjunction with an abrupt cessation of the current. It seems that the act of winding a conductor about an iron core makes the *moment of change* especially significant. But alternating current is *always* changing! So we might expect coils carrying AC to exhibit properties we did not encounter with DC.

Let us then coil our copper wire about an iron bar, like this; today such devices are typically called "inductors." Again we measure the voltage and current at different frequencies, calculating their ratio for each frequency. The graph shows the results.

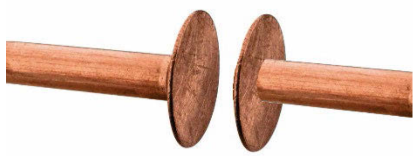




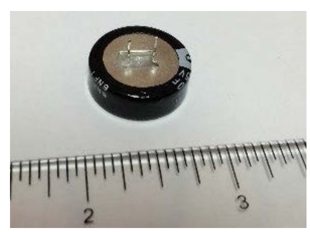
The ratio of voltage to current, which was *indifferent* to frequency when the wire was straight, is *directly proportional* to the frequency when the same wire is wound into a coil. How are we to interpret this behavior? To begin with, can we be sure that the ratio of voltage to current still means the same thing as it did before—the measure of resistance? Or does it now reveal a new property, one that emerges when a conductor is coiled up, especially about iron?—a property that reveals itself to changing current while being, apparently, invisible to steady current.

It will not have escaped you that coiling a wire, especially coiling it about iron, is precisely how we go about making an electromagnet. So we have good reason to suspect that this new property—if indeed it is new—is intimately connected with the magnetic field. I don't think we are yet in a position to speculate further about this property, still less to propose a name for it; but perhaps more guidance will emerge at a later stage of our investigation.

Let us next consider what happens when we *cut* a conductor instead of coiling it. In contrast to coiling, which we might have expected to make *no difference*, the action of cutting would seem to make *all the difference*. For when a conductor is cut, we regard it as no conductor at all, since no current can then flow. But this view reflects only the limited standpoint of steady current; for if you were to flatten out the exposed faces of the cut wire, making them larger and larger—again, a mere change of shape—you would

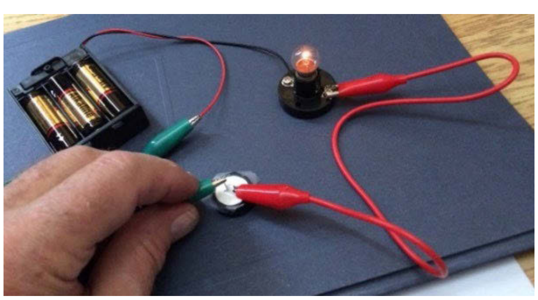


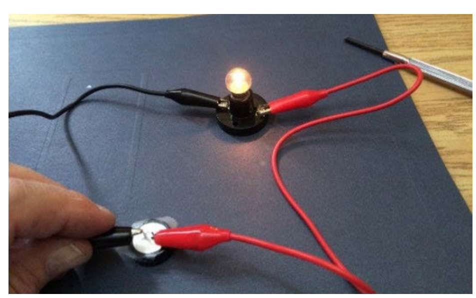
form a set of parallel conductive surfaces—a *capacitor*. Thus every severed wire is, in effect, a capacitor. And while it may be true that a capacitor cannot sustain steady current, there is certainly a transitory current while it charges, and again when it discharges. Let me demonstrate this.



This device is a capacitor, one so highly miniaturized as to contain many square inches of parallel conductive surfaces, spaced microscopically close together. And let me pause here to marvel at the remarkable degree of miniaturization that is achievable today. When I was in high school, a capacitor having the same electrical value as this one—one Farad—would have been nearly the size of my bedroom!

When I charge the capacitor with a battery, the flashlight bulb shows that current flows temporarily, diminishing as the capacitor gains charge and finally ceasing altogether. And when I then *discharge* the capacitor, the current again starts out at a maximum, and diminishes as the capacitor loses charge. So you see that a capacitor

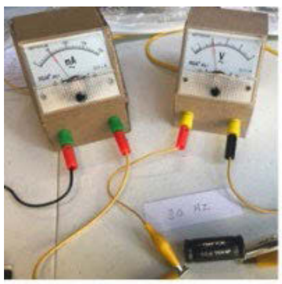
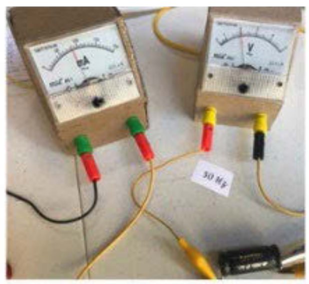
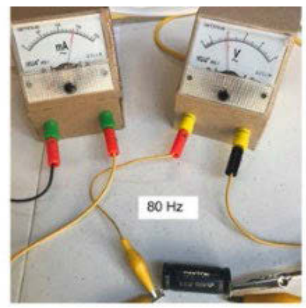


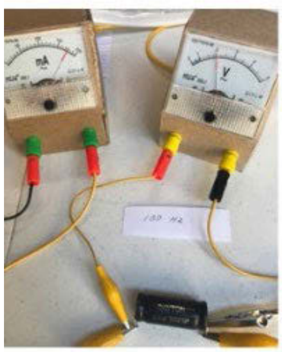
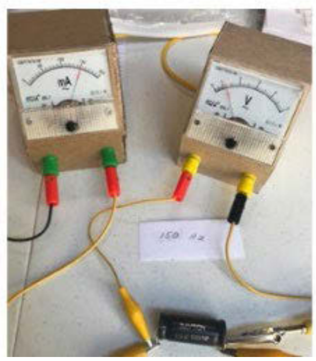
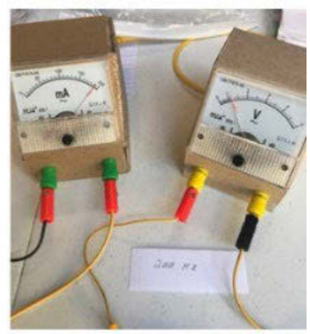


can indeed sustain an electric current by repeatedly charging and discharging. Since this is exactly what must happen if it is subjected to an *alternating* voltage, a capacitor—even though it is essentially a *broken wire*—ought nevertheless to pass alternating current.

