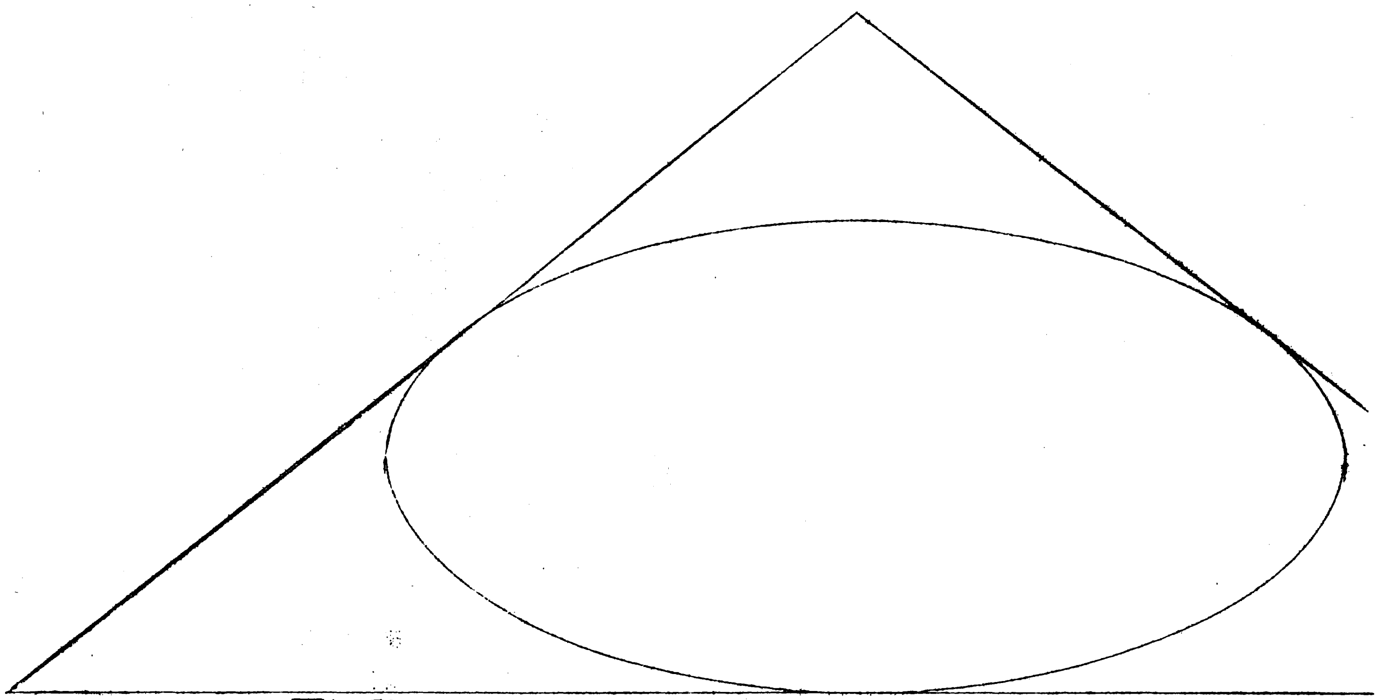

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A NOTE ON THE VIVIANI SCHOLIUM

(On the Inclined Plane)

by

Eva Brann

In the midst of the "Treatise on Naturally Accelerated Motion" presented for discussion in the Third Day of Galileo's Two New Sciences, there is to be found a long interjection known as the "Viviani Scholium", which is so fraught with confusingly inchoate riches that few Junior Mathematics Tutorials will ever find the time to do it justice. Last year Mr. Otis and I, and off and on some other people, spent a few afternoons attempting simply to establish the course of the argument. Mr. Klein gave us his notes to use; later I looked at some -- an infinitesimal part -- of the background literature. Perhaps this note, presenting the results, will serve to make Viviani's note somewhat more accessible to those who, liking beginnings best, wish to study it a little more.

Translation and Text

Dover: Galileo Galilei, Dialogues Concerning Two New Sciences, Dover Publications, New York, trans. H. Crew and A. De Salvio, 1914. Since this is the edition in common use, references will be to its pages and paragraphs. Since, beginning with the title,[3)] it is not so much a translation as a paraphrase of Galileo's carefully worded text, adjustments have frequently been made from the major edition:

N.E. Le Opere di Galileo Galilei, reprinted from the National Edition, Florence, 1933, Vol. VIII, p. 214ff.

Author and Context of the Scholium

In 1639, the year after the publication of the Two New Sciences, Galileo wrote to Castelli, a chief pupil (N.E. p. 23):

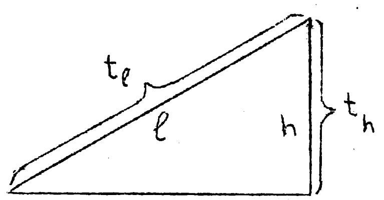
> "The opposition voiced many months ago by the young man who is at present my host and pupil [Vincenzio Viviani] against that principle [that the final velocities of bodies descending planes of different inclinations but the same height are equal] supposed by me in my treatise on accelerated motion, made it necessary for

> me, for the purpose of persuading him that such a principle was admissible and true, to think about it in such a way that I finally, to his and my delight, happened -- unless I am mistaken -- to come on the conclusive demonstration, which has been, so far, communicated to more than one person here. Of this he has now made a development for me, which, since I am in fact deprived of my sight, I would probably have gotten mixed up with respect to the figures and letters which are required here. It is written in dialogue, [a.] as Salviati's part; thus it is possible, if ever my Discourses and Demonstrations should be printed anew, to insert it immediately below the Scholium of the second Proposition [b.] of said treatise, on fac. 177 of that impression [Leyden] , as a theorem most essential to the establishment of the science of motion [c.] advanced by me."

a. The form of the Viviani Scholium, is not exactly that of other "scholia" (as notes developing something in the text are called), for the Galilean scholia on the propositions are in Latin, as is the mathematical text itself. Rather it is in dialogue form, spoken in Italian, and carried by Salviati. This last point marks its great importance, for most of the dialogic "interruptions", present intriguing false starts, alternate informal proofs, etc., and are initiated by Sagredo, the bright pupil, rather than by Salviati who speaks for "our Academian",[1] Galileo (Dover, p. 180). The dialogue in the colloquial tongue has the general purpose of widening the view, of carrying out the meaning and of giving a likely and probable[2] basis to the mathematical tract and its strict *mos geometricus*[3]. In modern jargon, the dialogue might be called the meta-language of the new science.

