On Incommensurability

This is an attempt to think about a question that is considered settled and so I must warn you that I may seem to be wasting our time. Like other such ventures you may encounter at St. John's it is less concerned with what we might be said to know either these days or at any time and more interested in how we think we know it. It is also, as a Wednesday Afternoon Lecture, a bit more of a work in progress than a Friday Night Lecture might be. You and I are in the enviable position of not having to worry what each other will think if you just virtually slip out of this virtual Hall. You are putting up with a good bit already in these odd times; if what is on your screen late on a Wednesday afternoon is not at least moderately absorbing then do something else.

Incommensurability in its most general sense describes the relation of two or more measurable things that have no common measure. We might say that the length of a road and the loudness of a shout have no common measure: the road is not twice as long as the shout is loud, or any other multiple or fraction of the shout's loudness either, though both length and loudness can be measured. Must the two things, road length and shout volume, have some common measure in order that we be aware of both of them at all? Do we have to be able to measure a thing to know it is there? We commonly suppose that the existence of things is found in the evidence of our senses. If I can see the road or hear the shout, then it is there. Seeing and hearing do not have to be the same thing in order to testify equally well to the presence of the things we see and hear.

Nobody is meanwhile scandalized that many things have some kind of relation of incommensurability to one another. The situation grows more interesting if two things that could easily be imagined to have a common measure can be shown to have none. Suppose I could show you two straight lines that cannot possibly have a common measure. They would not be lines in pencil lead or chalk; but if those are the only kind of lines you will accept you had better virtually slip out now. No matter how small the equal pieces may be that I can divide one line into, I would show that no such piece would ever fit a whole number of times into the other line. It seems unimaginable that this should be so. It may be even harder to imagine than the possibility – shall I call it the "opposite possibility"? -- raised in the *Meno* that all human excellence could be grasped as a simple unity.

Let us begin with what measuring is. It seems to be a laying out of something next to something else in order to tell which one is more in some way and which less. Euclid says that a small line measures a larger line if it fits into it some whole number of times. We can, using a looser notion of measurement, easily see who is taller when two stand side by side. But how do we know who will make a better ruler? If there is a unit by which leadership may be measured, it has not yet been discovered. Creon in Sophocles's *Antigone* knows that he is being measured by the hard times he must rule in, but he does not imagine that it might be in yielding to Antigone that he would show his worthiness to rule. "To what must I show myself adequate?" cries Creon. Or more literally "What is it? To what kind of happenstance do I arrive commensurable?" The Greek word translated by "commensurable" here, "summetros", means "sharing a measure with." Symmetric things in English might seem at least to have their parts in proportion to each other: things can be in balance and hold together by having a common measure. Creon seems worried that there may be situations he cannot rule. He is not wrong to worry: rulers need not only strength but understanding. The human soul itself could turn out to contain parts that share no common measure. Do our powerful fears and desires have any common language with our cautious reasonings at all? To find oneself incapable of judging a situation for want of a proper measure is perhaps not an uncommon fear. It might be like an anxiety dream in which you have to take an exam in a language you don't recognize.

Anxious democracies often invoke the importance of the things we all have in common, our commensurability; we say that is our strength, what unites us. But those things must be so often invoked because of the strength of what divides us: each of us is a unique self. Some of the things we want most cannot be shared. We find ourselves sometimes desperately wishing a very particular someone could see something in us that is to be found in nobody else. Sometimes we boast of how well we know that no-one else can die for us; it seems to make not only our deaths but our whole lives look at least for a moment like inalienable property and to give us permission to do whatever we want with every precarious moment we are alive. Have I persuaded you that incommensurability has much more to do with us all than we might usually think? Here is one last example, from the Hebrew tradition. Adam, speaking to God after God has tried every other animate possibility before finally solving the question of where to find Adam a companion and has divided the first human in two: "This, at last, is bone of my bone of my bone and flesh of my flesh!" Hear the deep relief. Are the Tyrant and the ordinary Citizen of one bone and flesh as well?

Socrates reminds us early in his conversation with Meno that Meno's father was a friend of Xerxes, the Great King of all Persia, and he means us to glimpse Meno's inner Xerxes, and our own. Meno has grown up knowing that he is two degrees separate from the absolute ruler of all the lands from Egypt to Turkey to the edge of India. Those who unashamedly take Xerxes as a measure in their drive for greatness are likely to refuse to be run-of-the-mill examples of the generically Human. Sophocles minces no words in titling his paradigmatic tragedy about a man who introduces himself as "Oedipus the Great": the play is called *Oedipus the Tyrant.* We each have something in us that wants to be a God, utterly unconstrained. Whether that leads to philosophy or tyranny or eternal salvation is not clear. Each looks, and may seek to look, incommensurable with our daily life.

Incommensurability and the Golden Section

My wonder at the Incommensurable was re-kindled a few years ago in a Freshman Mathematics Tutorial when we came to Euclid's proposition eleven of Book Two, which shows how to divide a line so that the rectangle formed by the whole line and the smaller piece of the division will equal the square on the larger piece. The more famous example is already hiding in I, 47, the Pythagorean theorem. In neither proposition does Euclid say anything explicit about the topic. But it somehow came to mind for me in Book Two. When we have learned how to make the division of the line in Proposition 11, we may wonder, even if we do not recognize what we can now do as dividing a line in the so-called "golden section": is there a numerical relation between the two pieces into which we divide the given line? Whether or not it exists among numbers the Golden Ratio has been found in countless examples of the great works of art and architecture that have survived their Ancient Greek makers. The Parthenon is only the most famous example. But is that ratio to be found among numbers? That is, could the two pieces into which Euclid's line is divided be measured by some common unit and would there thus be a fixed numerical ratio between the two pieces? Are there then two numbers whose relation to each other names a specific kind of beauty? Maybe so. We might on the other hand be disappointed to find beauty of any kind to be, as we say, formulaic. And yet, perhaps the example of maximum commensurability, instantly recognizable in its unsurpassed simplicity, is the ratio of the double. Aristotle is not afraid to say that the sound of the Octave is the embodiment of a 2 to 1 ratio and that it is not only agreeable but beautiful.