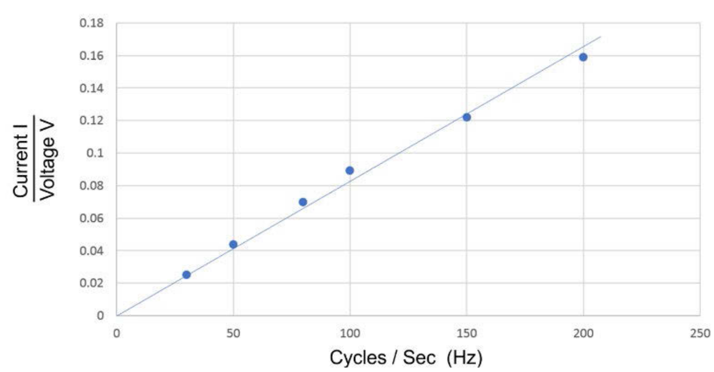
I wish I could mold our copper wire into a capacitor of practical size, the way I earlier wound that wire into a usable inductor; but since that isn't possible, I'll use a commercially-manufactured unit like this one**.** Let us then survey a range of frequencies with this capacitor, just as we did for the inductor. Again we measure both the alternating voltage applied to the capacitor and the alternating current that flows through it.

Here are the meter readings at six different frequencies ...

and this time, instead of graphing the ratio of *voltage to current*, I will graph the ratio of *current to voltage*—you'll see why in a moment...



We get a *straight line*—the ratio of *current to voltage* is proportional to the frequency. Therefore the ratio of *voltage to current* must be *inversely* proportional to the frequency. Thus the ratio of voltage to current—a ratio which was *independent* of frequency for resistance, but *directly proportional* to frequency in the inductor, is *inversely proportional* to frequency in the capacitor.

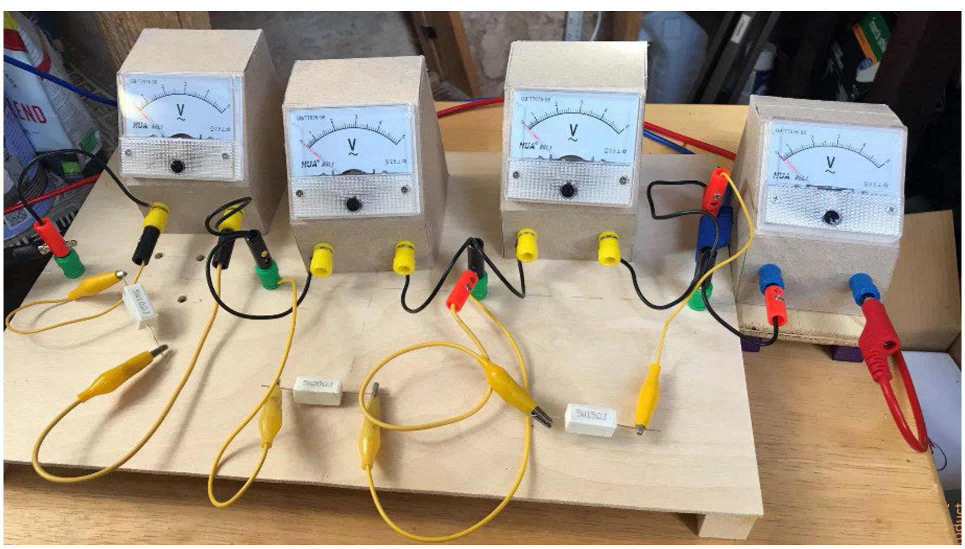
Does this behavior announce still another potentially new property? If so, it is a property that emerges when we create a gap in the conductor. But we know what happens when we apply a voltage across a gap—we create an electric field. So perhaps we have reason to suspect that if this is indeed a new property, it is bound up with the electric field.

{III}

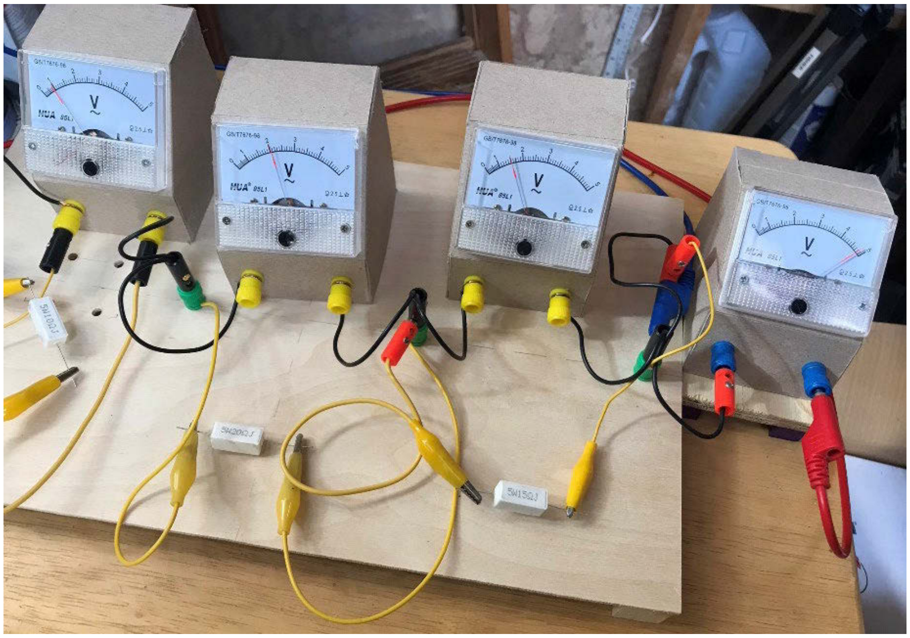
Our question is whether the inductor and capacitor displayed new electrical properties, or just the same old property of resistance, which for some reason varied with frequency in opposite ways. We saw earlier that the distinctive proportionality expressed by Ohm's Law was rooted in the conversion of electrical energy into heat. That is the deeper meaning of resistance: resistance is not just a synonym for "imperfect conducting power"; it denotes the specific property of a conductor that *converts electrical energy to heat*. The greater the resistance, the more rapidly must energy be expended in order to maintain a given current—and therefore the more rapidly will heat be produced.