The labyrinthine laboriousness of this conversation is, accordingly, in the most striking contrast to the simplicity of the mathematical result. But that is precisely the good of it -- here we can re-do some of the work done before classical physics became the fait accompli as which it is handed about.

b. The placing of the Viviani Scholium after the Latin scholium extends the fundamental rule of motion for free fall: $s \propto t^2$ (Th. II) and its inverse: $t \propto \sqrt{s}$ (Cor. II)[4)] to inclined planes, simply by extending to them the fundamental assumption $v \propto t$ (Dover, p. 167). But to find, as Theorem III does, how the times for the different planes will vary, namely that $t_l : t_h ::$ length : height, it is necessary to use also the second fundamental assumption, namely the equality of speeds at the bottom (Dover, p. 169); and it is this assumption which is now proved.



This "theorem" is an assumption made by Salviati in the dialogue (Dover, p. 169-172) introductory to the first theorem of the "Treatise on Naturally Accelerated Motion" -- once again: the speeds for one and the same body at the bottom of differently inclined planes of the same height are equal. Sagredo's "reason tells him at once" that it must hold for smooth, straight inclined planes (Dover, p. 170), and Salviati describes in addition an experiment showing it for curved planes. This assumption is now to be proved.

The proof is made via a lemma introducing an equilibrating body or counter-weight which measures the moment of the body in question by resisting it (II 4). This resistance itself is measured by the vertical distance the equilibrating body would fall if it were free within the limits of its "inextensible" connections in the system (see Note 25) and by its weight.

In other words, the science of the description of local motion, kinematics, is ultimately based on the science of weights and their equilibrating power and potential velocities, on statics.

Now the science of weights has, before Newton, a certain preeminence[5)] for these various reasons:

1. It is basic to mechanics, and therefore practically useful; in particular, it has an immediate civil relevance, as in warfare and public works.[6)]
2. It is mathematicizable in geometric, i.e., in classical pre-analytic terms; that is, diagrams can be drawn of statical situations whose visual symmetry immediately represents the conditions of equilibrium (cf. E. Mach, The Science of Mechanics, Open Court, 1960, pp. 14-15, on Archimedes' statics). Its traditional presentations must therefore have been more satisfactory to Galileo than those of the science of motion[7)].
3. It is the nature of statics that it may measure bodies as if they were a sort of cause, i.e. in their role (at least within the system) of effecting other bodies -- as exerting force to cause motion. This approach is called dynamics[8)]. Now because an account is always a little more satisfactory as it ceases to be merely descriptive and includes a clearer notion of that which is held responsible (Physics 194 b 16 ff.) Galileo tried from his youth on to tie his kinematics in with statics (as in his early treatise On Motion, trans. Drabkin pp. 20, 38 ff.), and to connect both, in turn, with the heavenly motions so as to achieve

a general dynamics:

> "If he [anyone] will advise me as to the motive power of these movable bodies [planets], I promise I shall be able to tell him what makes the earth move. Moreover I shall do the same if he can teach me what it is that moves earthly things downwards",

but

> "we do not really understand what principle or what force it is that moves stones downward, any more than we understand what moves them upward after they leave the thrower's hand, or what moves the moon around." (Dialogue, p. 234).

It is this unification of the physical sciences by means of a dynamic theory which Newton finally accomplished, and toward which the Viviani Scholium tends. The significance of the Scholium for Galileo's successors has, therefore, been not so much in the proof of the "essential theorem" as in the establishment of a dynamic notion of moment which could be interpreted as the acceleration on inclined planes.

The Terms of the Viviani Scholium

Impetus: this is the widest of all notions in pre-Newtonian physics -- it is at some time used for most of the terms below; the science of motion itself is, at its juncture with the science of weights, namely at the beginning of the Viviani Scholium (Dover, p. 180) called the "theory of impetus".

Its root PET, which turns up in Greek in πίπτω -- "I fall" and in πετο-μαι - "I fly", in Latin in peto -- "I make for or seek", shows the double way in which it is taken in the common human understanding, namely as a certain vehement internal desire or tendency expressed externally in a kind of spontaneous, seemingly inevitable, motion of which flying birds or falling bodies are the non-human paradigm.

This word, which is thus, like all common (i.e., true) words, perfectly at home in the Aristotelian world of ends, is then appropriated as a technical term to solve a problem which arises within this world. If it is supposed that something must be responsible (αἴτιον) for any change that occurs; in particular, in cases of change of place, i.e., in locomotion, the proper place for the body to be is its end in both senses of the word, and is therefore responsible for moving the body. This is called motion "by nature" (φύσει). But there are clearly motions contrary to what is natural, for instance, when a heavy body, which should be seeking the center of heaviness, is thrown in another direction. Now, once the interfering efficient source, say a hand, of this motion "by violence" (βίᾳ) is removed, what is responsible for the continuation of this motion, which has no proper end?

A traditional answer is that it is the impetus[9)] of a body. Then a host of complications arises, for on thinking out the implications of this answer, impetus appears to have to be both 1. a cause or source of motion which the body has in itself -- but this is precisely Aristotle's definition of nature in living things: ὅσων δ' ἡ ἀρχὴ ἐν αὑτοῖς τῆς κινήσεως, ταῦτα φύσει φαμὲν κινεῖσθαι, "Of those bodies which have a source of motion in themselves we say that they move 'by nature'". 2. an effect of the application of violence -- the Aristotelian opposite of nature: τὰ μὲν φύσει τὰ δὲ βίᾳ καὶ παρὰ φύσιν, "some things are according to nature, while others are by force or against nature" and παρὰ φύσιν μὲν οἷον τὰ γεηρὰ ἄνω καὶ τὸ πῦρ κάτω, "against nature [are such things] as clod-like things [moving] up and flame-like things [moving] down" (Physics VIII, 4).

Again, do not bodies which are accelerated naturally also seem to *acquire* a greater tendency to move, i.e., more impetus, in the very course of their movement? And does the *additional* tendency arise in some way by reason of the bodies' original impetus or rather by reason of the greater effectiveness of the end, as Aristotle might claim (*De Caelo* 277 a 28 ff.) -- is the new impetus self-induced or impressed? And so also in forced motions, what causes their decreasing impetus?

