THE MAGNIFICENT PENDULUMS[1]

Howard J. Fisher

Welcome, everyone; and thank you for joining me this afternoon to think about the magnificent pendulums. My talk will have five parts; this is

Part I. That Same Again Feeling[2]

There is something captivating about repetitive actions. What is it? Is it perhaps the *regularity* of such events?—think of the repeated *drip, drip, drip* of a leaky faucet, how it captures the consciousness, and may even drive a person to distraction. But mere regularity, I think, is not the reason.

The second hand of this clock[3] advances regularly every second; a prolonged encounter with it is rather boring than captivating. But the *pendulum* of a clock is a

different matter: its motion *does* tend to ensnare our awareness. The forth and back motion of the pendulum *draws us in*, but the endless forward progress of a clock's second hand puts us to sleep. A motion forward and back returns to its beginning and thus defines a *completed act*. Endless progress, in contrast, never reaches completion; it is forever only getting started. *Completeness*, I think, is what draws our notice.

That may be the reason why a repeated *sound* commands more attention than a repeated motion. The sight of our clock's second hand intimated no conclusion, no wholeness. But its sound alone, like that of the dripping faucet, begins to lay hold of our consciousness. Without the sight of a clock face to announce that there really is no end to circular motion, each tick of the clock can seem to be a whole in itself—and that, apparently, is all our consciousness needs to capitulate.

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[1] Transcript of a lecture given remotely at St. John's College, Santa Fe, New Mexico, on 18 November 2020.

[2] "That same-again feeling" was a much-lampooned phrase meant to describe the musical octave; it appeared in the College's freshman laboratory manuals from the late 1960's to the early 1970's. It's notoriety was no doubt greatly augmented by Gabrielle de Seife's delightful eponymous poem in *The Collegian*, no. 191 (October 1975).

[3] James Jones, London, c. 1790. My thanks to Don Lemons, who supplied a video of this fine instrument.

For all its appeal, though, the pendulum's motion is not easy to apprehend. Indeed, the "wholeness" of each swing seems, by comparison, to overwhelm our ability to grasp the successive moments of such motion. Let us see how much we can grasp with the unaided senses.

First, it certainly appears that every swing of this primitive pendulum takes the same time as every other swing; so that the body's repetitive motion marks off equal intervals

of time. And to the extent that each full swing returns the suspended body to the same position as that from which it began, it would be astonishing if every swing were *not* identical to every other in all respects, since from the same conditions we expect the same results.

But of course a swinging body does not, or not quite, manage to return to its starting position. Sometimes this loss is detectible in the very first swing; but even when it is not, we know that every swing must suffer *some* loss of motion, since the body eventually loses enough motion to make the decrease evident. So it is not clear that our sense of "same again" is entirely reliable.

I think we can also probably grasp that the pendulum *momentarily stops* at the extremity of each swing. Even so, however, it's not clear that we understand this by sight alone: for what we clearly see is that it reverses direction at the extremities; and Aristotle in his discussion of continuity shows how reversal necessitates momentary cessation. So perhaps our sight stimulates an intellectual apprehension so swift, and to which we are so accustomed, that we overlook it and attribute our judgment to sight, simply.

Another distinctive phase of the pendulum's motion is its *central* position, where we may persuade ourselves the pendulum achieves its greatest motion. Again, though, we may doubt whether we actually *see* this. For the symmetry of the pendulum's path would seem to necessitate such an inference; as would likewise our realization that the central position of a pendulum is also the *lowest* position—and therefore the place of maximum speed, as Galileo showed for falling bodies in general. Again, I am not questioning the truth of these conclusions, but only whether we apprehend them by sight alone.

Even with reason's aid, however, when it comes to the *intermediate positions* of the pendulum, what can we say of them? To be sure, we know how to measure. Suppose then we undertake an exhaustive series of measurements and so gain accurate knowledge of the pendulum's position at precise, individual times. Without some grasp of the *overall character* of pendular motion, these are mere isolated factlets. They present, as Socrates would say, a *swarm of bees* rather than the nature of Bee. What we really want is something like the *nature* of the pendulum—that is, if it *had* a nature, rather than being, as it is, not natural but artificial. But the very fact that an artificial apparatus invites us to look at it as though it *were* natural is already interesting.

You may have noticed something else as the pendulum's motion decayed: the time occupied by each of the small swings was not very different, if indeed at all different, from the time marked out by each of the original large swings. Let's investigate this more exactly.

Here are five swings from the first few minutes of the previous video. When we time

them, they occupy 6.27 seconds, for an average of 1.254 seconds per swing.