Why should we like the relation produced by doubling? This is the sort of question that may conceal more depth than we think: what does it mean that beauty could be a kind of order? The two sets of vibrations that coincide every second beat: ONE two, ONE two, ONE two, hundreds of times per second are a kind of hyper-allegro march or two-step of 440 against 880 beats per second that we hear as two pitches, a perfect mingling and repetition of Same and Slightly Other. We may not even perceive more than one pitch, the two blend so well. Two faces, neither especially noteworthy, placed side by side and seen to resemble each other will often make us laugh with pleasure; who knows why?

What can it be that makes twoness beautiful? "I am dying, Egypt, dying," repeats Marc Antony to Cleopatra, in Shakespeare's play, calling her "Egypt" in a rhetorical figure, as though the nation she rules has produced in her an image of itself, a kind of double. "Dying…" he repeats, "Dying." To say something twice is already to begin a poem, because in a well-crafted poem nothing is only itself, and everything implies everything else. If it were sufficient to call a thing by its name just once in order to say all that it is or all one may have in one's heart, then perhaps there could be no poetry. The verse structure of the ancient Hebrew Psalms, attributed to King David, is a kind of doubling or rhyming in which the sense of a line is repeated in different form to make a pair, or to reveal that One is Two and vice versa. One could call it "Octavic structure". The two beginning lines of Psalm 23 :"The Lord is my shepherd/ I shall not want", might be rendered in prose: " Since the Lord is my shepherd, I will have all I need." The first thought is completed and becomes one of a unified pair as we hear that to have the Lord as one's shepherd must involve freedom from want. The logical undeniability of the second line, given the premise of the first, retroactively transforms the first line into a proclamation of complete trust and proud allegiance.

"I am dying, Egypt, dying, only/ I here importune death a while, until/Of many thousand kisses the poor last/ I lay upon thy lips …"

says Antony; the first five words could be his very last if taken to mean: "I have killed myself because they told me you were dead," but he speaks again,
"… Dying…" that is, 'it is nothing I can draw back from now, it is fully real that I will soon cease to be real.' And perhaps he is asking himself and his love how a story that seemed so full of strength and new possibility has now brought inescapable death. The poetry of the drama perhaps plays with incommensurability here: the lines are spoken as Cleopatra and her two women servants are about to hoist Marc Antony up to the window of their tower. He is almost immovable and will very soon be unreachably far away, among the dead, and yet he and they are asserting the greatest nearness between him and Cleopatra that these great lovers have ever felt. Two are becoming one as they near the vanishing point.

If the question of whether it can be numbered does occur to us about the golden rectangle I think Euclid supposes that we ourselves may be able to supply the surprising answer in the negative with only a little reasoning. It is unfortunately not the kind of reasoning I am confident will succeed in this lecture. I have tried several times to write out a brief and perfectly clear account that shows the two pieces of the Golden Section have no common measure; but without the assurance that audience members could be free to pause the lecture and ask a question wherever something is not making sense to them I have felt that my attempts have failed. There may be an unexpected commonality between Plato and Euclid in that the real life of both authors' work only manifests itself fully in conversation, and in deeds. I will include my proof that the Golden Section makes two lines that have no common measure in the written text of this talk, which will be available through the Dean's Office, and maybe another short proof that the diagonal of a square has no common measure with its side. Neither is terribly difficult to follow. Even one of them might still take up too much of our time together today. For now I ask you to grant me that both these things are provable.

[Shall I belabor your sense of wonder with one more corollary? If line A and line B should be shown to have no common measure, then the line that is their sum could be measured neither by any equal division of A or of B. Add them together to make a new line and call their sum C. So we will have three lines, none of which shares a measure with either of the others. If we add A to C then we have a new line, D, the largest of the four, which likewise has no common measure with C and thus with any of the others, and so we may proceed ad infinitum, making new lines, each of which has no common measure with any of its components: an endless array of mutually incommensurable lines growing larger and larger forever. We could have proceeded by subtraction and produced the same result in the direction of the infinitesimal.]

Perhaps the beauty people say is to be found in the Golden Section and made visible in the dimensions of the Parthenon and of numerous Ancient Greek sculptures and vases, and paintings would be a kind of opposite to the beauty of the Octave. The Two to One ratio is commensurability at its clearest and simplest, expressed in the very smallest numbers, while the Golden Section is a ratio we cannot express but only continually approach in even the very largest numbers. Yet it too has a kind of sublime geometric simplicity: a line has been divided in two pieces so that the area made by a square on one piece and that made by a rectangle of the whole line with the other piece are equal.

In the language of ratios, which the Freshmen will learn very soon, the smaller piece has to the larger piece the same ratio as the larger piece does to the whole. In that same language the side of a square equal to a unit will have the same ratio to the diagonal of that square as the diagonal has to two of those units. The diagonal is the mean of One and Two, somewhat as the larger piece of the Golden Section is the mean between the smaller piece and the whole.

Measure and Counting

It was one of the complaints of the traditional moralists of Athens that Socrates was a Sophist, a teacher of slippery arguments by which to win debates in courtrooms or public assemblies and that he could make the weaker argument look like the stronger; that he could make you think that day was night or even was odd. These were some of the accusations brought against him in the trial that led to his death.