Again, is one to speak of impetus as 1. conserved when once implanted -- in modern terms, as "inertial" (as appears to be the case in accumulating impetus, i.e., in accelerated motions) -- so that bodies can come to rest only by accumulating a sufficient amount of counter-impulse or by being forcibly stopped; or 2. as slowly wearing off (as in decelerating projectiles), i.e., as self-expending.

And again, is the problem of the *durability* of impetus the same with or different from that of its *cause* and *origin*?

And so forth. These dilemmas point to a radical question: what *really* is in want of explanation -- that there is motion *without* the application of violence, or that there is motion *consequent* on it? Newton will answer that if the principles are to be *mathematical*, the question is wrongly put: all motion and all change of motion is *by force* (*Principia*, Definition III, Law II), but force, far from being a violation of nature is its law . Clearly this non-mathematical impetus is a hopelessly elusive notion except in its most common-sensical meaning which Galileo attempts to convey in the synonyms *ability*, *energy*, *tendency to motion*, and *momentum* (Dover p. 181, para. 3; p. 183, para. 2).

On the whole, Galileo came to prefer to leave the matter of impetus as cause unsettled (Dover, pp. 165-166), except that he clearly thinks of impetus as inertial (see Note 22) and -- insofar as it appears to be a consequence of velocity--as an effect, either induced naturally by the (undefined) action of gravity or impressed forcibly. For the rest, he deals with the quantitative aspects of impetus only.

Momentum is one such quantity of impetus (Dover, p. 184, para. 2), and a property or consequence of velocity (p. 167: momenti della sua velocità; p. 169: celeritatis momenta). Since Galileo has no concept of mass as quantity of matter (Principia, Definition I) separate from weight, Galilean momentum is not Newton's "quantity of motion", "arising from the velocity and quantity of matter conjointly" (Principia, Definition II), i.e., $m \cdot v$, but rather the quantity of impetus arising from the velocity and weight conjointly, i.e., $w \cdot v$. But since in Galileo's science of motion weight has no role (this is proved in the First Day, Dover, p. 62 ff; also Note 19),

$$\text{momentum} \propto \text{velocity},$$

simply, and they are interchangeable.

Moment, the impetus of static situations, is given no name separate from momentum by Galileo, and this is what makes the Viviani Scholium so difficult to unravel. Nevertheless the context and the qualifying adjectives of (II) and (III) mark the "momentum" of these parts of the Scholium as a different notion.[10)] Galileo defines moment in mechanics or statics (On Mechanics, pp. 151, 145):

> ". . .moment is that impetus to go downward composed of heaviness, position, and anything else by which this tendency may be caused."

(Note that here Galileo forms in words the "lever product" of weight and distance which in his mathematics he studiously avoids, never multiplying his means and extremes.) Since <ins>heaviness</ins> means proper or free weight and, for the inclined plane, <ins>position</ins> determines a vertical distance[11)] covered in a given constrained (mechanical) motion,

$$\text{moment} \propto \text{weight} \cdot \text{vertical distance};$$

and this corresponds to so-called "statical moment" in case's where, as in pulleys, the lever arms are turned down. What does Galileo mean by "anything else"? Probably the fact that since, the times being the same, vertical distance is as the mean velocity (see below under <ins>Velocity</ins>, and Note 13, Th. II), vertical moment can also be written as the corresponding final momentum, namely as $w \cdot v$, although in equilibrium situations,where nothing is actually in motion, the velocity is only incipient, or potential, or <ins>virtual</ins> (see Note 25). This kind of velocity is referred to as "the spaces which would be traversed by them [the bodies] in equal times" (Dover, p. 183, para. 1). <ins>It bridges the gap between moment</ins> along a plane <ins>and</ins> <ins>momentum</ins> at a point on it. Salviati makes this correction in the Scholium in (II, 6), This is the very part of the Scholium which is usually regarded as its crux: the momentum along the inclined plane could now be called a constant <ins>acceleration</ins> equal to the gravitational constant and reduced by the inclination: $a = g \sin \vartheta$ -- though Galileo does not do this.

<ins>Partial</ins> moment (Dover, p. 183, para. 2) is a yet somewhat different use of the word "momento", which coincides with the <ins>effective weight</ins>, i.e., that part of the weight apparent <ins>along</ins> the inclined plane (see Note 10). <ins>Total</ins> moment is then the "proper" or vertical or free-hanging weight of a body.

<ins>Resistance</ins> or <ins>force</ins> is the "<ins>un-natural</ins>" quantity used in Sections II and III as a measure of the "<ins>natural</ins>" momentum of a body. It is conceived as the "minimal force" sufficient to hold the body at rest (Dover, p. 182, para. 2), as the

impressed power impelling (Dover, p. 166: "virtù impressa", "impellente") the body upward just sufficiently to counteract its downward moment[12)]. Its measure, in turn, is a vertical weight, i.e., a weight's total moment. If this force, with a minimal addition (<u>On Mechanics</u>, p. 160), is exercised over a distance, <u>work</u> (F·s) is performed.

<u>Velocity</u>, (speed) is undefined in the Treatises on Motion -- it simply appears in Axioms III and IV of the Treatise on Uniform Motion (Dover, p. 155. An analysis of Galileo's treatment of the measure of speed is given in Note 13).

Velocity is the quantitative aspect of locomotion,[14)] called in medieval terms its "intensity". In Theorem I of the Treatise on Accelerated Motion, Galileo borrows a medieval

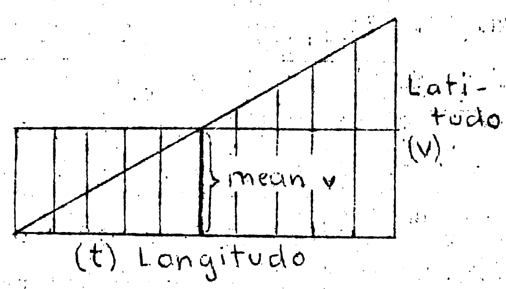


diagram [probably from Orême (14th century) who did not, however, apply it to accelerated motion, see <u>An Abstract of Nicholas Orême's Treatise on the Breadth of Forms</u>, St. John's Bookstore, and Clagett, <u>Science of Mechanics</u>, pp. 677-678] and represents the "intensities" of the motion by the "breadths" of a triangular "form" i.e., as heights erected ordinately on the base; this base is a "length" equally divided to represent equal periods of time (Galileo rotates the diagram -- Orême's diagram is closer to the modern convention which gives the horizontal to the independent variable t .) The sum of all the heights is taken as the area of the form (Dover, p. 214) and interpreted originally as a kind of <u>total effort</u> which is the result of all the intensity. Even Galileo is still unwilling, as the complicated reasoning of Dover, p. 175 shows, simply to form the algebraic product of the velocity and the time axis to get

$\frac{s}{t} \cdot t = s$, that is, to make the area a distance integral.