And here are five swings from much later in the same video, when the amplitude of

each swing has noticeably decayed. *Now* the time for the same number of swings is 6.17 seconds, or 1.234 seconds per swing. Not quite equal, but certainly very close. We will ask later on whether the difference is significant. For now, though, it appears that every undulation of the pendulum occupies pretty much the same interval of time, without regard to the size of the swing. This is the property called *isochrony*.

Galileo is said to have noticed this phenomenon when he was a student at Pisa. The Pisa cathedral was illuminated by large suspended oil lamps, which would occasionally swing back and forth. Sometimes the movement was small, other times much greater; but, small or large, the *time* occupied by each swing appeared to be the same.

The lamp shown here, part of an Italian promotional video, is called *Galileo's Lamp* by the tour guides. We can be confident that the real Galileo's Lamp was not

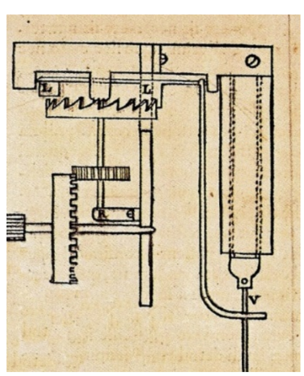
electrified. Nor does this lamp readily sway to and fro. In fact it isn't Galileo's lamp at all, having been installed about ten years after he supposedly made his observations. But it is nonetheless lovely to look at; and evidently somebody at the Pisa Tourist Office thought it was worth one minute and twelve seconds of video, so perhaps it is to us, as well.

Part II. The Pendulum Clock

A device that divides up time into equal parts is a *clock*; and you can see how valuable the property of isochrony is in a pendulum clock, since its accuracy will then not depend on every swing describing the same size arc. The problem for the clockmaker then becomes simply to *keep the pendulum swinging*. But how to do that without interfering with the regularity of its swings? Well, we know how, at least in principle; for it is what every parent has done when pushing a child's swing. If we time the pushes just right, the swing maintains its motion, either soothing or thrilling. But if we apply any of them too early or too late, the motion becomes erratic and the child soon announces its righteous displeasure.

In a classic pendulum clock the slow descent of a weight, or the protracted unwinding of a spring, is converted into a series of well-timed pushes by a triggering mechanism called the *escapement*, of which there are numerous designs. The very first pendulum clock—Huygens's clock of 1656—employed what is now called a *verge* escapement.

In Huygens's drawing, the pendulum V (at the right) is pivoted at the top and swings in and out of the plane of the paper. The heart of the mechanism is the *crown*

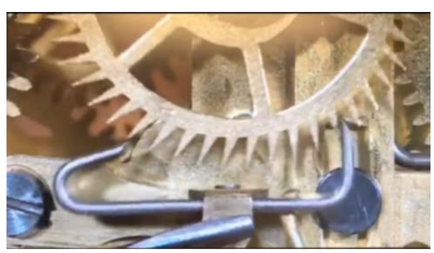


wheel, whose circle of vertical teeth gives it that name. A train of gears connects the crown wheel to a descending weight; and, were the wheel not periodically engaged by the angled “pawls” (marked L), it would spin without interruption. But as the drawing shows, the pawls are mounted on a horizontal axle that is coupled to the pendulum. As the pendulum oscillates, this axle rotates back and forth; and with each half-swing the pawls bear alternately against the vertical faces of successive teeth, so that each receives a gentle nudge as the tooth passes by. The axle transmits this nudge to the pendulum at the same point in each swing, thereby replenishing the motion which the pendulum would otherwise lose during each swing.

Let’s watch the mechanism at work.

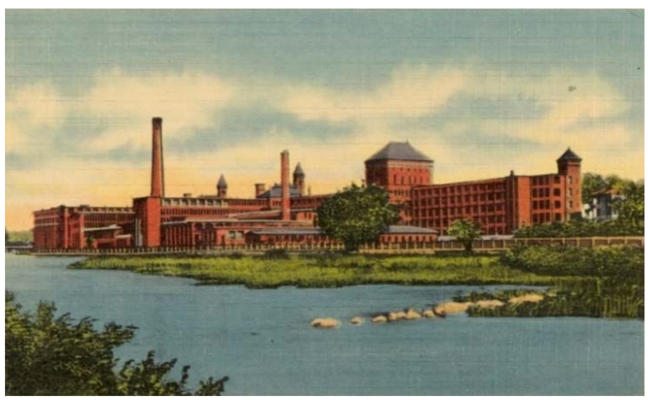
This verge escapement is part of a clock built nearly two centuries after Huygens’s; we are looking down upon the crown wheel. Notice how the pawls are brought alternately to bear against opposite teeth. Each quarter-turn of the axle corresponds to a half-swing of the pendulum. By the way, the verge escapement takes its name from this axle, called the “verge” from Latin *virga*, a rod or stick.