If then a current flows through a number of resistors, the heat produced by all of them together must be the sum of the individual quantities of heat produced in each; and therefore too the total energy expended must likewise be the sum of the energies expended in each. And hence the *voltage* across all of them connected end-to-end will be the *sum of the voltages* across each of them separately. Let me demonstrate this.

Here are three resistors, connected end-to-end across the generator terminals. As the photograph shows, I have a voltmeter connected across each resistor as well as a fourth voltmeter across the combination.



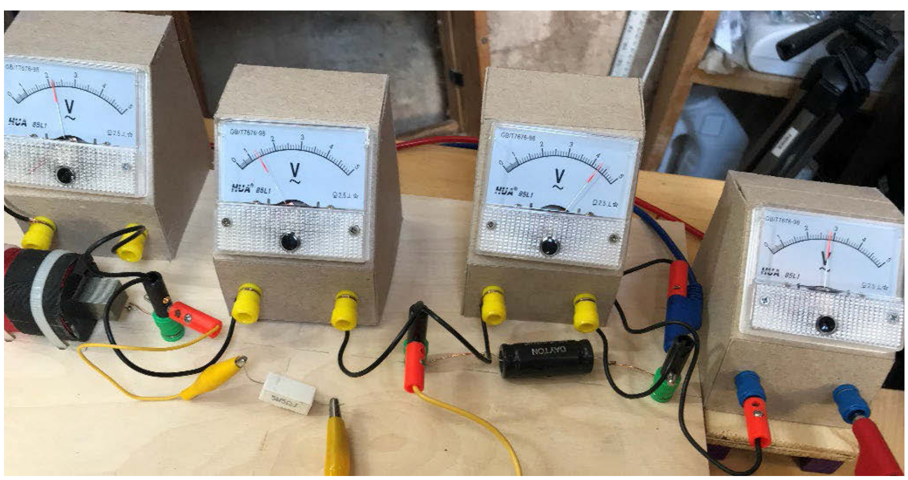
When we energize the combination, the voltage across all three together does indeed equal the sum of the three voltages separately—you see that 1.1 volts plus 2.1 volts plus



1.65 volts (the meters on the left) total 4.85 volts (the meter on the right), just as we had anticipated.

We see, then, what happens when resistors are connected together. Let us now see whether a resistor, an inductor, and a capacitor in combination behave the same way.

Here is the same setup as before, only I have now replaced the left-hand resistor with an inductor, and the right-hand resistor with a capacitor. But the three meters on the

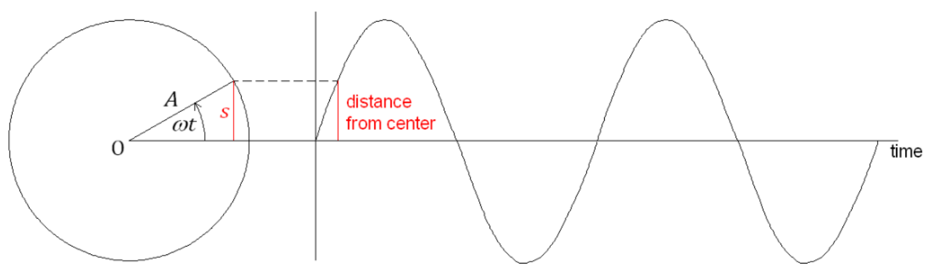


left register 2.2 volts, 1.3 volts, and 4.3 volts, respectively—while the meter on the right reads only 2.8 volts. The voltage across all together does *not* equal the sum of the three voltages separately! How is that possible—how can the whole not equal the sum of the parts? And yet it was equal to the sum of its parts for resistors—that is, for components that had shown themselves indifferent to a current's relation to time. Put another way, the logic of electrical conduction presented no surprises so long as time was irrelevant, but it seems to descend into absurdity when time becomes fundamentally involved.

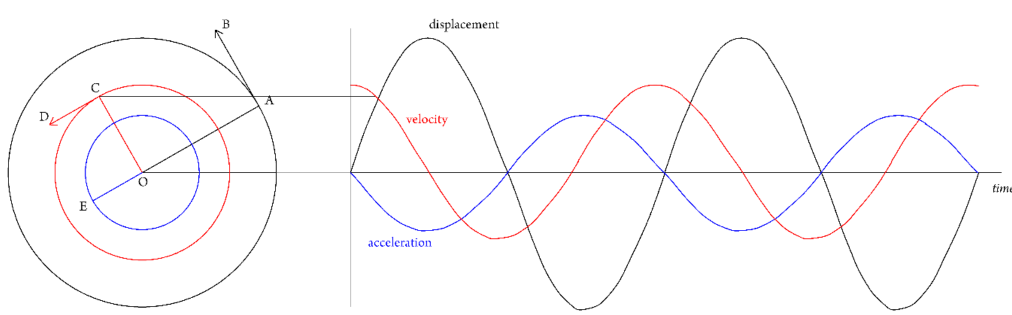
{IV}

Earlier we set out to *measure* alternating current; and at first that endeavor seemed no more than a technical challenge, which the rectifying diode enabled us to overcome. But perhaps we are now beginning to suspect that our need for a measuring device is merely the utilitarian aspect of something far more fundamental: what we really need is a *language*. For direct current, a language of *magnitude* sufficed: *how much* voltage and *how much* current pretty much summarized all that there was to say. But now with alternating current, these quantities are time-related in a distinctive way that their direct-current counterparts were not. Hence, as our attempt to measure AC voltage just revealed, a language of simple magnitude is inadequate to the task—so seriously inadequate, in fact, that it actually generated an antinomy!