Orême's diagram finds for all the successive intensities of a naturally accelerated motion in a given time one intensity, the mean velocity,[15)] which will accumulate the same effort or distance in the same time. This enables Galileo to convert any accelerated motion into a uniform motion whose velocity is the mean velocity of the accelerated motion. Thus for finite times, he has the average velocity, and if he had taken the times small enough he would have had instantaneous velocity,[16)] of which he certainly conceived (Dover, p. 164-165). Galileo therefore has four views of velocity
1. as intensity of motion, undefined within his science;
2. as a distance-time ratio, $\frac{s}{t}$ in uniform motion; as an average, $\frac{\Delta s}{\Delta t}$ in accelerated motion; as a ratio of infinitesimals (this understanding does not turn up in the mathematics, which proceeds finitely, except that the ordinates are <u>interpreted</u> to measure the velocities for instants of time, and that given equal areas are <u>assumed</u> to contain equal sums of ordinates, Dover, p. 173).

<u>Acceleration</u> is conspicuously absent from the Scholium. In general it does not often occur as a substantive in the <u>Two New Sciences</u>; Galileo speaks of "accelerated motion" or "increased velocity". So it is a modernism to think of him as establishing that $a = g = k$ in free fall, or that $a = g \sin \vartheta$ on inclined planes. For since continuously varying gravitational acceleration is unknown to him [he thinks for instance that motion is accelerated constantly up to the moon (Dialogue, pp. 223-224)], though he knows that on curved planes "acceleration proceedes with degrees much different" from that on straight planes (Dover, p. 172), and again that the accelerations on different straight planes differ (see Note 14), he does not conceive of the acceleration of any <u>one</u> motion

changing regularly -- hence he cannot be said to think of acceleration as a quantity, such as may be proved to be k in a given case, at all.

Motion is undefined within the new science of motion.[17)]

Key

$\propto$	- varies with
$\sim$	- is similar to
h	- height
l	- length
M	- momentum or moment
M_{AC}	- moment along AC
s	- distance
v	- velocity
v_C	- velocity at C
v_{AC}	- velocity over Ac, i.e., the mean velocity

The Argument of the Scholium

The argument of the Viviani Scholium is developed in four parts:

I. (Dover, p. 181) Fundamental observation: for one and the same plane, the final velocity increases and decreases with the angle of inclination.

II. (Dover, pp. 182-183) A lemma from mechanics: for differently inclined planes of the same height, the momenta vary inversely as the lengths.

III. (Dover, p. 184) The main theorem: for differently inclined planes of the same height the final velocities are the same.

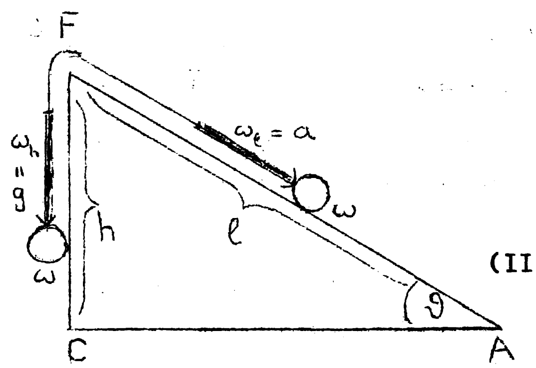
IV. (Dover, p. 185) Theorems III and VI of the "Treatise on Naturally Accelerated Motion" proved by means of the foregoing.

Formulaic Summary

The reasoning of the Viviani Scholium is too laborious for summary; moreover its usefulness is precisely that it reproduces all the reasonings underlying the beginnings -- one can only hope to explicate it by paraphrase. Instead, two formulae giving the gist of its argument, may be useful:

$$\text{(II)} \quad \frac{\text{moment}_{\text{inclined plane}}}{\text{moment}_{\text{vertical}}} = \frac{w \cdot h}{w \cdot l} = \frac{v_h}{v_l} = \frac{w_h}{w_l} = \frac{h}{l} = \sin \vartheta$$

where v_h and v_l are the vertical (virtual) velocities and ?
w_h and w_l are the effective components of two connected equal weights w along h and l. But, since, when the weights are disconnected, all these may be interpreted as accelerative effects where the vertical effect is a constant g,



$\frac{a}{g} = \sin \vartheta$, $a = g \sin \vartheta$,

and since $v = \sqrt{2gs}$ (Dover, p. 180),

$$\text{(III)} \quad v_A = \sqrt{2g \sin \vartheta \cdot l} \,, \qquad v_C = \sqrt{2\,gh} \;;$$

so that

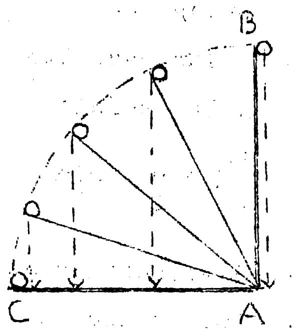
$$\boxed{v_C = v_A \,.}$$

I.

Dover, p. 181 para. 3 — ". . .the momenta or velocities of one and the same moving body vary with the inclination of the plane".

1. The telling fact in this beginning observation is that the variation is given non-quantitatively, i.e., non-mathematically, see (I, 5).

2. The fact of this variation is established by appealing to



ordinary experience of the two extreme cases -- that things dropped fall straight down while things laid on a horizontal go nowhere. The comparative effect of the in-between cases of the inclined plane are then extra- ? polated by a kind of "waived" or "thought" experiment.[18] The constants here are the body[19] and the length of the plane.