Many smaller clocks and watches employ some version of the *anchor* escapement, so called because of the anchorlike shape of the cog that carries the pawls. Here is a variation of the anchor design, in which filed edges of a bent rod constitute the pawls:



I hope you can see how the angled ends alternately stop, and are then pushed out of the way by, successive teeth—each time receiving a nudge that is transmitted to the pendulum. That escapement was widely employed in mantel clocks like this one:

The clock in this photo belonged to my maternal grandfather, Hyman Shengold, who was himself a watchmaker. I should explain that in the 1920's, a watch "maker" was really more of a watch *repairman*. Watches, and most clocks, were mass-produced in factories like this one in Waltham, Massachusetts.[4] But they required constant



adjustment and cleaning; and it was the "watchmaker" who performed these services, as well as more elaborate ones such as replacing springs and gears. The escapement of my grandfather's clock is not visible without disassembly; but the pendulum is. Here it is in operation.

[4] The title of this lecture intentionally echoes that of Booth Tarkington's novel *The Magnificent Ambersons*, which is set in roughly the same period of rapid American industrialization and mass manufacture as that during which the Waltham Watch Company flourished.

The pendulum is mounted on a ribbon of spring brass, which eliminates the friction and wear of a pivot and bearing, while being weak enough to exert negligible effect on the pendulum's period. The period of this model is .723 seconds.

Part III. Pendular Motion

We know how to keep a pendulum moving, but what precisely *is* the motion we are preserving? What kind of motion does a pendulum actually perform? As I noted earlier, this is not the kind of question we can answer by measuring a series of positions occupied at successive times. It is a question about what *essentially* characterizes pendular motion; and to that extent it is like asking about any of the characteristic behaviors of a cat: not a question to be answered by measurement, though measurements can contribute to finding an answer.

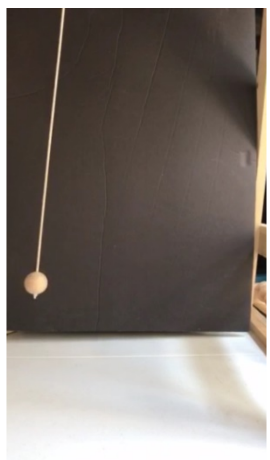
Compared to pendular motion, the two motions studied by Galileo in the *Two New Sciences* are simple indeed. *Uniform* motion, which traverses equal spaces in equal times, expresses its essential character in the single defining property of *constant velocity*. Once we know *that* property, particular circumstances, like the direction or the duration of the movement, are just ancillary details. The motion of bodies in *free fall*, similarly, has but a single defining property: constant *acceleration*, that is, rate of change of velocity with time. When the three friends depicted in Galileo's dialogue seek definitions for those motions, it is precisely these essential characteristics they are trying to identify.

Uniform motion is maintained, at least for limited periods of time, by bodies that glide on a horizontal plane. Uniformly *accelerated* motion, Galileo shows, is exhibited not only by bodies in free fall but by those that glide on an inclined plane. But *pendular* motion traverses a *circular* arc. Its acceleration not only fails to be constant, it does not even change at a *rate* that is constant. If we hoped to discover in pendular motion a *single definitive rate of change*, as Galileo did for uniform motion and uniform acceleration, we shall be disappointed. As we shall see later, *no* characteristic of the pendulum exhibits a constant rate of change.

When Plato's wise legislator undertakes to craft just laws, he looks—at least to the extent possible—to the Form of Justice. Just laws will—again, to the extent possible—mirror the Form; and to the extent that we can see the Form reflected in the laws, we will understand both what the laws aim for, and to what extent they fall short. But if we have no inkling of Justice itself, we will have nothing upon which to base our judgements except the allegations of itinerant ideologues or the passing fashions of our own circle of society (which is likely to be narrower than we believe it to be).

That, it appears, is our situation with regard to the pendulum: we do not see a Form to which its distinctive motion aspires. But what if we were to discover a kind of motion which *does* seem to be reflected in the pendulum's motion—and one which, moreover, appears to possess a degree of unity and intelligibility that the pendulum's own motion does not itself display? I believe we can all experience such an elevation of vision with the aid of a simple mechanism.