When Meno's slave has seen that doubling the side of a square has not given us the length to produce a double square, but rather has produced a quadruple, Socrates innocently asks him to say exactly how long the line would need to be to give us a double of the original square, or just to point to it if he prefers. Plato is well aware that there will be no possible naming of the size in any units that also measure the side of the square, and yet that it is very easy to draw a line, the diagonal of the original square and then to point to it as the right size for the side of a double square. But the side and the diagonal are incommensurable. This would be a good moment to consult a written-out proof, one or two of which I will append to this talk.

[Appendix: Euclid I, 47, which shows that the square on the hypotenuse of a right triangle is equal in area to the sum of the squares of the other two sides can be applied to an isosceles right triangle that is half of a square. Then the squares on its two equal sides will add together to make an area equal to the square on its larger side which we may call the hypotenuse or the diagonal of our half square. So the square on the hypotenuse has an even number of square units. This is only possible if the hypotenuse itself is an even number of units long. But if the ratio of side to diagonal is expressible in numbers then it has an expression in the lowest possible terms, as e.g. 3:2 are the lowest terms for 6:4. One or both numbers will be odd in such lowest-term expressions. Let our half-square triangle's sides in their relation to its hypotenuse be expressed in lowest terms. Since this case has a diagonal with an even numbered length, the length of the smaller side must be odd. But since the diagonal's square is even then we know its length must be also; now to be even is to be two times some number, for that is the definition of "even". So when we multiply it by itself to get the area of the square on the hypotenuse we will be making a square whose area is (2 x N) x (2 x N) where N is just whatever number it needs to be to give us, when doubled, the even numbered length of the hypotenuse of our original triangle. That means our square on the hypotenuse, or on the diagonal is (4 x N x N) square units. So each of the squares on the sides of the isosceles right triangle will be half of that, or (2 x N x N) square units. Aha! Each smaller side of our original triangle, since it has a length whose square is 2 x N x N, must after all itself be even for no even square can come from any but an even side. But we said it had to be odd because the hypotenuse was even and the sides of our triangle were to be expressed in lowest terms. If the side and the diagonal of a square have a common measure, i.e. can both be expressed as whole numbers of the same 'unit', then we have shown that the same line must have a length measured by a number that is both even and odd. Since there is no such number, we conclude that our premise must have been mistaken and there is no common measure.]

Something happens to a mind that has begun to follow geometrical demonstrations; something that looks like a choice as it regards the truths of different realms; maybe it shows itself most clearly in the response to the absurdity that results when the square and the diagonal are assumed to have a common measure. It does not look like peace of mind. The Imagination presses to remind us that we may divide the side or the diagonal as finely as we like, and insinuates that anyone with sense would see that somewhere in the realm of the very small there is bound to be a piece so small as to measure whatever other piece from wherever else we might assign it to. The Reason insists that no possible common measure can be found which does not involve the result that an even number must also be odd.

Aristotle points out that as to the role of wonder in Philosophy, although Socrates may say Philosophy begins in Wonder, we can see how in some cases wonder must give way to a kind of familiarity and that what would now really produce wonder would be a demonstration that diagonal and side were after all commensurable. More wonder would be there than in the known outcome, namely that they are not and never can be.

But I am afraid that the original wonder has not ceased to work on me, and even some familiarity with the proofs has not driven away a sense of aporia about their result. Is the incommensurability that lurks so near the beginning of Geometry really just something to get over? Would that make it like the contemporaneous horrifying discovery that lies could prevail over the truth among the audience of a fair and open discussion? Day can be made to seem night. One must still make up one's mind if that discovery means no persuasion by words is ever to be trusted, or if something like truth can somehow still be approached. That is the choice I am thinking of. If, as I am suggesting, Euclid well knows the problem of incommensurability, and even expects his more discerning readers to perceive it very early in his book, then we see it has not discouraged him. Is there a common unit that measures both our thoughts and the world? Let us return to this matter of measuring in its primal form of counting.

When we count we could be said to measure the "how much" of something in units. We want to know how big our herd is today, perhaps to see if any lambs were carried off in the night, and we lay down a kind of measuring stick called "one sheep" next to the herd and see how many we can find in it. The unit, one sheep, is not exactly like a fixed length: it may match a small lamb in one instance, a large ram in the next. They are equally sheep and there are two of them. Cattle ranchers talk about how many "head" of cattle are on their ranch, so that the head has become the unit, since cows, bulls, and calves each agree in having one head and so can be counted quickly by counting heads. Each particular sheep's head is not exactly like any other, any more than each sheep was. "A unit," as Euclid will say in Book Seven, "is that by which each thing is called 'one'." Actual unity is like perfect doubleness, not a thing one finds in one's hands, (although hands may seem quite a good example of the double,) but it is a way of seeing with the mind's eye and of talking. Things appear to come in kinds. A kind is a natural unity. It may not be saying too much to say that different kinds must always be somewhat incommensurable. Money is a fiction that lets us pretend that three days of the labor and materials and skills of shoe-making could be equal to one day of the labor and material and skills of house-building. Money and the market price allow us to set things as equal that in fact have no common measure. Euclid says that ratios can exist (only) among things of the same kind and that things are of the same kind which can by multiplication exceed one another. No number of houses can exceed a shoe, or even equal one, whatever Mother Goose may say.. Comparing houses to shoes is, as we say, like comparing apples to oranges. What lets us count heads or noses or whole sheep of different ages and sizes without difficulty is the notion of the pure unit: the oneness that any thinkable, nameable thing must have to be a thing at all. Things that are generically of the same kind can be counted by reference to ones that are all exactly the same as each other. Do we invent these units? Different assemblages of these ones are the different whole numbers. Each such assemblage has in addition to its component units a single unity peculiar to itself: three is one number and is different from all others. It is not possible even to be as a multiplicity without at the same time being a kind of one. Can numbers image The World of Being? It sounds absurd. Yet there are enough numbers for every individual being there is or ever has been. And perhaps there would turn out to be enough groupings of numbers to mirror the groupings of things by kinds, by larger and smaller categories. If everything that is real can be named and reckoned with by computers then we are already deep into the project of mirroring the entire world in numbers. Does it matter if we suppose that numbers are simply a convenient labelling device we have in some unspecified way dreamed up or if we think that the three of three horses and the three of three frogs are both able to be three by some active power, a unifying force at work on them as we might imagine the activity at work on making a horse be and stay a horse at all? It might matter quite a bit.

