On the Origins of Celestial Dynamics:

Kepler and Newton

(A revised version of a lecture given at St. John's College in May, 1971)

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I wish to consider two moments in the emergence of what we have come to call celestial dynamics or mechanics, a Keplerian moment and a Newtonian one, seeking to explore what the development of such a dynamics meant to its authors. David Gregory, a follower of Newton, writing in 1702, spoke of the new celestial physics that "the most sagacious Kepler had got the scent of, but the Prince of Geometers Sir Isaac Newton brought to such a pitch as surprizes all the world". Actually, the Keplerian dynamics and the Newtonian dynamics differ in important respects, but Gregory's singling out of Kepler and Newton makes sense. Kepler was the first to introduce a dynamics into the heavens in the sense of hypothesizing a quantifiable influence of one celestial body on the motion of another, and Newton's universal gravitation is the same kind of thing. Moreover, the mathematical results Kepler arrives at by pursuing his hypothesis coincide, almost, with Newton's results, derived from a different dynamics.

Meanwhile, in the period intervening between the appearance of Kepler's hypothesis in 1609, and the appearance of Newton's Principia in 1687, there were various attempts at proposing what may be called mechanical causes for the celestial motions, but none of them allowed of mathematical formulation, or led to an astronomical calculus, a way of predicting positions of planets. The egregious Mr. Hobbes imagined that, as the southern and northern hemispheres of the Earth differ with respect to the proportion of dry land and ocean, therefore the aethereal vortex or whirlpool that moves about the Sun, having now more solid land to press against and now more of the yielding ocean, would drive the Earth in a path differing from a circle, perhaps approaching an ellipse. M. Descartes figured out a reason why the suns or stars are off-center in their vortices, so that in our vortex the planetary paths are eccentric, but as in Hobbes' case, the hypothesis did not lend itself to mathematization; on the contrary, as Descartes believed the universe to be packed with vortices inclined at various angles to one another, vortices that fill all space and interact with one another by transference of matter and motion, any simple mathematical rule for the planetary orbits and motions becomes implausible.

In Kepler's and Newton's cases, we can ask how the dynamical hypothesis and its quantification come about, what they presuppose, what they mean to their authors. Let me begin with Kepler.

The first sprouts of Kepler's celestial dynamics make their appearance in his first venture into print, his Cosmographic Mystery of 1596, published when he was just turning 25. Since April, 1594, Kepler had been holding the position of district mathematician in Graz, with the task of teaching mathematics to the boys in a Protestant school, and making up an annual astrological calendar for the province, showing when to plant crops, and what to expect of the weather and the Turks. He was, let me mention, marvelously successful with his first calendar: the cold spell he had predicted was so grievous that herdsmen in the mountains lost their lives or their noses from frostbite, and the invasions of the Turks he had predicted were also grievous; the provincial magistrates therefore added a bonus to his stipend. But Kepler was not satisfied with this kind of astrological hackwork. Beginning on the Sunday of Pentecost in 1595, we find him concerned with, and indeed thinking without let-up about, three large cosmological questions.

At the start of the Cosmographic Mystery, Kepler says, "there were three things above all of which I sought the causes why they were thus and not otherwise: the number, size, and motions of the [planetary] orbs. That I dared this was brought about by that beautiful harmony of the quiescent things, the Sun, fixed stars, and intervening space, with God the Father, the Son, and the Holy Ghost." That is, Kepler sees the spherical lay-out of the cosmos, with the Sun at the center, and the stars at the periphery, as an image or signature of the triune God, the Creator, His Being, Knowledge, and Love. And with this vision in his head, he makes bold to seek the number, spacings, and motions that the Creator gave to the mobile bodies, the planets, occupying the intermediate space between Sun and stars.

Obviously, Kepler is at this point a Copernican. By no means does he have a thorough knowledge of Copernicus' theory. As he begins his speculations, he has not read and does not even possess Copernicus' book; he does not even know Rheticus' Narratio prima, the book in which, in 1540, three years before the appearance of the De revolutionibus, Rheticus had communicated to the world the major outlines of Copernicus' theory and given an account of its superiority over the Ptolemaic theory. Kepler says that he had learned partly from his teacher Maestlin at Tubingen, and partly from his own thinking, the mathematical advantages that Copernicus has over Ptolemy. The Copernican arrangement, simply by its lay-out, accounts for certain phenomena that are left unaccounted for, are left as coincidences, in the Ptolemaic arrangement. Why do the Sun and the Moon not retrograde, while the other planets do? Why do Mercury and Venus always keep relatively close to the Sun, while the other planets can be at any angular distance? Why are the superior planets always lowest in their epicycles, when in opposition to the Sun? For these questions and a few more, the Copernican arrangement provides an answer; the Ptolemaic does not.

By the time he had finished his Cosmographic Mystery, Kepler had apparently read (the book paraphrases a sentence or two from) the famous tenth chapter of Book I of the De revolutionibus, where Copernicus says, in his brief commendation of the heliocentric arrangement, "we find in this arrangement a marvelous symmetry of the world and a harmony in the relationship of the motion and size of the orbits, such as one cannot find elsewhere." But even before, Kepler was asking not merely in what the symmetry and harmony consist, but also: On what are they founded? How does man come to recognize them? And already at the start, Kepler has answers to which he will always adhere: The world carries in itself the features of the omnipotent creator and is his copy, his signature. To man, God gave a rational soul, thereby stamping him in His own image. It is with that soul that man can recognize the symmetry and harmony of the Copernican world. Seeing that spherical Copernican world in terms of an idea of Nicholas Cusanus, as a kind of quantitative representation of the indissoluble triune essence of God, Kepler is encouraged to raise and pursue his bold, naive questions.

One of the questions was not new. If you were Copernican, there were six circumsolar planets, not seven planets as with Ptolemy, since Copernicus leaves the Moon as a satellite of the Earth. Rheticus in his Narratio prima had explained this sixfold number by the sacredness and perfection of the number six: six is the first number equal to the sum of its factors. A little later in the 16th century Zarlino will be using this same idea to explain the role of the first six numbers in musical consonances; he will be the first musical theorist to include thirds and sixths among the consonances, as they needed to be included for polyphony's sake. Kepler will be the second such musical theorist, but here as in the case of the number of the planets he will reject the notion of particular numbers as causes. He rejects number-mysticism in that sense. Numbers, for him, are only abstractions from the created things, and therefore posterior to the Creation; they could not therefore be used by God as archetypal forms for cosmopoiēsis, the making of the world.