Here is a pendulum consisting of a wooden ball mounted on a slender rod. Its effective length is about 31-1/2 inches, which, at the elevation and latitude of Santa Fe,



causes it to swing out and back every 1.8 seconds, or 33-1/3 times per minute. Its motion is being continually replenished by a clock drive mechanism; except that in our technological era, the mechanism is electronic. The machinery that Hyman Shengold would have had to repair with individual brass gears and precision bearings is now a

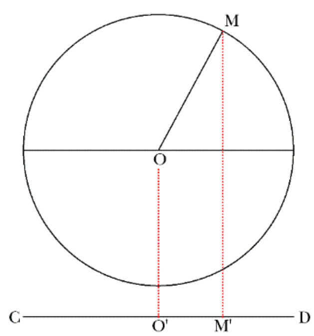
plastic frame and circuit board that, like so much else, is far cheaper to replace than to repair.

If you are old enough to remember long-playing records, which my generation called "LP's," the figure 33–1/3 will be familiar to you, for 33-1/3 revolutions per minute was the speed at which LP turntables revolved. I'm going to place such a turntable directly beneath the pendulum, with a small post mounted on the turntable directly beneath the release point of the pendulum; and watch what happens when we start both motions together:

One full swing of the pendulum coincides with one revolution of the turntable—and when we crouch down to view it from the side, the pendulum follows *at every moment* the motion of the post!—or, pretty close. Something very much like this, I imagine, was experienced by Plato's cave-dwellers when they turned their heads away from shadows and saw, for the first time, the objects that cast them. Pendular motion, we suddenly understand, is a (nearly) one-dimensional reflection of *regular circular motion*, which is two-dimensional. Can we be like Plato's wise legislator? Can we come to understand the essential character of pendular motion by looking at the Form, or paradigm, of that motion? Let us try.

Part IV. The Paradigm Circle

Let the circle in this slide depict the rotating turntable, with O representing the spindle and M the post. When we viewed the apparatus from the edge, we saw only the

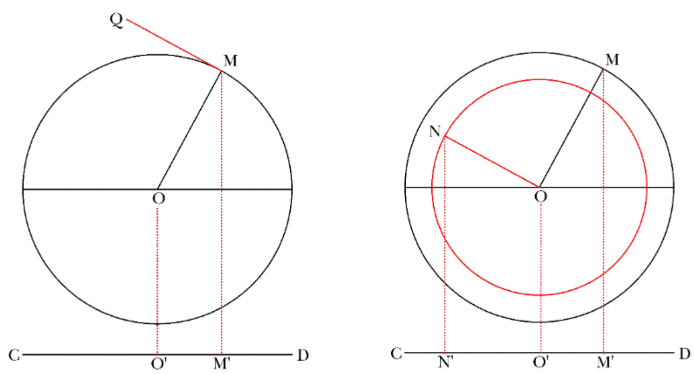


left-and-right motion of M; its toward-and-away dimension was lost to us. What we saw in the side view can be represented by a fixed point O′, the image of the spindle, and M′, the image of the post, moving left and right along the straight line between the extremities C and D, with central position O′. The length CD is the *amplitude* of the swing;[5] and of course it is equal to the diameter of the circle.

But at every moment, the swinging pendulum coincided vertically with M′. The line O′M′, which is the distance of the pendulum from its center, is therefore only an image of the radius of the circle; and the *motion of the pendulum* is but an image of *regular motion about the circle*. Taking regular circular motion, therefore, as the *paradigm* of pendular motion, generations of Johnnies have called this circle the Paradigm Circle. Since one full swing of the pendulum corresponds to one rotation of the Circle, the *period* of the pendulum swing (forward and back) is the same as the time of rotation; let us call it τ. We have at least part of what we were looking for: a unifying image with which to characterize pendular motion; and in particular we can interpret the ever-varying position of the pendulum by referring it to the strictly *regular* rotation of radius OM.

But we can do more.

At M draw MQ tangent to the circle and equal to the *velocity* of point M—or, what is the same, the *rate of change* of radius OM (in direction only, since OM is constant in



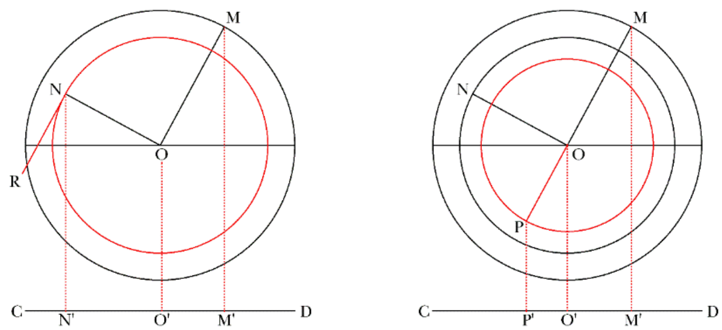
length). Then draw ON equal and parallel to MQ. Thus it is clear that ON, likewise, represents the velocity of point M.