Dover p. 181 bottom

3. The diagram above immediately suggests the reason for the variation: the velocities are acquired over the <u>heights</u> of the planes only.[20] And the <u>cause</u> of this, is the <u>common center</u> "toward which all heavy things conspire."[21]

4. This too explains why there can be no natural motion along the horizontal -- it has no vertical component. The horizontal, conceived as a small arc of a large circle about the common center, is, therefore, Galileo's <u>inertial line</u>.[22]

5. This non-mathematical beginning in ordinary experience and thought-experiment indicates that Viviani is saying, in behalf of Galileo, who is himself in the <u>Two New Sciences</u> extremely reticent about such matters (Dover, p. 166), certain all heavy things things fundamental and prior to the descriptive mathematical science of the Treatises on Motion. In particular, he is
? stating one of Galileo's fundamental corrections of Aristotle:[23] terrestrial bodies have not two natural motions, down <u>and</u> up (<u>On the Heavens</u> I, 268 a 11-26) but only <u>one</u>, <u>downwards</u>:
? ". . .it is impossible for a heavy body [and all bodies are heavy, being bodies] . . .to move itself <u>naturally</u> upwards. . ." (N.E. p. 215).

II.

1. We are now given as a lemma the <u>mathematical</u> statement, i.e., the <u>manner</u> of variation of the momenta, or -- as we shall say -- the moments, with the inclination, for which Salviati refers to Galileo's treatise <u>On Mechanics</u> (trans. Stillman Drake, Madison 1960, p. 175) written at Padua about 1600.[24)]

Dover, p. 182, para. 1

2. It is: <u>the moment of one and the same heavy body moving on different inclined planes of the same height varies inversely as their lengths</u>.

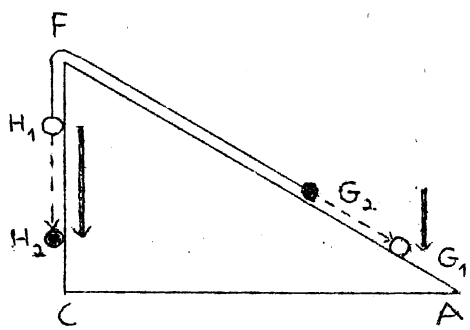
3. Salviati actually begins with a special, and as it turns out, normative case: he will compare the moment on the inclined plane with the unique, maximum moment along the vertical. So that he proves

$$M_{FA} : M_{FC} :: FC : FA.$$

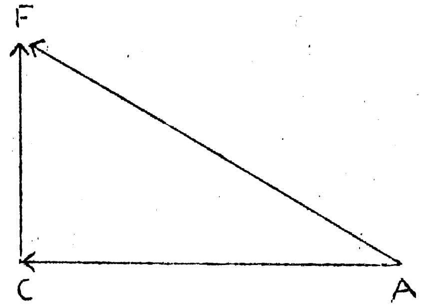
Dover, p. 182, para. 2

4. To do this, he introduces a <u>second</u> body, i.e., he exploits the science of <u>statics</u> (i.e., the study of the conditions for equilibrium). Now equilibrium obtains between connected bodies H and G if the

downward impetus of H = resistance (force) provided by G.



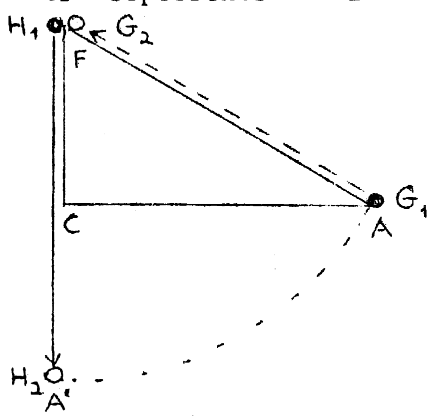
5. a. Hang a body H, (inexstensibly) connected with the original body G on the vertical side FC of the inclined plane FA. In each motion, over the whole or part of AF and FC, G and H must ascend or descend through the same distances (broken arrow). But their vertical rise or fall will differ as the solid arrow in the diagram.



5. b. For a body rising from A to F, Δ FCA can be viewed as a diagram of displacements, such that, in terms of vectors, which are here anticipated,

$$\overrightarrow{AF} = \overrightarrow{AC} + \overrightarrow{CF}.$$

And again, anticipating Newton's parallelogram of forces (Principia, Laws, Cor. I), since no relevant force is required to move the body through the horizontal AC (I, 4), $\overrightarrow{CF}$ represents the force required to raise a body over AF.

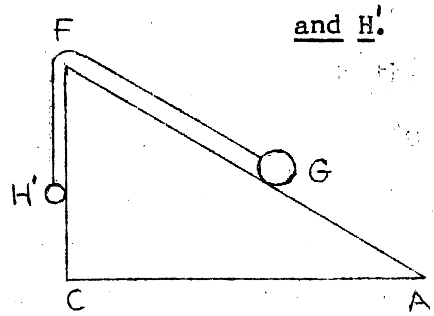


5. c. Now let G go over the whole distance AF equivalent to the vertical rise FC, and H must at the same time fall vertically through FA'; and the same ratio FC : FA holds for any part of such a motion. So that the tendencies to motion or potential vertical velocities (cf. under Moment and Note 25) are in the ratio

$$v_H : v_G :: FA : FC.$$

Dover, p. 183, para. 1

5. d. Therefore, for equilibrium to occur, these tendencies to motion must be opposed.[25] <u>This is to be done by means of the weights of G and H'.</u>



So that if weights are used such that $w_G : w_{H'} :: FA : FC$, the tendency to motion is counteracted and the static moments are equal.

Errata – Collegian, March 1963

p. 3, ¶ 2: $= c.$

p. 13, l. 5: $\frac{v_1}{v_n} = \frac{w_1}{w_n}$

p. 18, l. 4 from bottom: AB : AC

p. 19, l. 3: AC : AD

p. 31, l. 9: BC (1)

Dover, P. 183, para. 2

6. Now in equilibrium,

$$\text{impetus}_G = \text{resistance}_{H'} \quad (II, 4),$$

and

$$\text{resistance}_{H'} = w_{H'}, \qquad (II, 5d),$$

so that

$$\text{impetus}_G = w_{H'},$$

and H', whose total moment acts as weight along FC, is the measure of the partial moment of G along FA.

Furthermore

$$\text{total moment}_G = w_G ,$$

and therefore

$$\text{impetus}_G : \text{total moment}_G :: w_{H'} : w_G$$
$$:: FC : FA \quad (II, 5d)$$

and so Salviati returns to the single object G studied in kinematics (cf. Note 19). Note that though we should, to be exact, now again speak of momentum, these impeti along inclined planes are no longer, as were the momenta of free fall, similar to the velocities simply, but to the vertical velocities. However, and this is the crux of (II, 6), these velocities have now been reconceived in terms of weights, so that they are downward, whereas in (II, 5c) G rose while H fell. In short in a connected system $\overrightarrow{CF} \equiv \overrightarrow{FC}$ for unconnected bodies.