The second and third of Kepler's questions were new. In August of 1595 Kepler wrote to Maestlin, his former teacher at Tubingen, telling of his investigations, and asking whether he had ever heard or read of anyone who went into the reason of the disposition of the planets, and the proportions of their motions. In the margin, Maestlin wrote in answer: "No".

Let me remark here that no analogous questions are likely to arise in what can be called, and indeed came to be called, the Ptolemaic System, which was what Kepler had been officially taught at the university. By this term I mean not the set of planetary theories in Ptolemy's Almagest, but rather the world picture, current in the Middle Ages and Renaissance, according to which the planetary spheres are nested to fill exactly the space between the highest sublunary element, fire, and the fixed stars. There is no trace of this picture in the Almagest, but just four years ago it was discovered that it is given in Ptolemy's Hypotheses of the Planets, the relevant passage having been omitted from Heiberg's standard edition of Ptolemy, apparently from some confusion among the translators; most of the work, including this passage, exists only in Arabic MSS, of which Heiberg gives only a German translation. What I now say is based on this recovered portion of the Hypotheses. The Ptolemaic System, Ptolemy freely admits, involves conjecture, but he also insists on its plausibility, as did his followers through the Middle Ages and Renaissance. Tycho Brahe was still accepting it in the 1570's. The plausibility is as follows. Ptolemy gives certain arguments in the Almagest, and again in amplified form in the Hypotheses, for a certain order of the planets, beginning Moon, Mercury, Venus, Sun, and going on to the superior planets; I won't repeat the arguments here. (The Sun, note, is the central one of the seven planets or wandering stars.) He had a very good value for the maximum distance of the Moon from the Earth, determined from observations, namely 64 Earth radii. Assume now that the maximum distance of one planet from the Earth is equal to the minimum distance of the planet next above it; take from the Almagest the ratios of nearest approach to farthest distance for each planet, and start constructing outward, using the Ptolemaic order. After the Moon comes Mercury and then Venus. The maximum distance of Venus turns out to be 1079 Earth-radii, and the Sun is to come next. But there was an independent method for determining the relative distances of the Sun and the Moon, a way invented by Hipparchus, described in the Almagest, and using eclipses. The result of that method, reported in the Almagest, was that at closest approach to the Earth the Sun was 1160 Earth-radii distance, 81 Earth-radii beyond the highest point of Venus' orb. Is this a big gap? Ptolemy shows, in the Hypotheses, that by a very slight change in the data of this determination, a change within the limits of observational error, the Sun at nearest approach will be found to use up the extra 81 Earth-radii, and everything fits. Moreover, this is the only order in which the planets can be made to fit in such a sequence of nested spheres, using the Ptolemaic numbers. In further justification Ptolemy adds, "this arrangement is most plausible, for it is not conceivable that there be in Nature a vacuum, or any meaningless and useless thing."

This Ptolemaic System was very well known during the 16th century, owing to the description of it in Peurbach's Theoricae planetarum, which went through many editions. I suspect it was widely accepted as filling out the heavens, and allowing for the strange motions of these divine beings - motions which, according to Ptolemy, follow from the essence of the planet and are like the will and understanding in man. Copernicus, and also Kepler in the Cosmographic Mystery, explicitly reject this system, but I do not think any really forceful argument was made till Kepler showed, some years later, that the Hipparchic method for the Sun's distance was practically useless, a small error in the observations leading to an enormous error in the final result.

The question of the reason of the spacing of the planetary orbs does not, then, arise in the Ptolemaic System, because there is in fact no spacing. In the Copernican theory, on the contrary, there are unused spaces, not only a huge one beyond Saturn, separating the solar system from the stars, but also unused spaces between the hoop-shaped regions of space that the individual planets pass through in their motions. That is the effect of the economy of the Copernican system, the elimination of the large epicycles. Copernicus speaks of planetary orbs and spheres; whether he believed them to be real or imaginary remains a subject of scholarly debate. Kepler thought Copernicus believed the spheres to be real and solid, but in the Cosmographic Mystery he is already pointing out some of the difficulties with this conception. By what chains or struts is the Earth with its atmosphere held in its solid sphere? We are already in the heavens, and they aren't solid. But in either case, whether the spheres are real or not, there have come to be apparently functionless spaces, and the question can be raised as to the reason of the spacing of the planetary orbs.

Copernicus does not raise this question. He is apparently seeking to re-do not the cosmography of Ptolemy's Hypotheses of the Planets, but the mathematical astronomy of Ptolemy's Almagest, and he wishes to do this job consistently with the first principles of the astronomical art. A primary principle is that there must be only uniform circular motion; this is required if there is to be strict periodic[ity] if the motions are not sometimes to fail, owing to their dependence on a changing and thus changeable motor virtue. The intellect abhors such an idea, Copernicus says. The Copernican insistence on uniformity of circular motion will be taken up by later astronomers, by Tycho, Longomontanus, Bullialdus, and others, and echoed for a hundred years and more, by both heliocentrists and non-heliocentrists. Not only had Ptolemy failed to keep to the principle; but new phenomena, discovered since Ptolemy's time, showed that there was an inequality in the precession of the equinoxes that Ptolemy had not suspected. This was the trepidation, supposedly proved by observations of the Arabs collated with those of Ptolemy and Hipparchus. According to one scholar (Ravetz), it was this supposed phenomenon that pushed Copernicus into setting the Earth in motion. For, argued Ravetz, if the precession of the equinoxes is due to the motion of the stars, if this motion is non-uniform, and if the standard of time by which equality is judged is provided by the diurnal rotation of those very same stars, then the standard of time has been vitiated, and the entire system has become logically incoherent. The problem can be solved, Ravetz claimed, by giving the diurnal and precessional motions to the Earth; Rheticus seems to have agreed, for he says that he sees no other possible explanation. In fact, the uniformity of the diurnal rotation is vitiated slightly whether one assigns the trepidation to the Earth or to the stars. Sometime after 1588 Tycho Brahe convinced himself that the trepidation is not real, is the mere effect of the large errors in the times of the equinoxes that Ptolemy reports in Book III of the Almagest; and this is the conclusion also of modern astronomy. No, Copernicus' original motive appears to have been opposition to the Ptolemaic equant - that point, not the center of the circle, about which Ptolemy assumes the motion on the deferent circle to be uniform. This violated the first principle of the astronomical art, the assumption of uniform circular motions only. It is primarily with this idea in mind that Copernicus re-does the Almagest. He is not, on the whole, looking at the emergent system with the wild eye of a cosmologist; certainly he is not asking for the archetypal, apriori reasons in the mind of God that will account for the lay-out.