Then, just as O′M′, the image of OM, expressed the pendulum's *distance from the center* at any moment, so too O′N′, the image of ON, expresses the pendulum's *velocity* at that same moment. So we can know instantly that if the pendulum is at M′, its velocity is O′N′. In particular, it is obvious that the pendulum has maximum velocity at the center and zero velocity at the extremities. Of course we had already nourished intimations of this fact; but now we are gaining it through *clear sight*. We are *seeing* truth—or about as near to that we are likely to get unless we may someday join the banquet of Plato's blessed gods and, with them, "rise up to the vault of heaven" to be nourished by "gazing upon truth"—of course I'm quoting from *Phaedrus*, 247.

There is no need to stop at this level.

[5] In mathematical treatments, the amplitude is more often defined as half this amount.

At N draw NR tangent to the circle and equal to the velocity of N. And proceed, as before, to draw OP equal and parallel to NR; so that OP, too, is the velocity of N—that



is, the rate of change of ON, while ON in turn is the velocity of M. Then OP is the rate of change of the velocity of M—it is the *acceleration* of M.

Finally, just as O′M′, the image of OM, represented the pendulum's distance from the center, we recognize that O′P′, the image of OP, represents the pendulum's acceleration. Since M and P must always lie on opposite sides of center O, we instantly grasp—*by sight*—that the pendulum's acceleration is always in the direction opposite to the pendulum's displacement from the center. Moreover, it is always *proportional* to that displacement, since the circles revolve without changing their radii. Wherever the pendulum is at any moment, it is *accelerating towards the center* with an acceleration *proportional to its distance from the center*.

This proportionality has every appearance of being an *essential principle*, which the Paradigm Circle makes manifest when carried to its threefold development. For that reason I want to make a distinction now, whose importance will emerge later. The line O′M′ is not merely a distance; it is *the path that must be traversed* if the pendulum is to reach the center. At the present stage of our reasoning, we are supposing that path to be a straight line; but if we wish to apply the Paradigm Circle to a body that moves along a curve, the distance O′M′ would have to represent *curvilinear* or *arc distance*. That consideration, however, may serve to plant in us some doubt about the ordinary pendulum, since its path is, in fact, a curve.

Once we have appreciated the proportionality between *acceleration* and *distance to be traversed* as an essential principle of pendular motion, we can state it in a quantifiable way. Since each member of the threefold paradigm circle completes one revolution in one period τ, the speed of point M on the outermost circle will be

$$\frac{2\pi\,\mathrm{OM}}{\tau}.$$

And since that speed was represented by ON, we have

$$\mathrm{ON} = \frac{2\pi\,\mathrm{OM}}{\tau}, \quad \text{that is,} \quad \frac{\mathrm{ON}}{\mathrm{OM}} = \frac{2\pi}{\tau}.$$

In the same way, the speed of point N on the middle circle is

$$\frac{2\pi\,\mathrm{ON}}{\tau}\,;$$

and since that speed was represented by OP,

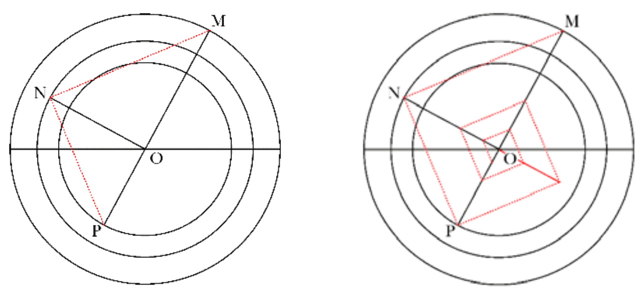
$$\mathrm{OP} = \frac{2\pi\,\mathrm{ON}}{\tau}, \quad \text{that is,} \quad \frac{\mathrm{OP}}{\mathrm{ON}} = \frac{2\pi}{\tau}.$$

Thus the quotients OP/ON and ON/OM are equal to one another, and each is equal to 2π divided by the period. Combining *ex aequali*, we have

$$\frac{\mathrm{OP}}{\mathrm{OM}}, \text{ that is, } \frac{\text{acceleration}}{\text{distance}} = \frac{\mathrm{OP}}{\mathrm{ON}} \times \frac{\mathrm{ON}}{\mathrm{OM}} = \left(\frac{2\pi}{\tau}\right)^2.$$

Thus the ratio of *acceleration* to *prospective path* is not only an essential principle of pendular motion, the *size* of that ratio actually determines the period.