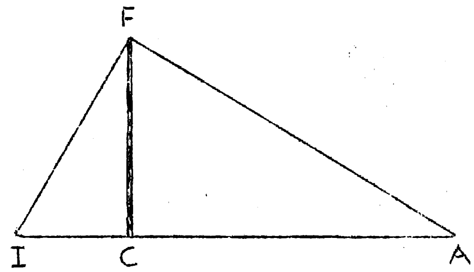
Now

$\text{impetus}_G = M_{FA}$, $\text{total moment}_G = M_{FC}$, by the definitions of these terms.

Therefore

$$\underline{\underline{M_{FA} : M_{FC} :: FC : FA}}$$

7. Sagredo completes the lemma by generalizing from (II, 3) to (II, 2) and *this is the mathematical form of impetus on inclined planes of equal heights.*



Since $M_{FA} : M_{FC} :: FC : FA$

and $M_{FC} : M_{FI} :: FI : FC$,

$\therefore$ $M_{FA} : M_{FI} :: FI : FA$, by *ex aequale* in perturbed proportion (cf. Heath, *Euclid*, II, p. 136).

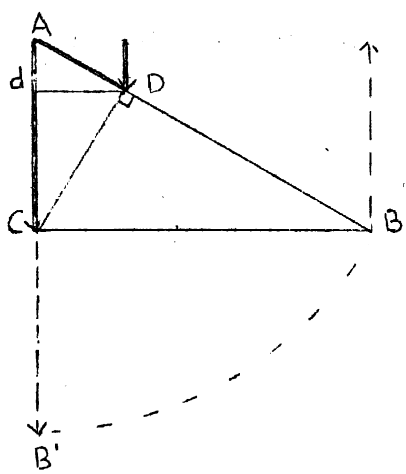
III.

Dover, p. 184, para.1

1. Finally Salviati proves the theorem: *If a body falls freely along smooth planes inclined at any angle whatever, but of the same height, the speeds with which it reaches the bottom are the same.*

2. Salviati first recalls a. the fundamental assumption which holds along one and the same plane of any inclination:

$v \propto t$ (Dover, p. 167; *Collegian* for February 1963, p. 1 ff.) and b. the fundamental theorem:



$s \propto v^2 \propto t^2$ (Th. II, p. 175.)

or $\sqrt{s} \propto v \propto t$ (Th. II, Cor. II, p. 180)

3. In $\triangle$ ABC, drop the perpendicular CD,

then $AB : AC :: AC : AD$ (as in Euclid VI, 8)

and $M_{AC} : M_{AB} :: AB : AD$ (II, 6)

$:: AC : AD$,

so that a body will traverse AC and AD in equal times. Why?

From here to "final velocities" the argument is not made explicit in the Scholium. It is: since the momenta are as the downward vertical velocities (II, 6),

$$v_{AC} : v_{AB} :: AC : Ad;$$

but for mean velocities, if $v \propto s$, t is equal, by the converse of Th. II, Note 13,

for

$$v_{AC} : v_{AB} :: AB : AC :: AD : Ad ;$$

but

$$v_{AB} : v_{AD} :: Ad : AC ,$$ these being the respective vertical distances.

$$\therefore\ v_{AC} : v_{AD} :: AD : AC ,$$

and this holds for the final velocities,

so that

$$v_D : v_C :: AD : AC$$

$$v_D : v_B :: t_{AD} : t_{AB} \quad \text{(III, 2 a)}$$

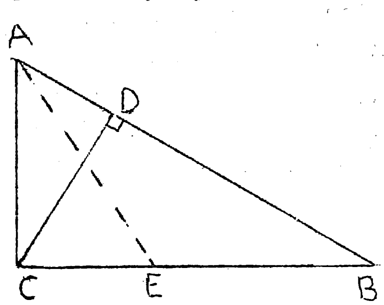
$$:: AD : AC \quad \text{(III, 2 b)}$$

$$\therefore\ v_B = v_C ,$$

and in general, the velocities on all planes AB, AE, AC, etc. will be the same for all points on a parallel to the base.

IV.

Dover,
p. 185
paras. 1, 2, 3



1. Theorem III from the above:

let $t_{AB} = AB$,

then $t_{AD} = AC$ (III, 2 b),

but $t_{AC} = AC$ (for $t_{AD} = t_{AC}$, III, 3).

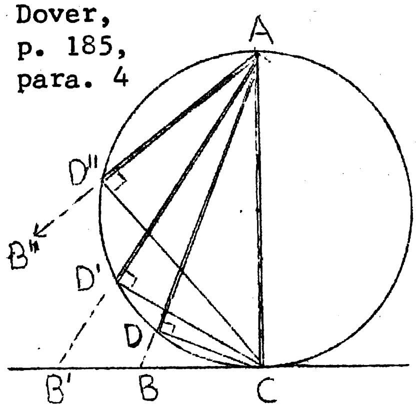
$$\therefore\ t_{AB} : t_{AC} :: AB : AC$$

Likewise

$$t_{AC} : t_{AE} :: AC : AE$$

$$\therefore\ t_{AB} : t_{AE} :: AB : AC, \text{ in general.}$$ [26]

Dover,
p. 185,
para. 4



2. Theorem VI from the above: Inscribe any $\triangle ADC$, $AD'C$, $AD''C$, etc. into a circle with diameter AC. Then, as in (III, 3), a body will traverse AC and all planes AD in equal times.[27)]

Here Saiviati closes his "digression" which is "verily of profit for the subject of motion" (N. E. 219).

Notes

1. Galileo often refers to his membership in the Accademia dei Lincei (Academy of the Lynx-eyed), an organization of men who, to quote from its constitution, take pains ". . .in the observation of natural phenomena and the book of nature [cf. Note 2] which is always at hand; that is, the heavens and the earth". (See Stillman Drake, Discoveries and Opinions of Galileo, Doubleday 1957, pp. 76-78).

2. "Probably basis": the first paragraph of the Viviani Scholium says that the principle in question is to be better established "con probabili discorsi et esperienze" (N. E. p. 214; cf. Dover, p. 180, which translates this as "on logical and experimental grounds" [1]). Cf. also Dialogue Concerning the Two Chief World Systems, trans. Stillman Drake, University of California Press, 1962, p. 229, where a fundamental motion theorem is after demonstration, said to be "reasonable and probable". So it seems that for Galileo the more fundamental a matter is, the more it has the aspect of a likelihood and even a "fantasy" (Dover, p. 166) -- an εἰκὼς μῦθος, a "likely story" (Timaeus 29 d 2).