Does this perhaps bring to mind an earlier enigma—one we experienced when we first tried to understand the *pendulum*? There too we encountered a phenomenon whose very essence was cyclical motion, but for which Galileo's languages of *constant speed* and *constant acceleration*—both of them languages of simple magnitude—were likewise insufficient. A new language was needed for the pendulum, and we discovered that language in the paradigm circle.



The circle has radius A. As it rotates, the vertical distance s ranges between positive and negative peaks according to the equation $s = A \sin \omega t$; and in the first semester Junior Laboratory we confirm experimentally that these changing values of s correspond to the changing displacements of the pendulum as it swings from side to side about its central position. We then deduce the pendulum's velocity and acceleration; and so construct the *threefold paradigm circle*, which articulates all three of those quantities.



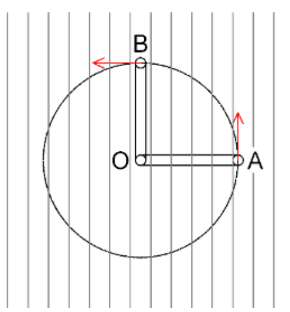
We may think of the paradigm circle as generating a sinuous pattern for each of them, as shown on the right in this slide. In this triple graph, time occupies the horizontal axis, while the other three quantities reside on the vertical axis; but the graph represents all four by *distances*—and thereby treats all four of them as *magnitudes of the same kind*—just as Galileo treated time and distance geometrically, and thus as magnitudes of the same kind. That is what I meant by a "language of simple magnitude." But note that the paradigm circle itself, unlike the graph which grows out of it, expresses time as an *angle of rotation*; and angle is not a "magnitude" in the sense of Euclid Book V—that is, capable of bearing ratio to another magnitude. As Euclid expresses it in Definition 4,

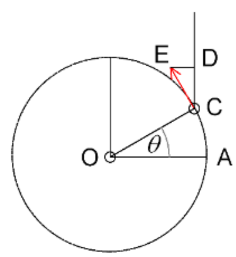
> Magnitudes are said to have a ratio to one another [if they] can be multiplied so as to exceed one another.

But angles, when multiplied, do not "exceed" one another—they rather return upon themselves! Five right angles does not "exceed" one right angle; it is geometrically *the same* as one right angle. Such is the native language of the paradigm circle; and that is why I wish to distinguish it from a "language of simple magnitude."

Returning, then, to the paradigm circle, we understand that it articulates a *species* of motion, independent of any particular device. That motion has been saddled with a variety of technical names, but let us simply call it "swinging motion." Now, what reason is there for thinking that the paradigm circle might prove illuminating for alternating current? Besides the obvious similarity between the periodicity of the paradigm circle and the cyclical character of alternating current, the mechanical AC generator practically incorporates a paradigm circle in its very construction.

For look at the way in which each leg of Faraday's rotating loop generates a current. We are viewing the loop along its axis of rotation; and when it rotates at constant speed,

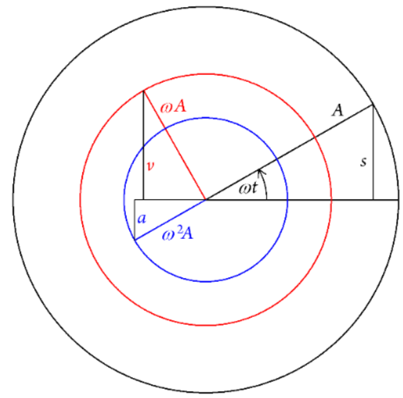




the number of lines cut per second depends on the loop's position. When the loop is turning through a small angle about the *vertical* (position B), each leg cuts a maximal number of lines of force; but when it is turning through the *horizontal* (position A), the moving leg cuts practically no lines at all. And in general, when the loop makes angle θ with a line perpendicular to the lines of force, the number of lines cut per degree rotation will be proportional to DE, the perpendicular part of its motion across the lines of force. But DE is proportional to the sine of angle DCE, that is, to $\sin\theta$. Thus the trigonometric function *sine* arises naturally in the context of electric generation, just as it appears naturally in the language of the paradigm circle.

Now just as Galileo's geometrical examination provides a basis for the equations $s = vt$ for uniform motion and $s = \frac{1}{2}at^2$ for naturally accelerated motion, the geometry of the threefold paradigm circle provides the equations of swinging motion. They are:

for displacement, $s = A \sin \omega t$,
which we saw already;



for velocity: $v = \omega A \cos \omega t$
and for acceleration:

$$a = -\omega^2 A \sin \omega t.$$

In these equations, A is the maximum extent of the swing, its peak value, typically called the *amplitude* of the swing; and ω is the speed of rotation of the threefold circle. Similarly, ωA is the peak value, or amplitude, of the velocity; and $\omega^2 A$ is the peak value, or amplitude, of the acceleration. Juniors obtain these equations from the paradigm circle in the first semester Laboratory; but if you'd like a brief résumé, that is also in the handout. The only factor we need worry about at this point is ω, the speed of rotation of the circle. In principle, we can express this quantity in any units we please—revolutions per minute, degrees per second, *quadrants per fortnight*, whatever we like—but our choice of units affects the forms of the equations. The equations I've given here assume that ω is measured in radians per second; and since there are 2π radians in a circle, ω will be 2π times the number of revolutions per second—that is, 2π times the frequency, or $2\pi f$.

Any mass mounted on a spring will exhibit swinging motion, as does this steel washer affixed to the end of a hacksaw blade.