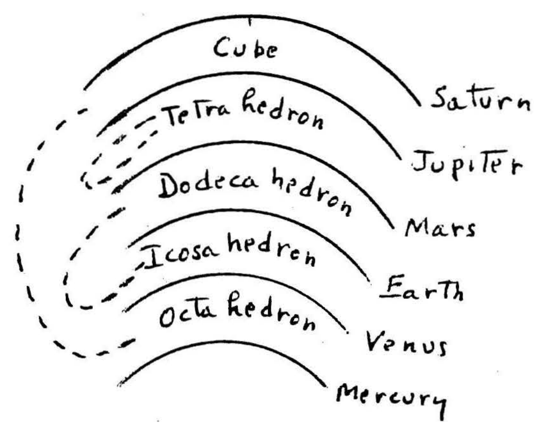
Between Copernicus' death in 1543 and 1596, the date of Kepler's Cosmographic Mystery, there were very few Copernicans who spoke out. The ill-fated Bruno; a poet or two in the entourage of Henry III of France; Benedetti, Galileo's precursor in mechanics; a mystically-minded Englishman named Thomas Digges - they were few.

On the other side was the overwhelming chorus of denunciation. Melanchthon, Luther's lieutenant and a professor at Wittenberg, referring to the Copernican doctrine, said, "really, wise governments ought to repress impudence of mind." Maurolycus, a very competent and indeed innovative mathematician of Messina, said that Copernicus "deserves a whip or a scourge rather than a refutation". Pyrrhonist sceptics like Montaigne and his followers were fond of citing Copernicus and Paracelsus to show that there can be found people to deny even the most universally accepted principles; this, to help show that we are so ignorant that it is even excessive to assert that we know that we know nothing. And Tycho Brahe wrote: "What need is there without any justification to imagine the Earth, a dark, dense and inert mass, to be a heavenly body undergoing even more numerous revolutions than the others, that is to say, subject to a triple motion, in violation not only of all physical truth but also of the authority of Holy Scripture, which ought to be paramount." And the list of denunciations could be greatly extended.

Kepler, then, turns out to be one of the early Copernicans, one of a handful, to speak out; he does so before Galileo, before his own teacher Maestlin. Maestlin praises Kepler for his first book, saying, "at last a learned man has been found who dared to speak out in defense of Copernicus, against the general chorus of obloquy." And Kepler's defence has a unique character, starting as it does from the notion of the spherical, Sun-centered world as symbol of God. It is this symbol, Kepler explicitly states, that encourages him to seek the reasons of the number, spacings, and ratio of motions of the planetary orbs. This symbol remains central in Kepler's thought; every one of his major undertakings and achievements seems to be relatable to it.

Let me mention in passing that Kepler's question about the spacings is not likely to arise for a follower of the Tychonic system, which resembles the Copernican except that the Earth remains stationary, and the Sun with the remaining planets moves about the Earth. In letters written in the late 1580's, Tycho says that he was induced to give up the Ptolemaic System by the discovery, from measurements of the parallax of Mars when it is in opposition to the Sun, that it is closer to the Earth than the Sun. This happens in the Copernican System, but not in the Ptolemaic; the Tychonic system accommodates the fact by preserving the Copernican spacings. Actually, Kepler found later that Tycho could not have determined, from his observations, the parallax of Mars; it was too small for observational discrimination by the means at his disposal. And poring over Tycho's MSS, Kepler concluded that some assistant of Tycho had misunderstood instructions and computed the parallax, not from observation, but from the numerical parameters of Copernicus' system. In any case, if you do accept the Tychonic system, then the path of Mars cuts across the path of the Sun, not impossible because Tycho knows by now from his study of comets that there are no solid orbs, but still inelegant. And the entire set-up lacks the centered symmetry that provoked the Keplerian inquiry.

The answer Kepler finds to the first two of his questions is well known; the discovery comes after he has tried many different schemes, and it comes, he tells Maestlin, accompanied by a flood of tears.



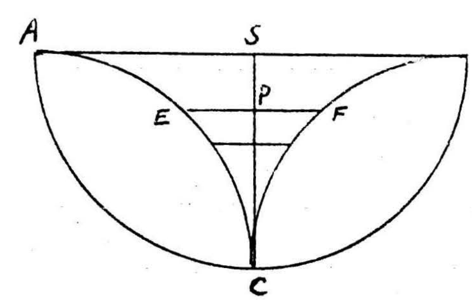
Saturn - cube - Jupiter - tetrahedron - Mars - dodecahedron - Earth - icosahedron - Venus - octahedron - Mercury. The structure is not built outward from the Sun: it is built inward and outward from the Earth's sphere, which divides the five regular solids into two groups. Cube, tetrahedron, and dodecahedron, Kepler calls primary; each has vertices formed by three edges, each has its own special kind of face - square, triangle, or pentagon. The secondary solids, octahedron and icosahedron, have their vertices formed by four and five edges, respectively, and have triangular faces. The octahedron is formed from the cube by replacing square faces by the points at their centers; the icosahedron is similarly formed from the dodecahedron. A similar transformation performed on the tetrahedron yields only another tetrahedron. Kepler speaks of the secondary bodies, octahedron and icosahedron, as offspring of the cube and dodecahedron, respectively, as fathers, and the tetrahedron as mother, from whom they receive their triangular faces. The tetrahedron meanwhile is hermaphroditic in its production of tetrahedra. Of the primary solids, the cube has to come first, because, Kepler says, it is "the thing itself", meaning, I believe, that it presents to us the very idea of corporification, the creation of body by the regular filling out of space in the three dimensions. The transformation of cube into tetrahedron is carried out by subtraction, replacing each square face by one of its diagonals; the transformation of cube into dodecahedron is carried out by addition, roofing over the cube, turning each edge into the diagonal of a pentagon.

Out of the 120 possible orders of the five bodies, Kepler can say that he has chosen the one that singles out, as a starting point, the very notion of corporification or the creation of body, that singles out the Earth's sphere as the very special place it is, the home of the image of God, and that, given these conditions, has the most complete symmetry. And it shows at once why the number of the planets must be just six; there are only five regular solids, as Euclid proves, hence only six circumscribing and inscribing spheres; the number has been deduced from the very idea of the creation of body, of the world, by an ever geometrizing, and let me add, echoing Kepler, a playful God. And man was meant to understand these things. Kepler says:

> As the eye was created for color, the ear for tone, so was the intellect of humans created for the understanding not of just any thing whatsoever but of quantities....It is the nature of our intellect to bring to the study of divine matters concepts which are built upon the category of quantity; if it is deprived of these concepts, then it can define only by pure negations.