3. The careful literary form of the Two New Sciences escapes people; for instance, compare the Elzevir and Dover title pages (Dover, pp. i and xv):

DISCORSI	dialogues concerning
E	TWO
DIMOSTRAZIONI	NEW
MATEMATICHE intorno à due nuoue scienze	SCIENCES

The point missed is that mathematical demonstrations are here given in the form of "Treatises" (N. E., p. 192) written in the universal Latin, and that this mathematical book is then read by the interlocutors (Salviati, Sagredo -- known in life as friends of Galileo -- and Simplicio the archetypal traditionalist -- named after the famous Aristotle commentator -- who always keeps a copy of De Caelo in his pocket, Dialogue, p. 136), as if they were turning over the "great book of nature" (Dialogue, p. 3), which is written in mathematical characters. [It must be by reason of this analogy that Galileo greets writing as "surpassing all stupendous inventions", while those who forego the "inquiry concerning nature" and think it necessary "to take refuge in reasonable speech" (Phaedo 96 b 8, 99 e 5) decry γράμματα, "scratchings" (Phaedrus 274 c ff.).]

The interlocutors then join in a discussion held in the tongue of daily life in which the intelligence disports itself freely and which is accessible to everyone, just as Descartes writes his Discourse on Method in French "in the hope that those who avail themselves of their natural reason alone, may be better judges of my opinions than those who give heed only to the writings of the ancients".

4. The Dover translation makes the Latin scholium apply only to Cor. II -- not so the original.

5. Stevin, whose famous anticipation of the contents of the Scholium is given below (Note 24, q.v. for full references to his Art of Weighing) calls the Art of Weighing "the most wonderful of all free arts", p. 93; [cf. our college seal:

facio	liberos	ex liberis	libris	libraque
I make	free men	from children	by books	and balance].

for it has the quality par excellence of the new sciences -- it kills wonder by explanation i.e., by bringing out plain and flat in diagrams (or formulas) the occult, hidden reasons.

Thus Stevin's most famous diagram (Note 24) appears on the title page of the 1586 impression of his *Art of Weighing* (p. 47) with the Dutch superscription "Wonder and yet no wonder" (cf. also Descartes, *Passions of the Soul*, Part II, Articles LXXV - LXXVIII).

Note that Kepler, too, in a most far-fetched way, brings in the principle of the balance to explicate his "librations", i.e., the planets' varying motions into the sun, which turn their circular orbits into ellipses (*Epitome* V, 1).

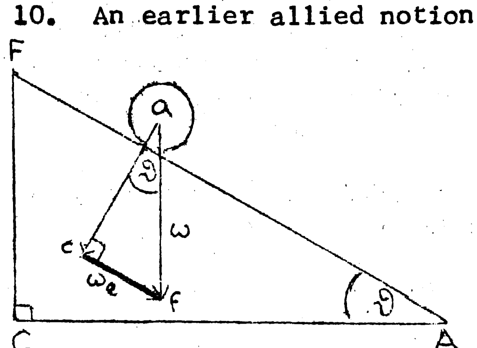
6. For instance, Stevin's *Practice of Weighing*, the complement to the *Art of Weighing*, is dedicated to the "Burgomasters and Rulers of the City of Nuremberg" (p. 293).

7. See Moody and Clagett, the *Medieval Science of Weights*, Madison 1952, p. 3 ff; Galileo, *On Motion and On Mechanics*, trans. I. E. Drabkin and Stillman Drake, Madison 1960, p. 135 ff.

8. By the usual systematic perversion of Aristotelian terms, since δύναμις is defined by Aristotle (*Metaphysics* 1046 a 11 ff.) as the "source of motion in another, insofar as it is other", where it is, precisely not a forcible cause: the first meaning he gives is that of the δύναμις τοῦ παθεῖν, "the capability of being affected".

9. For documents and discussion see Clagett, *Science of Mechanics in the Middle Ages*, p. 678 ff. and pp. 667-668, Koyré, *Etudes Galiléennes*, III pp. 91-97. II, pp. 9-21 and all of I. In Galileo, especially *Dialogue*, p. 194 ff.

10. An earlier allied notion is that of *gravitas secundum situm* -- "heaviness according to situation", also called *effective* (positional) *weight*, or (by Stevin, see Note 24) *apparent* (oblique) weight. In modern terms, if $\triangle$ AFC $\sim$ $\triangle$ afc, where $\vec{fa}$ is the measure of w, $\vec{cf}$ is the component of w effective along the inclined plane, i.e., $w_e = w \sin \vartheta$, see Clagett, *Science of Mechanics in the Middle Ages*, p. 676 *et al.*).



11. In levers, wheels, etc. "position" determines a *horizontal* distance from the line of application of w to the fulcrum or center, and the moment is w • horizontal distance. This is, in modern terms, rotational moment, cf. *Notes to Accompany the Reading of Newton* (Junior Mathematics Manual) p. 28.

12. Cf. also Dialogue, p. 213 ff. The measuring of moment by resistance depends, of course, on an implicit application of Newton, Principia, trans. Motte-Cajori, Berkeley, 1960, Law III; cf. also Dover, p. 255, para. 2.

13. The development of the measure of velocity, which is the tacit object of the Treatise on Uniform Motion (Dover, pp. 154-159), is very ingenious. (Uniform motion is, of course, motion which has only velocity [celeritas] and no acceleration [ad-celeratio]):

Galileo gives:

One definition, of uniform motion [motus aequabilis] :

For one moving object [mobile],

if $t_1 = t_2$, $s_1 = s_2$ for any t ,

where time is at the very beginning the independent variable.

Four axioms:

For the same uniform motion,

I. if $t_1 > t_2$, $s_1 > s_2$,

and conversely,

II. if $s_1 > s_2$, $t_1 > t_2$.

For the same time, with different uniform motions,

III. if $v_1 > v_2$, $s_1 > s_2$,

and conversely,

IV. if $s_1 > s_2$, $v_1 > v_2$.

The axioms serve 1. to introduce the basic undefined terms of the science: time, distance, velocity, and their indeterminate relations of more or less. In particular. Axioms III and IV introduce the notion of velocity as arising where different motions (namely motions with different velocities) occur in the same time, and implicitly, since time is the constant reference, as being distance covered per a particular time. 2. to assure the symmetry of all the relations by giving their converses. 3. to assure the applicability of the definition of "same ratio" (Euclid V, Def. 5), so that variations can be established in the theorems. The basic terms are thus established as Eudoxan magnitudes.