[The Freshmen will soon read a book in which Socrates offers a line divided in four as an image of all visible and knowable things; the primary twofold division is between knowable and visible, and each of those two parts is again divided in two "in that same ratio", however we may like to think of it, between knowable and visible. Within the example we have to represent the relation between visible and knowable simply as a matter of the sizes of our two pieces of an original line, which itself we may suppose to represent all that is. Should the visible be larger than the knowable or vice versa? If we draw a line, divide it first in two in any way we choose, and divide each piece in the same ratio as the first division, we will produce four lines, the greatest, the least, and the two in the middle. It can be easily proven that the two in the middle will be of equal size. In Socrates's example the upper division of the lower part of the whole line represents among the visible those things we would call “visible originals”: trees, people, animals, stars etc.; while the lower division represents “visible images” of those things: shadows, reflections, paintings, and so forth. The lower piece of the upper part of the original line thus must represent in the realm of the knowable the knowable images, which Socrates suggests are mathematical beings: triangles, lines, points, drawable figures and representable numbers, etc. while the uppermost piece of all will represent whatever could be the “knowable originals” or the actual things that are known, whether always through images or perhaps sometimes directly through themselves. The mathematical or “learnable” things would thus be the shadows of the knowables, and the quantity of images of the knowables would match that of the originals of the visibles. Or would they overlap? The things both knowable as images and immediately visible as things would then be the same.]

The appearance of things coming in kinds and our capacity to see and grasp it grounds most of how and what we think. When we recognize something at all we are finding it to be a part of some whole, an example or a fragment of a certain kind of unity. Here too we seem to have a choice: shall we notice this power we seem to have, give it a nod, and just get on with our work of understanding and re-shaping the world, or shall we try to dwell upon it and wonder at it? Maybe returning to it would modify our urge to reshape the world a little. On the other hand it may be so near to our root that to dig it up would leave us nothing to get near it *with*. Just the same, let us try a little right now.

When we encounter something that appears especially unified, that brings together many parts in many ways to make a One, we feel delight. We call it beautiful, whether it be a painting or a melody or a story, or a face, or a sunset. We locate unities within larger unities: person, family, city, nation, Cosmos. When we meet something that strongly resembles something of a different kind in some respect, we delight to bring the two together in our speech. We call it making a metaphor. "All Flesh is Grass". Is it a stretch to describe this behavior as a kind of counting or measuring? It seems to have little to do with the measuring about which Nietzsche complains. That, he says, is a devaluing of the only world we have, by the false invention of another better world somewhere else, whose beings we claim to glimpse and by which we judge our world and find it wanting. This measuring that lets us see how flesh is grass might be no slander on real flesh or grass, but a primary encounter with both.

People sometimes express disappointment that the earliest ancient examples of writing – the wedge-shaped marks in tablets of clay that were baked or dried in Mesopotamia many thousands of years ago -- seem to be restricted to inventories: mere lists of things or quantities of grain. But counting or measuring in a primal sense is the essence of our grasp on the world, at least as that grasp is found in language itself. We need not shy from calling those lists the first written poetry. The transition from listing names (i.e. units) and numbers of things to writing poetry seems very slight indeed compared with the transition from an unbroken sequence of immediate stimulus-reactions, to names and numbers of things at all. Indeed ancient poets as well as modern ones are notoriously fond of lists as such. Homer gives us the Catalog of Ships. The Hebrew Scriptures list who begat whom. And both are masters of metaphor and measure. Homer speaks unforgettably of the sword or spear blade cutting flesh as "Pitiless Bronze." If we are not explicitly measuring by unities, or counting -- "giving an account" as we say -- we are measuring still. Saul was by head and shoulders the tallest among the men of Israel. Thomas Hobbes says in *Leviathan* that *all* thinking is counting and computation: adding and subtracting. It is a surprising agreement with the Platonic insight that our capacity to see unity is what makes us human. And somehow we can see unity in what refuses to break into natural units. The Continuous must have a unity of its own: it is imaged in the line in which every point has another as near to it as you please. Could the Unlimited be another ingredient of the World, like wholeness being present in everything that is? It could never be visible without having already undergone some unification but it might always maintain its indeterminacy in a kind of refusal of any permanent allegiance to particular unity or identity. Nothing is immune to change and everything must decay. Today's Ponderosa Pine tree seems fully formed and vitally involved in being what it is; it seems almost to breathe if you look at it in the sun and the wind. Botanists will tell you that it does breathe. But some few years from now it will be lying on the ground, rotting into dirt, relinquishing the noble form that seemed completely to possess it.

In Incommensurability we seem to have found a thing we cannot imagine, if to imagine means to give a unified identity to something, but must nevertheless think as true. This is already remarkable and may encourage us to be more careful in distinguishing the imaginable from the true in other cases. The imagination is not a perfectly reliable guide even to the possible, let alone to the true. Is it the infinite divisibility of the line that leads our imagination and our reason in opposite directions? Perhaps we can know some things to be true regarding what is infinite without being able to imagine them. Moses Maimonides points out that Apollonius proves a curved asymptote approaches a limiting straight line so that the distance between them is forever diminishing without limit and yet without ever entirely disappearing. They can get infinitely nearer forever without meeting. Maimonides says that we cannot imagine this but that we can know it. Even to look down railroad tracks includes imagining we see that they meet at the distance of the vanishing point. We may get so used to knowing something, that we think we *are* successfully imagining it, or we may decide to discard our imaginations as any help at all to knowing, but both alternatives are likely to be mistaken.