All right; we are meant to see these things; but are they <u>true</u>? To know that, we must know that the distances in the construction jibe with the distances determined by the astronomers, and moreover, jibe rather exactly. Kepler at different times expresses the thought that the imposed forms might not fit the world quite exactly, but in that case he hopes to find reasons even for the deviations. The problem Kepler faces in testing his hypothesis is first of all to know what distances to take from Copernicus; the sphere of each planet must be of such a thickness as to accommodate the planet's approaches to and recessions from the Sun, but should one, for instance, allow space for Copernicus' equatorial epicycle, which sticks out beyond the planet's path at aphelion? And can one trust Copernicus' theories for Venus and Mercury, which involve some peculiar hypocyclic and epicyclic motions that keep time with the Earth's motion? Moreover, Kepler thinks it incongruous that Copernicus computes the planetary distances from the center of the Earth's orbit rather than from the Sun itself. It is with such considerations that Kepler begins his critique of the details of the Copernican theories. But disallowing the equatorial epicycles, shifting to the real Sun as reference point, Kepler is able to make a preliminary comparison of distances. The ratios for the intervals between Mars and Jupiter and between Venus and the Earth come out with zero per cent error; for the Earth-Mars interval the error is 5%, for the Jupiter-Saturn interval about 9%. For the Mercury-Venus interval, with Copernicus' numbers, the error is unfortunately 20%. Kepler persuades himself - on the ground of Mercury's very unusual situation and motion - that for Mercury the sphere to be used is that inscribed, not in the octahedron itself, but in the three squares formed by the twelve edges of the octahedron - the octahedron is the only regular solid that can be sliced through along its edges in such a way as to yield regular polygons. With this concession, the Mercury-Venus error is reduced to 2%; the largest error remains that for Saturn, whose distance is the greatest and therefore most difficult to measure; the next largest error involves the Earth, whose Copernican theory Kepler has reason to doubt; and the average error for all the intervals is but 3.3%. It is eminently conceivable that these errors should have turned out to be much higher; and Kepler has his initial moment of elation; later on, as he calculates, there are doubts, and then again moments of elation. He writes to Maestlin that he suspects a tremendous miracle of God. Maestlin, older, more cautious, widely known as a competent astronomer, comes to agree with him, comes to suppose that it will be possible to obtain the distances of the planets apriori. He assists extensively in the preparation and publication of the book, in which Kepler calls upon all astronomers to help in working out the details of the hypothesis. Among the readers were those who like Johann Praetorius of Altdorf said that even if the numbers came out exactly, it would not mean a thing: astronomy should go back to its practical business of predicting the planetary positions. Tycho's reaction was less hostile: of course there are harmonies, he said, but one must work out the planetary theories on the basis of exact observations first, before investigating the harmonies. Tycho understands here that the theories must employ uniform circular motion, in accordance with the Copernican insistence on that principle; and he understands, too, of course that the Earth is at rest.

This brings me to another theory that is contained in Kepler's book, one which Tycho will object to, and which even Maestlin finds, he says, too subtle. From the very beginning, Kepler had wanted to account not only for the number and spacing of the planets, but for the proportion of their motions. From the very beginning, he had noted that the periods of the planets follow behind the distances, so that the period of the planet twice as far from the Sun is more than twice as great. This observation had been one of Kepler's encouragements in the investigation of the reason of the distances, because, he says, if God adapted the motions to the orbs according to some law of distances, then surely He also accommodated the distances to some rule. The first mathematical rule Kepler proposes for the periods is given in a diagram: Note that the diagram is pretty. S is the Sun, ACB the sphere of fixed stars. AEC and BFC are quadrants of



circles with radius equal to the radius of the stellar sphere. To a given distance of a planet from the Sun, SP, Kepler imagines that there would correspond a vigor of motion proportional to the line EF. In the Sun would be the moving soul, and an infinite force of motion; at the periphery are the motionless stars, providing by their distance the space for the planetary motions, and by the non-uniformity of their distribution, a background against which the contemplative creature, man, can locate the planets. The difficulty with the scheme is that Kepler has no clue as to the radius of the cosmos, and without a value for that radius, there is no possibility of calculating the consequences of the hypothesis, and so subjecting it to empirical test. This hypothesis, for Kepler at this point, has a status similar to that of the other one about the five regular solids, in the sense that it arises from the same thought, of the world as symbol of God. The five-regular solid theory had the assumed fact of spacings to work with; this hypothesis has the assumed fact of an inverse variation of distance and speed.

Kepler tries another hypothesis for the motions which is more testable, and in a rough way correct, although it is not the right one (the right one is the third law that he will discover only in 1618). I shall not describe it. Here again Kepler is looking for a pure mathematical form, graspable because mathematical by the mind, and which when discovered will be a symbol of the creating and radiative activity of the Godhead. So he will speak of the decrease in motive vigor with increasing distance from the Sun as suitable; it was fitting that God should have arranged matters thus. On the other hand, he at one point paraphrases the *Timaeus*, on soul being distributed through the cosmos; and in a somewhat different vein, he also begins to compare the spreading out of the motive virtue to the spreading out of light from a center; light, as he will say later, is a kind of mediating thing, intermediate between bodies and souls. Kepler is the first to quantify light intensity. It is the quantification of the Sun's motive virtue, that will lead on to his celestial dynamics. He is already onto an important clue to it, in eliminating Copernicus' equatorial epicycle, which was totally incompatible with the five-regular solid theory, and in thinking about the individual planet as slowing up at aphelion, in some proportion that he is not sure of. Later on, in 1599, pursuing the same idea, he will discover the annual inequality of the Moon.

Between the time of completing the Cosmographic Mystery in 1596, and going to Prague to work with Tycho Brahe in 1600, Kepler became involved in the study of musical harmony, and a word must be said about this investigation as it relates to his study of the planets. Kepler loved polyphonic music, which he regarded as one of the most important discoveries of modern times, ranking with the compass and printing. In his Harmonic of the World, published in 1619, he will write:

> It is no longer a marvel that at last this way of singing in several parts, unknown to the ancients, should have been invented by Man, the Ape of his Creator; that, namely, he should, by the artificial symphony of several voices, play out, in a brief portion of an hour, the perpetuity of the whole duration of the world, and should to some degree taste of God the Creator's satisfaction in His own works, with a most intensely sweet pleasure gained from this Music that imitates God.