How do the equations of swinging motion apply to this device? Let us first attend to the inertial element, the *mass*. It must exemplify Newton's Second Law; so when we substitute the acceleration equation

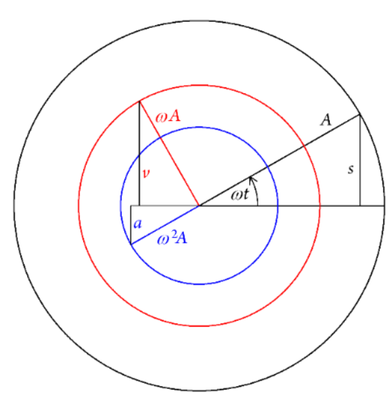
$$a = -\omega^2 A \sin \omega t$$

(from the blue radius), into the Second Law we have

$$\boxed{F = ma} = -m\omega^2 A \sin \omega t,$$

while the velocity equation (from the red radius) was

$$v = \omega A \cos \omega t.$$



The paradigm circle shows us that the blue radius precedes the red radius by 90 degrees of rotation—so that, likewise, the force precedes the velocity. So now we know, for an inertial body, how force and velocity are related in *time*. How do they relate in *magnitude*?

To see this, find the ratio of their respective amplitudes, that is, of their peak values independent of sign. We have

$$\text{the amplitude of force over the amplitude of velocity} = \frac{m\omega^2 A}{\omega A} = \omega m = 2\pi f m\,.$$

It is proportional to the frequency, f. But recall that when we measured the ratio of alternating voltage to current for the inductor, V/I, it too was proportional to frequency. We seem to have come upon an intriguing analogy between an *electrical inductor* and a *mechanical mass*. In particular, the ratio between the measured voltage and current in the inductor appears to have the same mathematical form as that between the amplitudes of force and velocity for a mechanical mass. If we write them explicitly in that same form we shall have:

$$\frac{force}{velocity} = 2\pi f m \ \ \text{(for the mass)}$$

and

$$\frac{V}{I} = 2\pi f L \ \ \text{(for the inductor)}$$

where L represents some characteristic of the inductor that is analogous to mechanical mass m. How seriously should we take this analogy?

Analogies present a particularly difficult problem for science. On the one hand, they can suggest hitherto undiscovered correspondences, as (perhaps) in this case. On the other hand, to the extent that science seeks to lay bare the actualities underlying the phenomena it studies, it cannot rest with mere similarities. Closely related to analogy is *metaphor*—which creates an even worse problem, by stating (either explicitly or implicitly) that one thing *is* another thing—while all the while knowing that it is *not*. If we say, with Hobbes, "Man is a wolf,"[5] we are invoking the wolf as the emblem of a truth about man; but if we really thought that man *was* a wolf, simply, the statement would be an empty tautology. Rightly understood, the metaphor requires us to live in two worlds at once—one in which man *is*, and another is which man *is not*, a wolf. Nothing, it seems, could more spectacularly fall short of the Cartesian ideal of clarity and distinctness than does *metaphor*.

Thus it may occasion considerable wonderment that Maxwell not only took the electro-mechanical analogy very seriously, but even introduced a metaphorical terminology to express it. He branded L as a new electrical property, which he named "electrical inertia." Thus did Maxwell answer our earlier question: our measurements with the inductor did not simply reveal a resistance that varied with frequency but a wholly different electrodynamic property.

Later investigators disdained what they judged to be capricious metaphor on Maxwell's part, and so replaced his colorful title "electrical inertia" with the soberer term "inductance." But—and I will say more about this later—I don't believe Maxwell employed metaphor in the way they thought he did.

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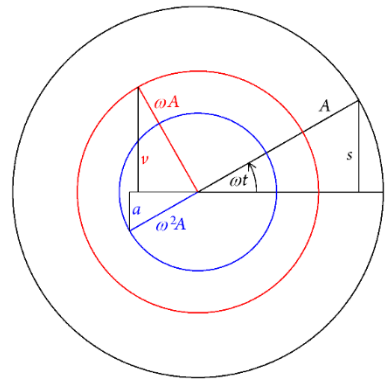
[5] Shortened from the Latin "Homo homini lupus," cited by Hobbes in his preface to *De Cive* ("On the Citizen"). It is a favorite example of Colin S. Turbayne in his book, *The Myth of Metaphor* (1962).

But we have almost lost sight of our mechanical vibrator! Let us return to it, and this time consider the elastic element, the *spring*. It is characterized by Hooke's Law; so substituting the distance equation, $s = A\sin\omega t$, into that law we have



$$\boxed{F = ks} = kA\sin\omega t$$

while the velocity equation is, again,

$$v = \omega A\cos\omega t\,.$$

The paradigm circle reveals that the red radius precedes the black radius by 90 degrees of rotation—so that, in this case, the *velocity precedes the force*. Such is the time-relation of force and velocity for elasticity and, interestingly, it is the reverse of that for inertia. And if we now examine the ratio of their amplitudes we find that

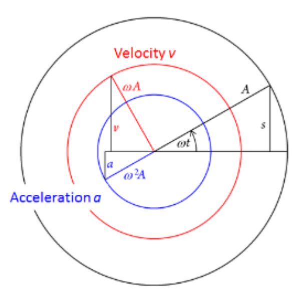
$$\text{the amplitude of the force over the amplitude of velocity} = \frac{kA}{\omega A} = \frac{k}{\omega} = \frac{k}{2\pi f}\,.$$

It is inversely proportional to the frequency—just like the ratio of alternating voltage to current for the capacitor. This argues a striking analogy between the electrical properties of a capacitor and the mechanical elasticity of a spring; and once again Maxwell will take the analogy seriously. Expressing the mechanical and the electrical ratios in the same mathematical form yields:

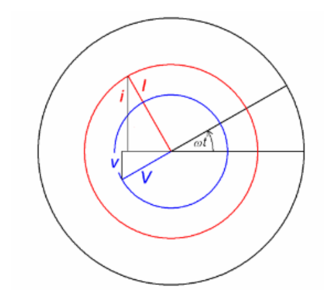
$$\frac{\text{force}}{\text{velocity}} = \frac{k}{2\pi f}\ \text{(for the spring)} \qquad \text{and} \qquad \frac{V}{I} = \frac{1/C}{2\pi f}\ \text{(for the capacitor),}$$

where $1/C$ represents some property of a capacitor that corresponds to k, the coefficient of elasticity of a spring. Maxwell had his own reasons for expressing the constant C in reciprocal form, but he identified C as another new electrical property, altogether different from resistance, and whose inverse he styled "electrical elasticity." The modern term for this property, here too fleeing in horror from dreaded metaphorical whimsey, is *capacitance*.[6]

Now if we have drawn sound parallels between inertia and inductance, on the one hand, and between elasticity and (inverse) capacitance, on the other, then we must not hesitate to represent the electrical quantities by the same paradigm circles as we did the mechanical quantities. The inductor's paradigm circle, then, will incorporate the blue and red radii—which will now represent voltage and current, respectively. And notice



Acceleration and velocity of mass m

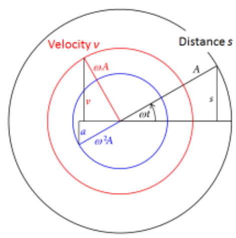


Voltage on and current through inductor L

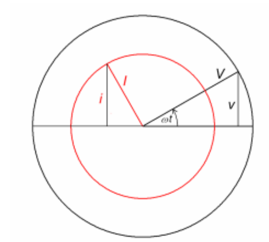
[6] The terms "inductance" and "capacitance," like so many other modern electrical names, were originated by Oliver Heaviside.

that *voltage precedes current* in the inductor; and therefore—since they do not rise and fall together—they cannot be proportional! Ohm's Law cannot hold for the inductor—at least, not on the instantaneous level.

Next, for the capacitor; its paradigm circle will embrace the black and red radii, which will represent voltage and current for the capacitor. Here, *current precedes*



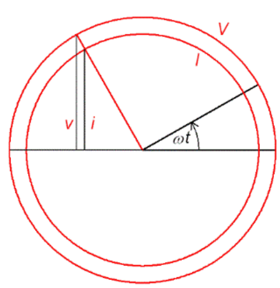
Displacement and velocity of spring



Voltage on and current through capacitor C

voltage, so again they cannot be proportional; and thus for the capacitor, too, Ohm's Law cannot hold on the instantaneous level.

When our circuit contained only resistors, Ohm's Law held for both DC and AC; and the individual voltages added up just as we expected them to do. But when an inductor and a capacitor were involved we came upon the seeming absurdity that the whole voltage was less than the sum of its parts. That perplexing development threatened to call our entire understanding of alternating current into question. But now that the paradigm circle has given us a language for measurement we did not then possess, we can comprehend why AC voltages add up for resistance but not for inductance and capacitance. The clue lies in our earlier observation that the ratio between AC voltage and current for a resistance did not depend on frequency, in contrast to those ratios for inductance and capacitance, which did. The frequency dependence we saw with inductance and capacitance arose because their voltage and current were *not synchronized*—one of them inevitably led or lagged the other. Then for resistance, where the ratio does not depend on frequency, we may infer that voltage and current *are* synchronized.[7] In the paradigm circle for resistance, then, the radii representing



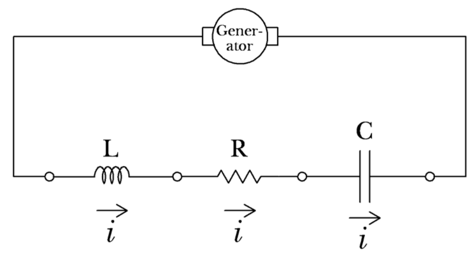
Voltage and current for resistor R

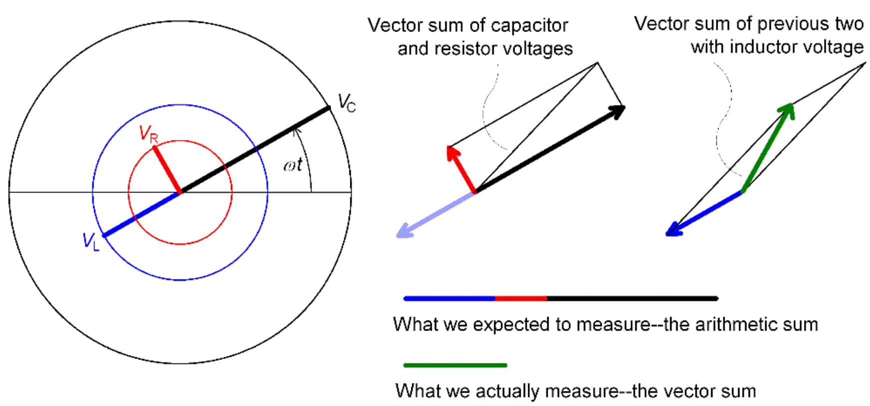
voltage and current will coincide. But voltage and current are synchronous *only* for the property called resistance; and we can now distinguish rigorously between that property and the properties of inductance and capacitance, which belong not to

[7] The mechanical analog to *resistance* would therefore appear to be a force that is proportional to *velocity*—such as the viscous force (whose action, by the way, also generates *heat*).

conductors as such but to the magnetic and electric fields that accompany the currents they carry. Evidently those properties were always present, and indeed must exist to some degree in every electrical circuit; but we never noticed them until alternating current made them manifest.

Let us now resolve the seeming paradox in our earlier circuit containing inductance, capacitance, and resistance in combination. Since the three components are connected end-to-end, the same current *i* must pass through each one at every instant;[8] so when we draw the paradigm circles for each of them, the radii representing their currents will all be identical in length and angle. We can then superimpose the circles upon one another with their current radii lined up, to obtain a composite paradigm, shown on the left in this slide.