Nor is that all. If this is a feast for the intellect, like that of which Zeus and the blessed gods partake in the *Phaedrus*, there is here abundance of nourishment. Since the ratios OP:ON and ON:OM are equal, the triangle formed by joining MN and NP is a right triangle.



And it is clear that we can continue the pattern to construct an unending rectilinear spiral, finding more and more circles-within-circles, without limit; and these will represent an unlimited number of *rates of change* of *rates of change* of *rates of change*, on and on, without ever reaching an end. *Now we understand why we found pendular motion so challenging to grasp*. When we began, we knew of only one way to understand motion; that was Galileo's way of *finding a constancy underlying the change*—as when he identified first *constant speed*, and then *constant acceleration*. But that method, when applied to the pendulum, turns out to be an *infinite* task. But the discovery of the Paradigm Circle—may I call it Plato's way?—invites us instead to discern a Unity instead of an infinite sequence.

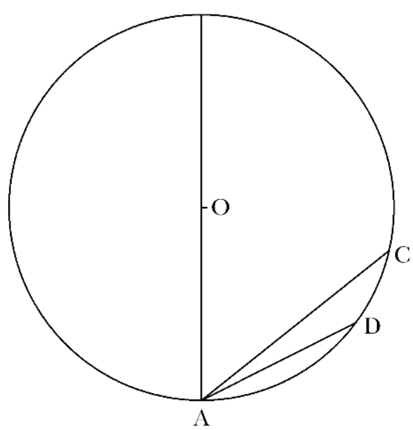
But where, in all this, is that marvelous property of *isochrony*, which Galileo observed in the swaying lamp and we witnessed in our earlier pendulum video? There, you will recall, the pendulum's swing took the same amount of time, regardless of its amplitude. The Paradigm Circle expresses isochrony by *turning with the same speed* regardless of its size. But *does it* necessarily turn with the same speed regardless of size? Well, we could point to the fact that the definitive ratio which determines the period, the ratio of acceleration to distance, makes *no reference* to the radius of the circle; and therefore, we might argue, the speed with which the circle turns should be independent of the pendulum's initial displacement—and *that independence*, we may contend, is what secures isochrony.

This seems to me a disappointingly weak argument. The absence of any connection between the speed of rotation and the size of the circle shows only that isochrony is *consistent* with the paradigm. But I would have expected such a universal and apparently fundamental characteristic of the pendulum to be *embedded* in the paradigm. That it is *not*, is to me just as astonishing as if we were to behold the Platonic Form of Cat but fail to find anywhere in it the propensity to chase mice. Then is isochrony, after all, *not* an essential characteristic of pendular motion? Yet *if* not, how does it come to be such a universal property among pendulums?

Part V. Natural Acceleration

Long after the young Galileo's encounter with the swaying cathedral lamp, he made systematic studies of the pendulum;[6] these, however, while meticulously confirming the *fact* of isochrony (allowing for the effects of air resistance), did not undertake to *explain* it. Nor, perhaps, should we expect him to do so, since his propositions in the Third Day deal with uniform acceleration; while the pendulum obviously undergoes *varying* acceleration, since it traces out a path having varying slope. Nevertheless, Proposition 6 of the Third Day (page 221) seems at least suggestive of pendulum isochrony, since it establishes isochrony of descent along chords of a circle.

Galileo's drawing is shown on the left; and the proposition may be paraphrased this way:



Let AC and AD be chords of the same circle. If bodies are released from rest at C and D, they will descend along chords CA and DA in equal times.

This does not bode well for the pendulum; for if the pendulum bob is suspended from center O, it will descend along the circular *arcs* CA or DA, not along their chords. But if the descents along the *chords* are isochronous, it is hard to see how descents along their *arcs* could be isochronous too. And if you recall the pendulum timing test we made earlier, the period for the larger swing was in fact greater than that for the smaller swing by about 1.6%.