For the production of polyphony one needs to be aiming at thirds and sixths as consonances, and these intervals involve the ratios 4:5, 5:6, 3:5, 5:8. The ancient derivation of the consonances, as you have it for instance in the Timaeus, does not account for these ratios. The trouble with Plato and the rest, Kepler says, is that they didn't listen carefully enough, before setting out to make their theory. Kepler sets out to make a new theory, without invoking the causal efficacy of numbers. His solution involves the regular polygons constructible with straight-edge and compass, which divide the circumference of the circle into equal parts. If one imagines the circle stretched out into a straight line, and transformed into a monochord, one has the divisions giving the consonances required for polyphony, including thirds and sixths, fundamentally because of the constructibility of the pentagon. The pentagon depends for its construction on the division of a line in extreme and mean ratio, the golden section, and if you are familiar with that division, and know how it can be indefinitely reproduced, by subtracting the smaller from the greater segment, or adding the greater to the whole, you may understand why Kepler views this division as imaging sexual generation, and you will thus gain an explanation of the tender feelings that accompany thirds and sixths in polyphonic music. Kepler did not suppose, and I do not believe that any theorist before him supposed, that the inquiry into the physical conditions for the production of certain intervals would account for the shades of feeling that those intervals arouse in consciousness. On the one side we have instruments like the monochord, from which we can get numbers; on the other, we have subtle perceptions of harmony, dissonance, restoration of consonance. There is a strange correspondence between soul and the bodily; but the bodily, in Kepler's view, does not account for the psychic.

Kepler's explanation for this correspondence takes us back to the sphere, image of the triune God. By creative radiation from the center, one gets the straight line, the element of bodily form. A straight line, rotated about one of its points, describes a plane, representing in this image the bodily. When the sphere is cut by the plane, the result is a circle, the true image of the created mind, which is assigned to govern the body. As the circle lies both on the sphere and in the plane, so is the mind at the same time in the body, which it instructs, and in God as a radiation which, so to speak, flows from God's countenance. Since now Kepler conceives the circle as the bearer of pure harmonies, and believes these harmonies to be based in the nature of the soul, he comes to speak of the soul as a circle, supplied with the marks of the constructible divisions, an infinitely small circle, a point equipped with directions, a qualitative point. This no doubt is metaphor or symbol, but it is by such means alone that we can understand (insofar as that is possible) how body, soul, and God are related.

The harmonic divisions of the circle apply, of course, in the heavens, as well as in music; it is from these divisions that Kepler develops his astrological doctrine, and also his harmonic theory for the planetary eccentricities. I cannot take time to describe these here. The five-regular solid theory Kepler comes to see as inexact, an archetypal form used to determine the number of the planets, then slightly modified in order to jibe with the harmonic theory of the eccentricities.

A playful God, ruled by the necessities of geometry, may be forced to such expedients.

All these parts of Kepler's work are omitted, to say the least, from the corpus of scientific knowledge recognized today. Meanwhile, his great achievement in remaking planetary theory, accomplished first for Mars in the years 1600 to 1605, is praised, sometimes on the mistaken grounds that it is purely empirical. It is not. It involves assumptions that are rejected today. Alternative paths to the so-called Keplerian laws are conceivable, but neither could they have been purely empirical. On the other hand, Kepler's study first of optics and then of the motions of Mars in the years 1600 to 1605 does lead to the development of a possibility already present in his thought. He is the first to quantify the intensity of light, in accordance with the inverse square of the distance from the source. (This is a purely apriori derivation, involving no experimentation.) He does not regard light as material or corpuscular; that would have meant Epicurean philosophy, which like most good Christians of the time he abhorred. Rather, he says, light is quantified according to surface, not according to corporeality. It is one of a group of immaterial emanations, whereby bodies, which are isolated from each other by their bounding surfaces, are enabled to be in communication with one another. Another such is the motive virtue issuing from the Sun, and in about 1602 Kepler finds out how this also can be quantified, its intensity varying inversely as the distance from the Sun; the empirical support consists in what is known as the bisection of the eccentricity, which he had been able to verify from Tycho's observations in the case of Mars and the Earth. A further step is taken in 1605 when he discovers that that component of the planet's motion whereby it approaches and recedes from the Sun, can be regarded as simply a libration, or what we would today call a simple harmonic motion: this, he says, smells of the balance, not of mind. And the elliptical motion that results from combining the libration with the vortical motion about the Sun - this also, he says, smells of matter, not of mind.

In no way do these developments disturb Kepler's fundamental view of the world.

They introduce something that one can perhaps call mechanism: matter turns out to have inertia in the sense of being sluggish, and it turns out to be pushed by an immaterial something in an incomprehensible way. Unexplained by the quasi-mechanism are the initial conditions, the sizes of the orbits and their eccentricities, and these must be works of mind, harmonically determined. Kepler's Harmonic of the World of 1619 will remain his final testament. And indeed it is through the spherical symbol, ultimate source of the archetypal harmonies, that Kepler was first enabled to accept Copernicanism, and then, developing the emanative aspect of the symbol, to banish from the sky the celestial intelligences, ultimate relics of paganism (as he calls them), and to regard the planets as material, subject to quantifiable forces that man from his moving platform can measure.

Kepler wanted to dedicate his Harmonicē mundi to James I of England. For years, very naively from a political point of view, he had looked to this monarch as the hope of Europe, the one who could bring a religious peace out of the strife of Reformation and Counter-Reformation. The relevance of the Harmonicē mundi to this end was that it was a work of the liberal arts, the arts of peace as Kepler called them, setting forth the principles of the harmonies with which the world had been adorned by its Creator. Kepler thought that, could men but see these things, they would be raised above the level of doctrinal dispute. Unfortunately, no one in the 17th century that I know of accepted either Kepler's dynamics as a whole (Leibniz undertook to re-vamp it), or his harmonic even in part. And as the book first appeared for sale in the market stalls, the Thirty Years War had already begun its terrible course.

....

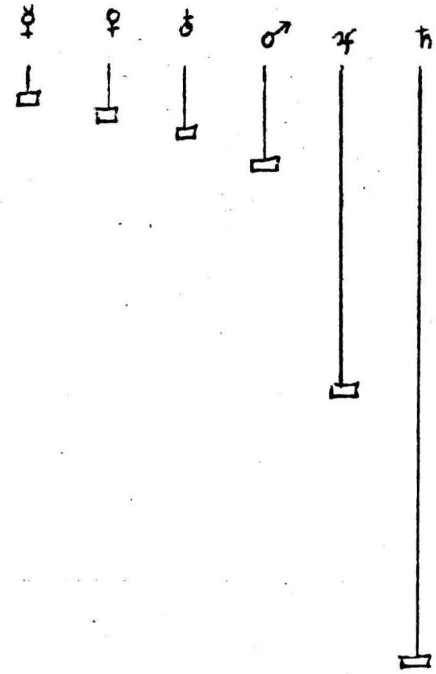
Let me turn abruptly to Newton. We do not expect to find effusions about the celestial harmonies in Newton's writings. True enough, in the second edition of the Principia, explaining his rule of induction, Newton says that Nature is ever consonant with itself; and so we might imagine it as emitting some single, deep organ tone. But this is from the second edition, 1713, and the first edition, 1687, does not contain the rule of induction. The third book, the System of the World, rather begins in that first edition with a set of propositions labelled "hypotheses".