Each of the radii represents the peak voltage of one component: the black radius that of the capacitor, the red that of the resistor, and the blue that of the inductor. Our initial expectation was that all three would add together arithmetically, to form the total. But the paradigm circle shows us that the arithmétic sum of the peaks is irrelevant, because the voltages *do not attain their peaks at the same time*. Instead, we must take their vector sum. First, compound two of the voltages together to obtain their resultant as shown in the middle diagram; then compound this last with the remaining voltage as in the right-hand diagram; the resultant, in green, is the actual total—and, as we see, it is considerably less than what we originally expected to measure, and now we know why. Do you think we could have gained this understanding had we not been blessed with the special language of the paradigm circle?

❖

I mentioned earlier that Maxwell's names for the new electrical quantities introduced a metaphorical terminology that later generations quickly disavowed. But Maxwell did not fashion those metaphors in the usual way, which typically subordinates one term to

[8] As Maxwell states in the *Treatise*, sect. 61, electricity does not *accumulate*.

another. For example, when in Act III, Shakespeare's Romeo refers to the stars as "night's candles,"[9] he is looking to everyday artifacts in order to characterize the heavenly luminaries. Rhetorically, Romeo is subordinating the celestial order to human craft. Thus when Maxwell called the distinctive property of the inductor "electrical inertia," readers who thought of "inertia" as a strictly mechanical term might well have assumed that Maxwell meant, similarly, to subordinate electrical understanding to mechanical understanding. Perhaps that was the reason for their intense aversion: lest electrical science seem to be constrained by mechanical precepts.

But Maxwell makes clear in the Treatise[10] that he places "mechanical inertia" and "electrical inertia" on an equal footing—both of them only reflections of a still more generalized "inertia," neither mechanical nor electrical in nature—and pointing to an even more universal physics which, he thought, ought to encompass both electrodynamics and mechanics as special cases. From that point of view, metaphor is to be embraced, not disdained—for when we are working at the boundaries of established thought we have no choice but to think metaphorically, and thereby to think in two worlds at once. Dialectic, not subordination, is the heart of Maxwellian metaphor.

The paradigm circle declines to treat time as a Euclidean magnitude, and thereby combines rest and motion in a single image. In Platonic terms, that would constitute a kind of mediation between the world of the unchanging Forms and the phenomenal world. Poetically, it constitutes metaphor, because, like Maxwell's, it too stands in two worlds at once.

Plato's *eidos* is the avowed aim of philosophical thought, not of routine thought. But perhaps all thought would be philosophical if it were not so easily deflected from its higher aim; and Socrates is quick to admonish us against such casual deflection. Although in the *Republic* Socrates concentrates much of his criticism on the power of art and poetry to distract from the philosophical pursuit,[11] a youthful disillusionment, recounted in the *Phaedo*, [12] taught him also that an intense interest in nature—in *how things work*—can perhaps be as much a distraction from the philosophic goal as is an uncritical love of poetry.

It is easy to take these strictures against art, poetry, and natural science as constituting fixed Platonic dogma. But doing so requires that we ignore some very powerful Socratic imagery, both in the *Republic's* Cave and in Diotima's discourse in the *Symposium*. The predominating imagery in both, I find, turns more on aspiration than rejection: once we have seen *fire*, it seems we are bound to seek the *Sun*.

To the extent we seek true understanding, we really do dwell in two worlds at once. By means of myths, with their intrinsic metaphors, we manage to do so without losing our sanity. And Socrates, the master of myth and metaphor, knows that better than anyone. ■

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[9] *Romeo and Juliet,* III.5

[10] Part IV, Chapter V

[11] *Republic* Book X, 607b

[12] *Phaedo* 96a–100d

Handout for lecture on *Alternating Current*

I. THE PARADIGM CIRCLE AND THE EQUATIONS OF SWINGING MOTION

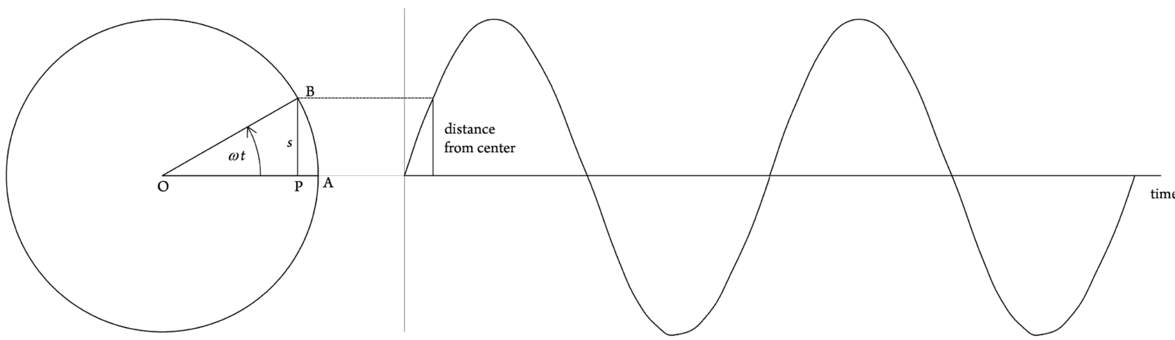
Consider a circle with center O and radius OB, and from point B drop the perpendicular BP, so that $\mathrm{BP} = \mathrm{OB} \sin \angle \mathrm{AOB}$. If the circle rotates at the rate ω, then $\angle \mathrm{AOB} = \omega t$; and we can write

$$\mathrm{BP} = \mathrm{OB} \sin \omega t$$

or, letting $s = \mathrm{BP}$ and $A = \mathrm{OB}$,

$$\boxed{s = A \sin \omega t\,.}$$

This is the first equation of swinging motion. As the circle rotates, s repeatedly ranges between equal peaks A in opposite directions, each peak being equal to the radius OB. A is customarily called the *amplitude* of the swing.