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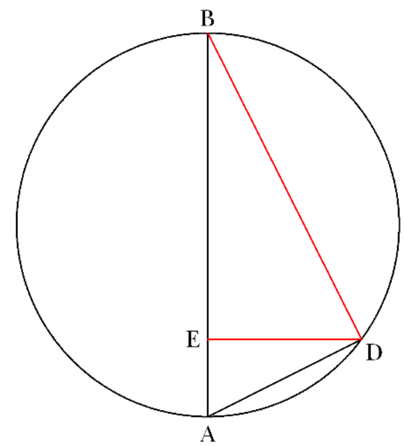
[6] Two New Sciences, 139–141, 277.

Nevertheless, as arcs become smaller, the more nearly do they coincide with their chords;[7] and so simple pendulums ought to become *more nearly isochronous the smaller we make their arcs*. This is an inference we can make directly from Galileo's Proposition 6, and it surprises me that Galileo did not.[8]

In fact, one of Galileo's proofs for Proposition 6 (the one he describes as based on "mechanics") reveals a proportionality between acceleration and distance reminiscent of the proportionality we found in the Paradigm Circle.



For consider any chord, say, chord DA. Join BD, as in the diagram on the right; and drop DE perpendicular to diameter AB.

Now Galileo had already established that the acceleration experienced by a body gliding on chord DA is to the acceleration of free fall as

$$\frac{EA}{DA}.$$[9]

But the acceleration of free fall is constant. Therefore the acceleration along DA is *itself* proportional to EA / DA. And, by similar triangles,

$$\frac{EA}{DA} = \frac{DA}{BA},$$

while diameter BA is constant for a given circle. Therefore the acceleration along chord DA is *proportional* to DA. That is, acceleration is proportional to distance, just as we found in the Paradigm Circle, except now for uniform rather than for varying accelerations. Apparently it is not the Paradigm Circle itself, but the *proportionality between acceleration and distance* that underlies isochrony. We effectively built that proportionality into the Paradigm Circle when we supposed the circle to rotate at a fixed speed. But now we see that the same proportionality can obtain, with the same isochronous results, even when there is no periodic motion at all. The Paradigm Circle is indeed "paradigmatic" for pendular motion—even though the simple pendulum doesn't quite live up to it. But the roots of *isochrony* appear to lie elsewhere—in the realm that Galileo calls "mechanical"; that is, the motions of bodies insofar as they *act on*, or *are acted upon by*, other bodies.

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[7] Compare Newton, *Principia*, Lemma 4 and Lemma 7 (the "microscope" lemma).

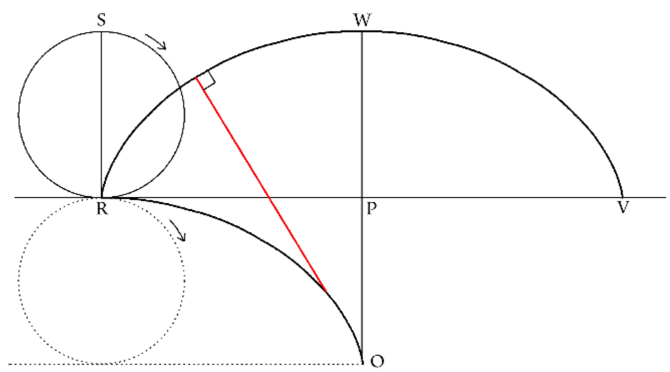
[8] "So far I have demonstrated without transgressing the boundaries of mechanics; but I am unable to demonstrate that the arcs [DA] and [CA] are traversed in the same time, which is what I seek." Galileo to del Monte, 29 Nov 1602, trans. in Paulo Palmieri, "Galileo's Experiments with Pendulums: Then and Now" (2007). http://philsci-archive.pitt.edu/3549/. Accessed 16 July 2020.

[9] For us this is an obvious consequence of reckoning the components of vertical acceleration; compare Newton, *Principia*, Corollary 2 to the laws of motion. Alternatively, by a corollary to Galileo's Proposition 3, the times of descent along EA and DA are as their lengths, respectively. But the final velocities, which are as the accelerations and the times jointly, are equal, by his Postulate. Then the accelerations are reciprocally as the lengths of the planes. Q.E.D.

Since it turns out that the simple pendulum does not actually execute “pendular” motion after all, perhaps we should abandon that name? The term that is now standard for the motion defined by the Paradigm Circle is “simple harmonic motion,” a title that is, to most present-day readers, simply opaque.[10] In contrast, a body that *executes* simple harmonic motion has a name that is easy to understand; it is called a “linear oscillator,” after the proportional, or *linear*, relation between acceleration and distance.