Shortly after the publication of the first edition, Newton began a series of revisions, pertaining particularly to the early part of Book III. He wrote a series of scholia to accompany those propositions, 4-9, which lead to the establishment of universal gravitation. I wish to read to you from the proposed scholium to Prop.8.

> By what proportion gravity decreases in receding from the Planets the ancients have not sufficiently explained. Yet they appear to have adumbrated it by the harmony of the celestial spheres, designating the Sun and the remaining six planets ... by means of Apollo with the Lyre of seven strings, and measuring the intervals of the spheres by the intervals of the tones. Thus they alleged that seven tones are brought into being ... and that the Sun strikes the strings. Hence Macrobius says, 'Apollo's Lyre of seven strings provides understanding of the motions of all the celestial spheres over which nature has set the Sun as moderator.' And Proclus (commenting) on Plato's Timaeus, 'The number seven they have dedicated to Apollo as to him who embraces all symphonies whatsoever, and therefore they used to call him ... the Prince of the number seven.' Likewise in Eusebius' Preparation of the Gospel [Eusebius was one of the Church Fathers] , the Sun is called, by the oracle of Apollo, the king of the seven-sounding harmony. But by this symbol they indicated that the Sun by his own force acts upon the planets in that harmonic ratio of distances by which the force of tension acts upon strings of different lengths...
>
> The same tension upon a string half as long acts four times as powerfully, for it generates the Octave, and the Octave is produced by a force four times as great. For if a string of given length stretched by a given weight produces a given tone, the same tension upon a string thrice as short acts nine times as much. For it produces the twelfth [i.e. an octave plus a fifth] , and a string which stretched by a given weight produces a given tone needs to be

stretched by nine times as much weight so as to produce the twelfth....

[Let me briefly review the mathematical relation here. Imagine a series of six strings with lengths proportional to the distances from the Sun to the six planets; let equal weights be hung on the strings; we thus obtain six different tones - very dissonant with one another, let me add, but Newton does not mention the fact. These tones betoken different forces, which can be measured by taking strings of <u>equal</u> lengths and hanging on them different weights, so as to give the same tones.



Any two of the weights will be inversely as the squares of the corresponding lengths. Newton continues:]

> Now this argument is subtle, yet became known to the ancients. For Pythagoras, as Macrobius avows, stretched the intestine of sheep or the sinews of oxen by attaching various weights, and from this learned the ratio of the celestial harmony. Therefore, by means of such experiments he ascertained that the weights by which all tones on equal strings [were produced] ... were reciprocally as the squares of the lengths of the strings by which the musical instrument emits the same tones. But the proportion discovered by these experiments, on the evidence of Macrobius, he applied to the heavens and consequently by comparing those weights with the weights of the Planets and the lengths of the strings with the distances of the Planets, he understood by means of the harmony of the heavens that the weights of the Planets towards the Sun were reciprocally as the squares of their distances from the Sun.
>
> But the Philosophers loved so to mitigate their mystical discourses that in the presence of the vulgar they foolishly propounded vulgar matters for the

> sake of ridicule, and hid the truth beneath discourses of this kind. In this sense Pythagoras numbered his musical tones from the Earth, as though from here to the Moon were a tone, and thence to Mercury a semitone, and from thence to the rest of the planets other musical intervals. But he taught that the sounds were emitted by the motion and attrition of the solid spheres, as though a great sphere emitted a heavier tone as happens when iron hammers are smitten. And from this, it seems, was born the Ptolemaic System of orbs, when meanwhile Pythagoras beneath parables of this sort was hiding his own system and the true harmony of the heavens.

I have to say: Newton's interpretation of the ancient texts is not a little dubious. Contrary to what all 17th century Copernicans believed, the early Pythagoreans were not heliocentrists; Philolaus, the first Pythagorean to write down doctrine (for which he is supposed to have been appropriately punished), did not in fact know the Earth to be round, and his Central Fire was not the Sun. Again, so far as anyone knows today, the law relating weights and string-lengths for different musical intervals was first discovered not by Pythagoras but in the late 1580's by Vincenzo Galilei, the father of Galileo Galilei. Indeed, the discovery of this law, which can be verified very precisely if one has a good ear (and Vincenzo was a musician) - this discovery may have been what set Galileo on his course of experimentation, seeking exact numerical ratios in nature; he started with pendulums (again, weights hung on strings), and proceeded to motion down inclined planes, in order perhaps to analyze the motion of the pendulum. But the incorrectness of Newton's interpretations is not my concern here. The sheer volume of the manuscripts, the many variants and revisions, in all of which Newton is seeking to show that the ancient philosophers before Aristotle understood the Newtonian System of the World, demonstrates that these views were important to Newton. Can we make that fact intelligible to ourselves, or must we conclude simply that it is one of the queernesses of genius?

I want to speak briefly about the discovery of universal gravitation. I have recently changed my mind on this matter. My previous argument (which I unfortunately published) was that before 1684 Newton did not have his 'proof' of universal gravitation, therefore was uncertain about the universality. I now believe that my interpretation was wrong with respect to the 'therefore' clause. The idea of universal gravitation can seem more paradoxical than we perhaps realize. For a long time, since the 1720's, the picture was, that Newton already in 1666 had all his principal ideas, and was held up from producing his masterpiece by the lack of a good value for the Earth's radius, or according to a 19th-century suggestion, by the lack of a certain mathematical theorem. That interpretation is supported by no solid evidence whatsoever; it is a mirage which can be explained; but my point here is only that it is wrong. All of Newton's statements about planetary motion, made before 1679, imply either Descartes' theory of vortices, and/or an aethereal theory to keep the planets from receding from the Sun. Newton uses Descartes' term, _conatus recedendi a centro_, the term which Huygens in 1673 replaces by the term _centrifugal force._ Newton's thought about planetary motion during these years, like Huygens', remains confined to Descartes' analogy of the stone in the sling. There is no evidence that, before 1679, Newton ever conceptualizes the orbital process as the falling of the planet out of the rectilinear path it would follow if left to itself, towards a central attracting body.