The rotational speed ω may be expressed in any angular measure—degrees per second, rotations per minute, or whatever. But it is advantageous to choose *radians per second*,[13] since doing so will simplify a number of subsequent expressions. Then if BC is the velocity at the circle's circumference,

$$\mathrm{BC} = \omega \cdot \mathrm{OB}\,.$$ [14]

Now construct a new circle with radius OD equal to BC, and let it rotate along with the first circle. Then just as the vertical component of radius OB generated a record of the swing's *displacement from the center*, so too the vertical component of OD generates a record of the swing's ever-changing *velocity* v. We have

$$v = \mathrm{OD}\ \sin \angle \mathrm{AOD}$$

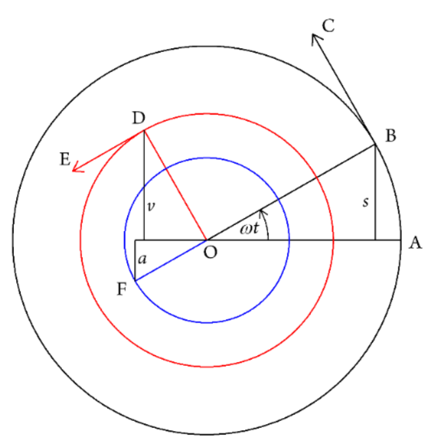
or, by similar triangles,

$$v = \mathrm{OD} \cos \omega t\,.$$

But $\mathrm{OD} = \mathrm{BC} = \omega \cdot \mathrm{OB} = \omega A$, so

$$\boxed{v = \omega A \cos \omega t,}$$

the second equation of swinging motion.



Similarly, let DE be the velocity at the circumference of the circle through D; then

[13] The *radian* is that angle which, when at the center of a circle, subtends an arc on the circumference equal to the circle's radius. Since the circumference 2π times the radius, there are, necessarily, 2π radians in a circle.

[14] If angle θ is expressed in radians, it will subtend an arc $\widehat{\mathrm{AB}}$ such that $\widehat{\mathrm{AB}} : 2\pi r :: \theta : 2\pi$, from which $\widehat{\mathrm{AB}} = r\theta$. Then if the angle increases at the rate ω, the arc similarly increases at the rate $v = r\omega$. Thus, *circumferential velocity* is simply the product of the radius and the rate of rotation.

$$\mathrm{DE} = \omega \cdot \mathrm{OD}\,.$$

If we go on to construct a third circle having radius OF equal to DE, what will that circle represent? The first circle specified position, while the second circle specified velocity—the *rate of change* of position. Then the third circle, which was constructed in precisely the same way, must represent *acceleration*, the rate of change of velocity. The vertical component of OF therefore generates a record of the swing's acceleration, so that

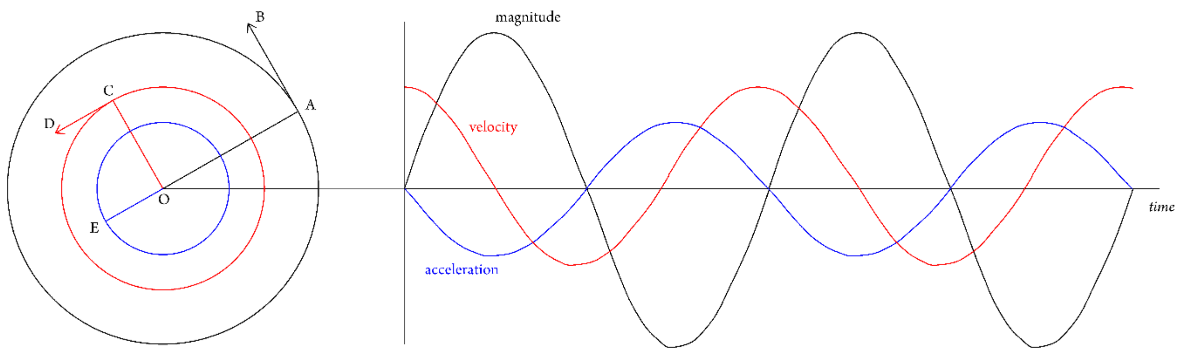
$$a = -\mathrm{OF}\sin\omega t\,.$$

But $\mathrm{OF} = \mathrm{DE} = \omega \cdot \mathrm{OD}$, while $\mathrm{OD} = \mathrm{BC} = \omega \cdot \mathrm{OB} = \omega A$. Substituting,

$$\boxed{a = -\omega^2 A \sin\omega t\,.}$$

This is the third equation of swinging motion. Notice the negative sign; it is clear from the diagram that a and s must have opposite signs.

The *threefold paradigm circle* generates a combined record of the position, velocity, and acceleration of swinging motion, like this:



II. AVERAGE CURRENT PASSED BY A RECTIFYING DIODE

One cycle of alternating current having peak value I_p may be represented as

$$I_p \sin\omega t$$

as t increases from 0 to the time of a full cycle, which is $2\pi/\omega$. But since the rectifying diode blocks half of each cycle, current will pass only while t increases from 0 to π/ω. The quantity of electricity passed during this time will therefore be

$$Q = \int_0^{\pi/\omega} I_p \sin\omega t\, dt = -\frac{I_p}{\omega}\cos\omega t\Big]_0^{\pi/\omega} = \frac{2I_p}{\omega}$$

so that the average current will be this quantity divided by the time of a full cycle, or

$$I_{av} = \frac{2I_p/\omega}{2\pi/\omega} = \frac{I_p}{\pi}\,.$$

Thus a DC meter equipped with a diode rectifier and connected to a source of alternating current will read $1/\pi$ or .318 times the peak value of that current. Commercial AC meters, however, are calibrated differently; in order to secure consistency between AC and DC calculations of power they do not read the average current but about 2.22 times the average. But since their readings are still proportional to the average values, my use of commercial meters does not affect any of the reasoning in the lecture.

H. Fisher