What about the wider implications of incommensurability?

There are many. One seems to be a fundamental distinction between the continuous and the discrete: two different kinds of magnitude, represented in our thinking here by lines, which are continuous and by numbers, which are assemblages of discrete units. Of course a line may be divided into as many equal parts, or artificial units, as we please, or as few as two, so it is not in every way unavailable to the language of number, but it has no natural unit and can be thought of apart from the notion of an assemblage of units. It may be this absence of natural units that is at the heart of incommensurability. When we want to measure distances in the physical world we begin by inventing a unit length like the inch: roughly the top joint of a king's thumb. Everything that has a length must thereafter submit to being so many thumbjoints long, measured to the nearest half-thumb-joint. But that original thumb joint reveals its peculiarity when we ask how we would measure its length. Strictly speaking it has none since there is no agreed-upon unit by which we would measure it. It is one unit long. How big that unit is cannot be determined. Would we say, "One is one"? Calling One a number might be like claiming to know how long an inch is: it only works for us as a measure in multitudes of itself. If we really want to say anything satisfying about how long an inch is we must invent a centimeter and say it is 2.5 of them, but then we cannot say what the 'absolute' length of the centimeter is.

So we do seem to make, or find, a multiple thing, number, amid things, namely units, each of which contains no multiplicity. Can the One be incommensurable with the numbers of things it counts? Socrates on the day of his death says that he gave up the study of natural science when he realized he still did not understand how one and one made two. Do we understand it? We may simply not know how to come any closer to understanding unity and so we proceed to get farther away, to make progress in some direction rather than seek we know not what from the origins. We might remind ourselves that although lines are limited or determined by points, they are not made of them. Wherever there can be two points we can think of a line that joins them but since the line can always be divided then there is always another point between any two so that if a line were made of points it would have as many as we like and we could add as many more without expecting it to change in length. That is not the way a wall is made of bricks at the very least. Is it the way a brick is made of clay? We can think so many remarkable things if we do not linger too long at the beginning that perhaps it would be wrong not to get on with our deductions and further explorations merely because we do not really understand unity. Perhaps there is room for both directions of thought?

Let me offer another image of a kind of incommensurability. Do we know what allows us to use words? Can there be untranslatable words? What do we mean when we say that a particular word, say, of Greek or French, really has no equivalent in English? We might like to say that such a word in its own language brings together as parts of the same whole several different thoughts or meanings which nowhere exist together in any single word of English. We can still list those meanings and instruct the learner to think them together; and we may have the learner's experience of beginning to feel as if after all the different meanings do deserve to have their own single word to unite in. We may start to think that "Deinos", the Ancient Greek root of our word "Dinosaur", doesn't have to be heard as: "Terrible but possibly also in other cases "Wondrous" and in yet others "Clever and Effective". We begin to hear "Terrible AND Wondrous, Clever AND Effective" all at once, in a way not really captured by our own recent and over-used "Awesome". But perhaps our minds have jumped a gap or discontinuity between Greek and English rather than finding a common unit of measure, and we are briefly thinking in Greek?

Perhaps all foreign words are strictly speaking not incomprehensible but yet never perfectly translatable?

I should address a doubt. What shall we say to someone who tells us we are making mountains of molehills and that there is no real problem with translation or even with expressing the diagonal in terms of the side? The Doubter will say that if the side is called "one" then the diagonal may be called "the square root of two", or "that number which when multiplied by itself will give us an answer of two". We may ask if the doubter can tell us how many times we will be multiplying it by itself and receive the somewhat mysterious reply "1.4142 …" with the further explanation that the dots represent a continuing fraction that never actually ceases. We may feel as if something is peculiar about a number that can never finish being named. But any number ending with a *finite* fraction will when multiplied by itself give us an answer that is either bigger or smaller than two. Inventing a symbol that means "find the number which when multiplied by itself gives the number under this sign" and calling it a "square root sign" does not guarantee that there *is* such a number corresponding to any number I put under the sign. For 49 we find 7 but for 2 we find "1.4142…" and the dots go on forever. If this infinitely continuing fraction is our way of reconciling the continuous with the discrete, or letting the same number be even and odd, we may wonder what might be getting lost. Perhaps there are no two things so close that the mind cannot find a gap nor so far apart that the mind cannot find a bridge? What about Being and Nonbeing, or life and death? Or Right and Wrong? Are they incommensurable once and for all? What is at stake for us when we try to know?

Incommensurability and Meno

I want to turn now again to the *Meno* for some help with the question of how we might come to know that something is true or of how we might learn. Why does Socrates use the example of proving something about the diagonal of a square when he wants to encourage Meno to suppose that it is possible to learn, and even perhaps to learn how to be good?

My thoughts are not terribly well-organized on this topic but let us begin with some possible connections. Virtue is proposed to us in the *Meno* and elsewhere in the dialogues as having four parts: Courage, Moderation, Justice, and Wisdom. Sometimes it is suggested that none of these can be separated from the others, that Courage without Wisdom is mere rashness, or Moderation without Justice mere cold selfishness. They are compared to the parts of a face, unable to exist as themselves except when all together. Suppose we imagined them as a square in which their equality and co-dependence might be imaged. Then when Socrates asks what unites them or plays the role of that by which each deserves to be called a virtue, we could imagine that he is asking if there is another line that touches each and all of them. That line could be a circle around the square, or it could be the diagonal. It is a line which by making two triangles in its division of the square would prevent the collapse of the square under pressure, as carpenters all know. It is inside the square yet bigger than any of the lines whose particular arrangement makes the square. It turns out – and you may as I have said want to consult a written version of this lecture to see why this is so – that its precise size is not nameable or measurable in terms of the sides of the square; and yet it has a perfectly well-defined size. How did Socrates's and Meno's attempt to define and unify human excellence lead to a mathematical problem about incommensurability and disharmony?