Now this does not mean that during these years Newton rejected the possibility of attractions and repulsions, as possible physical causes. He was not a Cartesian; he did not believe space to be identical with matter, and all transfer of motion to be by contact. He was familiar with Gassendi's counter-argument, according to which not everything that is, is substance or accident; thus time and space need not be the accidents of anything, but may independently subsist, and so space need not be the space _of_ something (namely body). This may not have satisfied Newton, but given Torricelli's experiment with the barometer, he was willing to grant the vacuum, and while this does not in itself lead to the granting of real attractions and repulsions, it opens up the possibility of hypothesizing them. No doubt, to hypothesize such forces was to depart from the accepted norm of natural philosophy established by

Descartes. But Robert Hooke was doing it, and Newton began doing it, speaking of the sociability and unsociability of bodies in chemical reactions and cohesions. The forces he considered seem to have been forces acting over very small distances; his alchemical experiments were probably meant to find out about them.

In 1679 comes the famous exchange of letters between Hooke and Newton, a polite fencing between bitter enemies. Here Hooke explicitly proposes that Newton work out the path of a body under an inverse-square attraction that pulls the body away from its rectilinear trajectory. So far as the evidence goes, this is the first time that Newton faced the planetary problem in such a form. And under this provocation, he makes the great discovery that a force of attraction, directed toward a fixed center, implies the equable description of areas, Kepler's so-called second law. He applies this law, which allows him to use area to represent time, to the ellipse with center of attraction in the focus, and finds that the force follows an inverse-square law.

Previously I thought that it was Hooke who first placed in front of Newton the idea of universal gravitation, so that if Newton had not grasped it before, he did so now, and proceeded to look for a way to test it. But the fact is, that Hooke himself did not believe gravitation to be universal. He had generalized gravitation more than any previous author. Earlier authors like Kepler had regarded attraction as belonging to <u>cognate</u> bodies. So Roberval could talk of a lunar gravity, a terrestrial gravity, a solar gravity, a jovial gravity, and so on. Let me quote Hooke' view in 1678; he is explaining a hypothesis about comets:

> I suppose the gravitating power of the Sun in the center of this part of the Heaven in which we are, hath an attractive power upon all the bodies of the Planets, & of the Earth that move about it, and that each of those again have a respect answerable, whereby they may be said to attract the Sun in the same manner as the Load-stone hath to Iron, and the Iron hath to the Load-stone. I conceive also that this attractive virtue may act likewise upon several bodies that come within the center of its sphere of activity, though

> 'tis not improbable also but that as on some bodies it may have no effect at all, no more than the Load-stone which acts on Iron, hath upon a bar of Tin, Lead, Glass, Wood, etc., so on other bodies, it may have a clean contrary effect, that is of protrusion, thrusting off, driving away ...; whence it is, I conceive, that the parts of the body of this Comet (being confounded or jumbled, as 'twere together, and so the gravitating principle destroyed) become of other natures than they were before, and so the body may cease to maintain its place in the Universe, where it was first placed.

Now Hooke is an inductivist of a sort, but induction is not here leading to universal gravitation. In his correspondence with Newton in the following year, Hooke suggests that Newton may be able to think of a cause of the gravitating principle: now in Hooke's understanding, and I think in Newton's, too, to say that was to imply that gravitation is not universal, for the material cause of gravitation could not itself be subject to gravitation.

In view of the passages cited and others I shall refer to later, I suspect that the idea of a truly universal gravitation became effectively present only after Newton had discovered the 'proof'. Why propose a theory which, by its very nature, precludes any mechanical explanation, which seems to preclude being tested, and which, moreover, as Newton actually suggests once he has begun to entertain it, would seem to put the calculation of a planetary orbit beyond the power of any human mind?

There is the problem, also, of explaining Newton's delay for five more years after 1679. The best explanation, I believe, is that Newton does not yet think he has discovered anything very important, and sees no direction in which to pursue his discovery. Then Halley appears, probably in August of 1684, and persuades him that his discovery of the logical relation between inverse-square law and Sun-focussed elliptical orbit is important, and that he should publish it, to secure the invention to himself. Newton sets to work, and we have a series of MSS which can be arranged in temporal order on the basis of internal evidence.

In the first MS, there is no sign of the notion of universal gravitation. Newton speaks of gravity as one species of centripetal force - the term "centripetal force" making its first appearance here (it is Newton's invention). There is no hint of the problem of perturbation, the disturbance of the orbit and motion of one planet by the attraction of another planet. The inverse-square law is derived from Kepler's third law as applied to the planets and to the satellites of Jupiter and Saturn, and the orbits are simply said to be elliptical. The entire development, I believe, is up in the air, in the sense that Newton does not know the cause of the attraction, does not know how exact Kepler's third law may be (he had questioned its exactitude at an earlier date), and is merely proceeding mathematically without knowing what may underlie the derivation: it could be something that might lead to the results needing to be qualified.

To mention just one possible explanation, one that Newton had thought up in the 1660's and proposed to the Royal Society in 1675: the action of the Sun on the planets might be due to the inrushing of a subtle aether, which would serve as fuel for the Sun's burning. A similar but different aether might be rushing into the Earth to produce terrestrial gravity; this aether might be transformed chemically within the Earth, then issue forth as our atmosphere. The satellite systems of Jupiter and Saturn might be sustained by similar circulations of aether. These several centripetal forces would be explicable mechanically, that is by impacts; gravitation would not be universal.

In the second MS the notion of perturbation appears: Newton is now assuming that all the bodies of the solar system attract one another, just as Hooke had before. Can the planetary orbits still be said to be elliptical? Hardly, if the ellipses are drawn badly out of shape by the perturbations, the attractions of the different planets on one another. What must be done is to evaluate the relative magnitude of these perturbations. How is that going to be possible?

Newton does it by considering the accelerations of the satellites of Jupiter towards Jupiter, of the Moon towards the Earth, of Venus towards the Sun. Each satellite is being accelerated towards the body round which it goes, and that acceleration depends on the power of the central body to attract, and so may be able to serve as a measure of that power. Of course, to be comparable measures, all three satellites ought to be at the same distance from their central body, and they aren't. But we can shift them in thought to the same distance, by using the inverse-square law. What we get, then, are the comparative attractive powers of Jupiter, the Earth, the Sun. That of the Sun is overwhelmingly larger than the others.