The simple pendulum, then, fails to instantiate the Paradigm Circle; but there is a pendulum that succeeds. Both Newton and Huygens were able to show that the path which genuinely secures isochrony of descent is not the circle but the *cycloid*, a curve generated from the circle.

In Huygens’ construction,[11] as circle SR rolls clockwise along the horizontal RV, the point R fixed on it traces out the curve RWV, a cycloid. Alternatively, if the dotted circle rolls clockwise along *its* horizontal, a point R fixed on *it* will describe cycloid RO.



He then proves that any tangent to the lower cycloid cuts the upper cycloid at right angles. That perpendicularity is the property which enables a suspended body to swing in a cycloidal curve, as we can see by inverting the picture.

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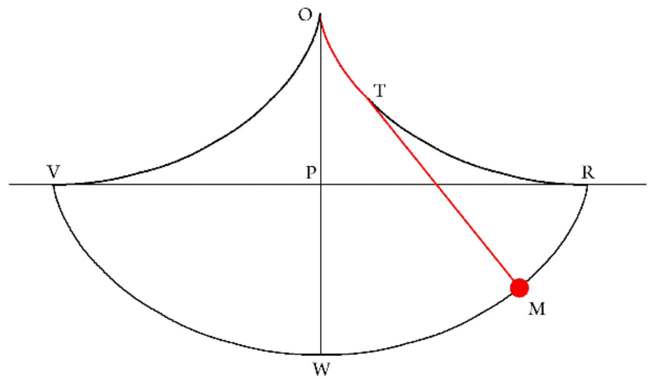
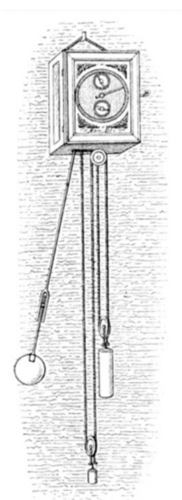
[10] W. Thomson and P. G. Tait, Treatise on Natural Philosophy, I. i. §53. “Simple harmonic motion... Such motions [are] approximately those of the simplest vibrations of sounding bodies,... whence their name” (Oxford English Dictionary).

[11] Blackwell, Richard J., trans.: *Christiaan Huygens’ The Pendulum Clock or Geometrical Demonstrations Concerning the Motion of Pendula as Applied to Clocks*. Iowa University Press (1986). Compare Prop. XIV, p. 50.



Huygens adapted this geometry to produce the first pendulum-controlled clock.[12] Although mechanical clocks had existed since at least the fourteenth century, using a variety of regulating mechanisms, only Huygens's cycloidal pendulum incorporated a *theory* of isochrony. In Huygens's clock, the bob M is suspended from O by a flexible strip, but it is constrained between two cycloidal ears, OR and OV. As the pendulum bob swings to the right, the strip bears against one ear to become tangent at point T; the bob therefore traces out a path on the cycloid RMWV. From the inclination of the curve at each point M we can show that the acceleration at that point is proportional to the arc WM. And since it is *that very arc* which the pendulum must traverse in order to reach the center, the cycloidal pendulum achieves that essential proportionality which the Paradigm Circle ordains but which the simple pendulum can only approximate.

Here is a drawing of Huygens's first clock, built in 1657. Its pendulum really *is* isochronous, even for wide swings such as the one depicted; and Huygens's pendulum



clocks achieved far greater accuracy than did earlier clock mechanisms. But was that improvement due to the *isochronous path* of the pendulum, or to the *pendulum* itself, regardless of path?

It turned out that in actual cycloidal clocks, friction of the suspension strip against the cycloidal ears, together with other problems that came about with large swings and

[12] Galileo designed, but never constructed, a pendulum clock escapement mechanism.

their resulting high pendulum speeds, outweighed the practical advantages of Huygens's theoretical improvement. It was less troublesome on the whole to restrict ordinary simple pendulums to small deflections.

Is there a cautionary lesson here? Huygens's pendulum is a more perfect instantiation of the Paradigm Circle than the simple pendulum—but the simple pendulum functioned better. Suitably restricted, the simple pendulum turned out to be an alternative technological path, one equally directed towards the Paradigm, but less obviously so—and which, perhaps for that reason, Huygens overlooked. That may serve to remind us that the work of ameliorating practical affairs is subtle. If our efforts are bent too doggedly upon mere *imitation* of the perfect, they may prove superficial, formulaic, and distracting. Granted that *insight*, or even a mere *glimpse*, into the Timeless is what the soul craves, knowing when and how to blend that vision with the logic of the Cave is an art in itself, one that well deserves to be both cultivated and cherished.

Thanks, everyone.