The road leads through Power. From the first they have disagreed on a fundamental level: Socrates wants to know what human excellence is, and Meno insists that the most needful thing is finding out how to acquire it. Socrates seems to promise that really knowing what it is or at least making a real attempt at learning that might be the only way to begin acquiring it and Meno fears that insisting on insight will lead to paralysis; and so he seems to recommend settling for anything that looks a lot like the path to acquisition: say, Power. If you have power, then you can enact whatever looks excellent to you, but without it, no quantity of insight will help; you will be a victim or a bystander, no real doer. Socrates helps us and Meno to see that Meno is after all more interested in power than in excellence; since everyone wants what is good or excellent but few seem capable of acquiring it, it must be, thinks Meno, that those few are the powerful; and a corollary must be that what is excellent is what can be acquired by power: gold and silver and a place in the councils of the city. Meno and the reader are shown the consequence of the definition of Virtue that says it is "For the one desiring fine things to have the power to get them".

Socrates does not here suggest that a more important difference among people than the division into who has or lacks the power to get good things might lie in the question of what things are really good and what others only look good. Perhaps he is not surprised that the distinction between those who only think they know what is good and those who really do know is not a familiar one for Meno. Like most of us Meno thinks it is easy to know what good things are. He also thought it was easy to imagine that some of the unhappy or the unlucky might actually want for themselves things they knew were bad for them. Socrates must carefully remind him what would really be involved in wishing to harm oneself without any counterbalancing benefit of any kind, namely a kind of absurdity or impossibility; and then Meno admits we all suppose we are choosing what is best, so that everyone can be said to share the desire for fine things, or at least for apparently fine things.

This equality in desiring apparent goods leaves the struggle for excellence to be, as we mentioned earlier, a matter decided according to who has the power to get or get at those apparent goods: Gold and Silver, and honors and powers and offices in the

city. Meno uses the verb “porizesthai” or “ to achieve … procure… make progress ” as the third infinitive we translated in the phrase “ To desire fine things and be able to get them.”
“ Poros”, the noun in that verb, is cognate with the English word “ford”, as in “ you can cross the river at the ford”. A Poros in Greek may refer to any number of stratagems or devices for accomplishing one’s goals. It is a word whose privative form “aporia” has great resonances with incommensurability. Aporia is the condition of being without resources in the face of something. It can describe simple lack of money or that more general difficulty that we describe as “feeling completely at a loss”. We are at an impasse and can see no way across some barrier. Here we are near to incommensurability. Meno first uses the word in the dialogue to say that when one knows the different virtues appropriate to old and young, men and women, slave and free, one will never be in any aporia about saying what virtue is. The simple connection of Poros, resource, to money lets Socrates remind Meno that for all the importance of Porizesthai or the “ being able to GET for oneself …” those fine things that virtuous people want; and for all the ways that money lets you have access to nearly anything you might want, there could be situations in which *not* Poros, resource, is crucial to virtue, but precisely Aporia, resourcelessness. If the only money to be had in a certain situation was money unjustly acquired, then Meno agrees an Aporia of money would then be virtuous. One might go another step and say if the only action that could be taken in a particular situation had to be action taken in complete ignorance of what a truly just outcome would look like then inaction might be preferable. Meno turns out to be more familiar with Aporia, at least in thought, than one might suspect of a very ambitious and not very scrupulous young aristocrat. The things he thinks about are strongly marked by the possibility of aporia. What kind of thinking does he like? He says he likes the way Gorgias explains the functioning of the body’s senses by reference to effluences or “outflows” from the objects sensed, which outflows can be

compared to very small shapes constantly crossing the space between say a sweet-smelling flower, and my nose. If the tiny shapes fit the pores of my nose then I will sense aroma. Other shapes, e.g. those conveying sounds, will not fit my smelling pores but instead will find paths through my ears and I will hear things. Socrates's Gorgian example ends with him noting that this mode of explanation can be adapted to all of the senses and perhaps many other questions as well. Maybe it explains too much? The element of the Incommensurable is very prominent. Socrates says the little shapes are "symmetroi" with the pores of their proper sense organs, that is they have a common measure. But smells are incommensurable with ears. To survey the microscopic world for a moment through this lens, we must be ready to see a constant flow of all kinds of possible sensations, tiny shapes crossing one another's paths in all directions constantly and bouncing away from doorways not designed to let them in or slipping neatly into passages through which they fit as smoothly as an old key. A simple test is always automatically going on amid the outflows and the bumps of the myriad tiny shapes, "Are you the right shape to pass?" is asked and answered thousands of times per second. Socrates describes Meno's pleasure in this explanation as a response to the High Tragic Manner, in which, as he says, this account appears. High Tragic Manner?

What is that likely to mean? Does Meno, whose father knew Xerxes, the tragic hero of Herodotus's History, does Meno have reason to wish for a world built on Tragedy? If he wants to be excellent, to win praise for his power and fine possessions, to be honored as a Homeric Warrior is honored, is it more comfortable than facing the impasse of your actual knowledge or ignorance of your own powers, to think that the best warriors are just the right shape from birth and that if you are fated to be great you will find a fitting passage? If you are already among the leading families of Thessaly then your fate is clearly calling you, and if your father narrowly missed becoming a Persian Satrap over some of Greece, maybe that has been saved for you! Those who treasure power above all things will recognize you because it takes one to know one, and they will recruit you. You and they will be ready to betray each other if that is the path to greater power, but meanwhile you are commensurable and you both need allies.