But do we really have attractions here or not? Thus far there has been no evidence that Newton's aethereal theory for the planets is wrong. What then happens, I think, is that Newton realizes a consequence of something he has been assuming. In his derivation of the comparative attractive powers of Jupiter, the Earth, etc., he has been assuming that the quantity of matter of the satellite or test body didn't (if you will forgive a pun) matter; it didn't matter what mass it had; it was accelerated to the same degree anyhow; the differences between the masses of the test bodies could be ignored. Is that right?

Is it so on the Earth? Did Newton know the downward acceleration of all bodies on the Earth, at a given place, to be the same? Not at this moment. Earlier we know he had assumed the rates to be slightly different for different bodies, depending on their micro-structure, and the way the downflowing aether affected them. Now, in the third MS, Newton sets out to test the constancy, and this is the most precise experiment reported in the Principia. He takes equal weights of nine different materials; encloses each of them - gold, salt, wool, wood, and so on - in boxes of equal size and shape, to make the air resistance the same; and uses these boxes as the bobs for nine different pendulums, with very long but equal suspensions. The pendulums, he says, played exactly together for a very long time. The accelerations of these different materials, he concludes, cannot differ from one another by more than one part in 1000. Essentially the same experiment, the Eötvös experiment, has been performed in this century with a precision of one part in one billion. Another way of stating the result, you may know, is that inertial mass is proportional to weight.

At this point in the manuscript series, there appears for the first time in history, so far as I know, a statement of Newton's third law of motion, the equality of action and reaction. Let me now put these two results together, Newton's Eötvös experiment, and his third law, as they are put together in the Principia. The first implies that bodies on the Earth are accelerated downward by a force that is strictly proportional to what Newton now calls their mass, by which he means their resistance to being accelerated. If the same thing holds with respect to Jupiter, with respect to Saturn, and with respect to the Sun, then one can compare the attracting powers of these different bodies in the way we have already seen: by taking a test body, it doesn't matter of what mass, placing it at a fixed distance from the attracting body, and seeing how much it is accelerated. Newton couldn't do this physically, as we've said, but assuming the inverse-square law he could find from the actual acceleration of a body at one distance what the acceleration would be if the satellite were placed at any stipulated distance.

Now comes the final step. Since the mass of the test body can be ignored, in the comparison of the attracting forces of two bodies, one can use each as a test body for the other. Then

$$\frac{\text{A's power of attraction}}{\text{B's power of attraction}} = \frac{\text{acceleration of B}}{\text{acceleration of A}} .$$

By the third law of motion, these accelerations are inversely as the inertial masses:

$$\frac{\text{acceleration of B}}{\text{acceleration of A}} = \frac{\text{mass of A}}{\text{mass of B}} .$$

Putting the two results together,

$$\frac{\text{A's power of attraction}}{\text{B's power of attraction}} = \frac{\text{mass of A}}{\text{mass of B}} .$$

All right, that's it. The gravitational force is proportional to both the mass of the attracting and the mass of the attracted body. Inertial mass belongs to bodies merely because they are bodies. Therefore gravitational force goes with all bodies; all bodies attract gravitationally. Gravitational attraction is therefore inexplicable by any mechanical model of matter in motion. The mechanical philosophy, Newton concludes in the 1680's and 1690's, is dead; he has rediscovered the ancient mystic Pythagorean truth of the harmony of the spheres. Gravitation, he concludes, is the result simply of the immediate action of God.

There was a tradition in 17th-century England, pursued particularly by the so-called Cambridge Platonists Henry More and Ralph Cudworth, having to do with the prisci theologi or ancient theologians, Hermes Trismegistus, Orpheus, Pythagoras, Thales, Plato, and so on, whose pagan wisdom, it was claimed, was really derivative from that of the Hebrew prophets, especially Moses. More and Cudworth developed their interpretation of these ancient doctrines into a justification for a new and revolutionary natural philosophy. Newton, influenced by these men in earlier years, now believes he has found the right interpretation of the ancient wisdom precisely because he has found the right natural philosophy. And so he writes:

> Since all matter duly formed is attended with signs of life and all things are framed with perfect art and wisdom and nature does nothing in vain; if there be an universal life and all space be the sensorium of a thinking being who by immediate presence perceives all things in it, as that which thinks in us, perceives their pictures in the brain; those laws of motion arising from life or will may be of universal extent. To some such laws the ancient philosophers seem to have alluded when they called God Harmony and signified his actuating matter harmonically by the God Pan's playing upon a Pipe. ... To the mystical philosophers Pan was the supreme divinity inspiring this world with harmonic ratio like a musical instrument and handling it with modulation, according to that saying of Orpheus "striking the harmony of the world in playful song". But they said that the Planets move in their circuits by force of their own souls, that is, by force of the gravity which takes its origin from the action of the soul. From this, it seems arose the opinion of

> the peripatetics concerning Intelligences moving solid globes. But the souls of the sun and of all the planets the more ancient Philosophers held for one and the same divinity exercising its powers in all bodies whatsoever. ... All [their gods] are one thing, though there be many names.

And so Newton goes on to argue, using passages from Plato and Lucretius and many other ancient writings, that the philosophers of antiquity, Thales, Anaximander, Pythagoras, Democritus, and so on, were really agreed upon the atomicity of matter, the inverse-square law of gravitation, the universality of gravitation, and further, true mystics that they were, held the true cause of gravity to be the direct action of God. The unity of physical, moral, and theological wisdom is thus shown to have been present in the beginnings of the world, transmitted from Adam and Eve. That unity and that wisdom were gradually lost, after the corruptions of the sons of Noah; but now they have been recovered and restored by Newton, who thus takes his place among the prisci theologi, the ancient theologians. Newton is even able to find in the biblical book of Daniel the prophecy of his, Newton's, rediscovery of the truth.

So the first beginnings of a mathematized celestial dynamics came, with Kepler, out of a trinitarian symbol, the three-foldness of the Sun, spherical shell of stars and intervening space in a Sun-centered world; Kepler had his main idea from the beginning. With Newton it was different, and the crucial justificatory discovery came late, with a precise experiment to test the exactness of the constancy of the acceleration of gravity, and a new realization of the meaning of that constancy. And in a world that has now lost its geometrical center, Newton accepts this discovery as a revelation of a mysterious, omnipresent, unitarian God, to discourse of whom from the appearances, as he will tell us in the General Scholium to the second edition of the Principia, does certainly belong to Natural Philosophy. But the most famous statement of the General Scholium, presented there as the outcome of inductivist caution, "I do not contrive hypotheses" (hypotheses, that is, as to the cause of gravitation) - this statement disguised rather than expressed the deeper ground of Newton's original and I suspect persisting view, that gravitation was indeed universal, and the result of the direct action of God, so that no hypotheses for it could be successfully contrived.