Another piece of thinking Meno likes is the argument that you cannot usefully seek to learn anything because either you know or you don't know and so you cannot learn what you already know but you cannot even recognize what you do not know. So await your fate in the confidence that you already have the right stuff. It's all or nothing at all. The tragic flavor is a flattering spice that suggests that the great human beings must be prepared to do and to suffer things perhaps not bearable for ordinary humans, and that this is why we remember them and follow them and tell their stories. Amid the chaos some shapes are arriving where they fit; praise and blame may be beside the point. The importance of well-directed effort or the possible guidance of insight look like idealistic distractions on a stage full of opportunities for immediate deployment of strategies: cast what grappling hooks you have in all directions and pull in the biggest fish you snag. Plato is showing us a young Meno shortly before he seized what must have seemed the perfect opportunity: an expedition led by the younger brother of the Great King of Persia, intending to take the throne from his incompetent elder. Xenophon, a contemporary of Plato and like him a student of Socrates, has written in his book *The March Back* of Meno's corrupt and violent attempt to become a Satrap in the Persian style and of the awful fate it led him to. We may suppose that Xenophon's report was known to Plato's first readers.

That this young Meno even wants to talk about how virtue is to be gotten and kept is a positive sign that he doesn't yet simply suppose that his job is to grab every gift of nature or fortune that comes in his reach. He somehow is open to the thought that he might need to do something else for himself, maybe improve himself somehow, and that Socrates could help him. But the Tragic element perhaps overwhelms him. That Tragedy is finally built on the foundation of incommensurability, the incommensurability of Gods and humans, that may be what will keep Meno safe from the danger of being truly changed by his talk with Socrates. Tragedy for Meno may be about the near-miss when a mortal comes along who could almost be mistaken for an immortal. There is no room on earth for humans who do not die. The great human beings who nevertheless do not yield or resign themselves to being less than the Gods will be wonders in their lifetimes and will be long remembered after their deaths. The shape that is our lot may limit us to either having what it takes or not but a certain kind of defiance of Fate *is* possible. The very thing that seeks to diminish us, our mortality, can be embraced by the tragic protagonist and can transform us. The insistence by the tragic character that she alone will define herself even if it should cost her life nearly makes her into a God, and it simultaneously kills her. Medea murders her children, her husband's new wife and father-in-law, taunts Jason and mounts a dragon chariot on the roof to fly to Athens, where she will bear what few can: to lead what remains of a wrecked and miserable life all her own. Something like this may be what Meno loves, hidden in the answers of Gorgias. He does not seem to love learning for its own sake.

It is striking how Gorgias's science anticipates our own: two millennia later our biology is still deep in the process of describing the docking of different tiny chemical particles and molecules in an elaborate process of sending and receiving signals and instructions according to what effluent shapes fit what receptors. And it is still more striking what a difference remains between the recognition that an effluence fits an opening and the experience of understanding a thought. To the Greek listener or reader a similarity of sound appears between the "Aporia" or being at a loss that Socrates will praise as an indispensable part of learning when he helps the slave boy recollect how to double the square, and on the other hand the the "Aporrhoe", or flowing outward, that constitutes the whole Gorgian account of how we perceive and possibly even of how we know. Maybe the nearness in sound of Aporia and Aporrhoe is intended to point toward the thought that the Gorgian/Scientific answer always leaves us where we started: on the outside of what may be going on. If we can see color, we say, it must be because little bits of something are flying into our eyes, something we might as well call "color particles" or color photons or color waves if you prefer; but what allows the arrival of these little shapes to become our experience of color remains dark. We posit that something about them carries what we end up calling "color" and that when it arrives in our eyes, or perhaps bumps something in our eyes that can send something that arrives in our brains, well then, color has arrived. Leibniz says that no matter how much we may imagine enlarging the physical pieces of the brain and nervous system, we will not thereby have achieved more than a larger picture of particles moving other particles. But where, he asks, will be our own understanding *from the inside* of thoughts and sensations in all this sequence of actions and reactions? We may get to a place where we can say, in effect:

"When this exact sequence of synapses firing takes place you are remembering your first grade teacher"; but all we will be doing is correlating two separate events: your own experience of memory, and a neurophysiologist's observations of events among your synapses. I do not mean to speak ill of the enterprise of Science, which surely has shown us many real beauties and marvels; and which is by no means simply identical with Technology; but I wonder if there are important differences in kind among ways of doing what we call knowing. We have now "known" for well over a century that fire happens when particles of combustible substances, like carbon and oxygen, combine in such a way as to release light and heat, and often other gases and particles. We have gone on learning many more details about smaller and smaller particles involved in the process. We have also claimed to know for much longer than a hundred years that it is a crime to set your neighbor's house on fire; but we have never ceased from arson. I make bold to say that no matter how thorough an account we can learn to give of fire, it will make no difference to the ways we treat our fellow human beings or indeed any of the living beings of the world. If no other mode of knowing is available to us than the techno-scientific, I am afraid we are doomed.

How welcome a kind of knowing of Incommensurability might be to us if it should guarantee justice and equality! Beyond any property I might own, I own myself as an insoluble mystery which may decline to be judged by any other standard than its own. If our dignity is our irreducible otherness from all others then most of our moral experiences will be determined by ways we do not fit in. Resistance and bravely saying "No!" will become the unmistakable marks of human goodness. But maybe this, too, like Gorgias's answers, solves too much.

The simple incongruity of having no common measure with others can stand for an inalienable freedom and an infinite value but how do we know we are not still flattering ourselves in the High Tragic manner? Is it not precisely commensurability we seek when we propose with Socrates that the effort to learn makes us better? All Nature is akin, he says. This is the opposite of severing ourselves from a world of indifferent collisions, or of what is worse, of seeking to become similarly indifferent ourselves. If we are bound to act for the sake of what is or seems better, then that may be a clue to how the cosmos acts.

These are only speculations about Meno and about ourselves. Without them I would not know how to begin reading Plato. The Aporia which Meno will call " numbness" in his image of Socrates the Torpedo Fish has its counterpart in the Gorgian Theory of Everything: in the word "Aporrhoe", or effluence, outflow. Everything is only connected to everything else by this constant outflow or stream of shapes by which each thing shares its visible, audible, smellable, tasteable, touchable, knowable self with the various human organs of perception which happen to be commensurate with it. One wonders where the inexhaustible source for such constant outflow can be located, and why it never runs low. One may also wonder if all that seeming incommensurability conceals a suppression of genuine differences of kind in things. Have we really seen very deeply, or heard any divine harmonies when we assert more or less a priori that sight and hearing are at bottom both the same, just matter in motion? There may be more numbness involved in this conclusion than in any temporary impasse that Socrates and his conversations-partners suffer in their attempts to learn.

The binary simplicity of the life of an effluence is perhaps part of its attraction to Meno and to us; either a shape fits what it hits or it doesn't. It is like being told either you already have what it takes or you never will. You are spared the wandering about in some gray area while trying to find common measures or small steps of approximation. You don't have to start trying from where you are to get someplace different. You do not have to examine opinions to see what may be partly true in them and where that may lead. As in the argument that you cannot learn what you already know nor what you have never encountered, there seems to be a kind of absolute separation between the wealthy possession of knowledge and the impoverished condition of ignorance. Neither one is quite understandable beyond the simple model of property ownership: if you know something then you have it, if not then you don't. And having means chiefly having the power to exchange one thing for another, to trade up, as we say. So Meno hopes by his conversation with Socrates to end up with some answers in his back pocket that will confound anyone he should have to debate; he is challenging Socrates with the answers he has memorized from Gorgias: " have you got anything stronger than this?"

If on the other hand it is possible to learn by experience that knowable things are not inert possessions but after all have a kind of life, then knowing must be different from simple possession of property, which one can do while sleeping; knowing must be an active practice, and one must be seeking to take on some of the life of what one wants to know. All nature, as Socrates suggests, must be akin for the learning that interests him to be possible. However true it may be that each being must somehow differ from all others, that no two snowflakes or leaves are ever identical, it is still finally commensurability that we seek. That the example chosen to give us hope about our capacity to learn is the diagonal of the square, whose ratio to its side is not expressible in common units, must be especially important. What may it mean? It seems to say that there is more than one kind of intelligibility; the lack of a common length measure for diagonal and side does not preclude knowing things about the diagonal and its relation to that side. The counting of discrete units is not the only path to knowledge. The discovery of the line that will let us double the square depends on seeing more than what is there; we must begin to see areas, two-dimensional beings in order to address a question about finding one dimension: the length of a line.

The diagonal is not in the end confoundingly hard to know; it is the side of a square twice as big as the one whose diagonal it is. If it is to be measured we must refer to a second dimension, not to a line but to an area. The Greek language uses the word "dunamis" or "power" in geometrical contexts somewhat as we might use "squared" or "to the second power" in English. A line may be said to be equal "dunamei"(the dative form of 'dunamis') or "in power, by means of its power" to some area, meaning that the square on that line is equal to the area in question. The diagonal can said in Greek to be "in power" the double of the square it divides. The echo of Meno's word for his ideal of the raw power of the tyrant who can grab whatever he wants and hold it is not accidental. But what is echoing what?

The suggestion that the diagonal is knowable by its power to become a double square is a fertile one. It makes us wonder if virtue has a kind of life in it since living things are partly known by their power to duplicate themselves, to grow or reproduce. Does the recognition of virtue necessarily involve a beginning of reproducing it or a desire to see it exercise its power? When we encounter someone good, we are in fact moved to imitation. Virtue would then be essentially active; it would be what it can become rather than a simple inert quantity. This could be connected to the difficulty in saying what exactly it is or even in describing it. A contemporary philosopher has written a book called "The Fragility of Goodness", but perhaps another could be written on the Power of Goodness. What if the more knowable a thing were the more beautiful, and alive it were? Would the most unified and beautiful of all naturally generate an entire world as a kind of octave of itself? We might then expect the nearness of such a thing to possess active power, to contain a kind of life that sustains and increases itself. Knowing would not mean to behold something over there in calm clarity, while deciding whether to take it or leave it. It would be to feel the effect of the nearness of life, to be drawn to imitate and be informed by order and pattern, so as to become more nearly unified oneself and more aware of the unity of the world. Would such an experience make us suppose that beautiful speeches could have the power to bring about beautiful deeds?

One Greek word for "Rascal", gleefully appropriated by Rabelais many centuries after its birth, is "Panourgos", which comes from two words meaning "all", and "work". A Rascal is someone who acknowledges no limits, who will "do everything" or do anything to have their way. We say, typically about a villain, that he or she would "stop at nothing" on the way to fulfilling their wicked plans. Turn this inside out and suppose a being so fully limited, which we will conjecture could mean so good, as to be in a sense fully at rest: it would not need to do anything in order to be as it was, and it would lack nothing so that no motive would exist for its taking some particular step, nefarious or otherwise. We might suppose it to be already fully active all the time and indeed to be the principle of all activity everywhere, but in a way that while pervading everything would have nothing to prove and so no need to undertake any new act. It would remind us of Achilles telling Phoinix that he has no need of human honor and hence no need to do any mighty deeds, but that he has honor enough from Zeus simply by being who he is. So humans who resemble this conjectural opposite of a rascal look like Gods, and the full version of such a being might be God. Human Excellence or Virtue would then likewise resemble God in being as self-contained as a social being might be able to be while remaining social. It might be better-defined than most things are, and hence more knowable, if definition is chiefly a matter of limits. Socrates suggests a little of this as he asks Meno about the presumed behavior of a virtuous human.

What if becoming virtuous really is a matter of making the effort to learn, not how we can master things but how things really are; and what if finding that out involves discovering that things are more orderly and beautiful than we can ever fully imagine?