Six theorems:

For one moving object,

I. $t \propto s$, if v constant

II. $s \propto v$, if t equal

III. $v \propto t$, if s equal

For two moving objects,

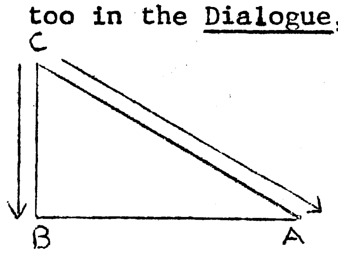
IV. $\frac{s_1}{s_2} \propto \frac{v_1}{v_2} \cdot \frac{t_1}{t_2}$

V. $\frac{t_1}{t_2} \propto \frac{s_1}{s_2} \cdot \frac{v_2}{v_1}$

VI. $\frac{v_1}{v_2} \propto \frac{s_1}{s_2} \cdot \frac{t_2}{t_1}$, so that $\underline{\underline{v \propto \frac{s}{t}}}$.

Theorems I - III establish the variations when each of the terms v, t, s is in turn held constant as reference. Theorems IV - VI then give the compound variations where none of the terms are constant, i.e., for two motions, i.e., the motions of two different objects, each with their own v, t, s. Thus, applying Theorem VI to a moving object with two motions, $v \propto \frac{s}{t}$, and so <u>the first "derived" magnitude</u> of physics is shown to have the dimensions $L\,T^{-1}$. It remains unclear within the Treatises whether this <u>is</u> velocity or the <u>measure</u> of velocity just as it is unclear whether velocity <u>is</u> motion or its intensity (see the manual <u>Variation and Units</u>, pp. 11-12 for compound variation and pp. 18-21 for the interpretation of $\frac{s}{t}$).

14. If it has no others -- Aristotle notwithstanding (cf. Note 17) -- then velocity <u>is</u> motion and rest is simply v = 0. For Galileo, however, motion and velocity are not yet explicitly the same; e.g., in Theorem IV of the Treatise on Uniform Motion, the <u>motions</u> are said to be uniform, but the speeds different, i.e., the rates of the change are each constant but of different amounts, as if the constancy were an attribute not of the rate of change, i.e., the velocity, but of the motion itself. So too in the <u>Dialogue</u>, pp. 24-26, bodies with "faster motion" are shown to have "equal velocities", i.e., a body will arrive at A and B with equal velocities (III), but will have spent more time over CA than over CB; hence it has "faster motion" on CB, which turns out to be <u>greater acceleration</u> (p. 25), which again is viewed not as merely the

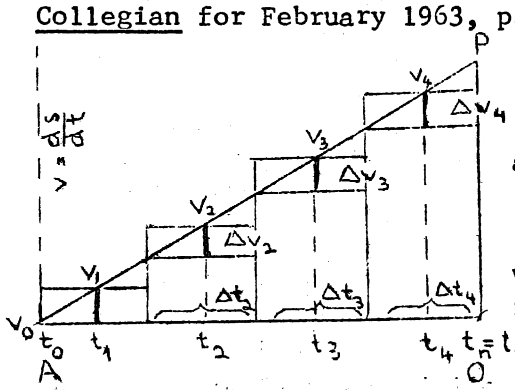


rate of change of the velocity, but as belonging directly to the primary concept, motion; indeed, to view it otherwise produces the very modern notion of change of change, impossible for Aristotle (Physics V, 225 b 15). Newton, too, takes acceleration as "change of motion"; indeed for him no other aspect of motion is observable except relatively, while accelerations can be an absolute effect (Principia, p. 12).

The Newtonian "quantity of motion" (mv - Principia Def. II) does not occur in Galileo, see the section on Momentum.

15. Cf. the Mean Value Theorem, I. Niven, Calculus, an Introductory Approach, p. 90, which is analogous.

16. Purely for exercise, one might re-conceive the Galilean diagram in terms of the moving mathematics unknown to Galileo. For this Sagredo's diagram seems best (but see C. Levin, Collegian for February 1963, p. 8):



If AO is subdivided n times, as $n \rightarrow \infty$, $\Delta t \rightarrow 0$; and the mean increments $\Delta v_1, \Delta v_2, \Delta v_3, \ldots \Delta v_n$ will approach the instantaneous increments $v_1, v_2, v_3, \ldots v_n$ at the instants $t_1, t_2, t_3, \ldots t_n$; these increments will all lie on the line AP whose formula is $v = \frac{ds}{dt} = at$ (cf. Junior Mechanics Manual, p. 8).

17. What is defined is the uniformity of a motion, that is, really, unchanging velocity. [The word "uniformity", incidentally, comes from the representation of intensities by "forms", cf. under Velocity.] This confounding of the measurable attribute with the thing itself makes a beautifully indicative beginning to modern physics.

For contrast, here is Aristotle's definition of motion: ἡ τοῦ δυνάμει ὄντος ἐντελέχεια, ᾗ τοιοῦτον, κίνησίς ἐστιν, "the fulfillment of that whose being is in its capability, insofar as it is such a thing, is motion" (Physics III, 201 a 10 ff.); thus motion, or taking the widest sense of κίνησις, change, is being-at-work to come into one's own.

18. See E. Mach, The Science of Mechanics, Open Court; 1960, p. 32 ff., R. S. Bart, Notes to Accompany the Reading of Newton's Principia, Annapolis 1957, p. 24.

19. Since it is characteristic of Galilean kinematics that its rules are independent of the bodily characteristics (see under <u>Momentum</u>), in particular of the quantity of matter of the object whose motion is described, this object is, in the Treatises themselves, usually called simply a "mobile"; however, as soon as causes are in question, as in this Scholium, it becomes a "grave", a heavy thing.

20. The mathematical rule relating velocity and height follows from Corollary II of the second theorem (Dover, p. 180): since by the corollary $t \propto \sqrt{h}$, and by the fundamental assumption (Dover, p. 167) $t \propto v$, therefore $v \propto \sqrt{h}$.

21. For Aristotle the "common center" is only accidentally identical with the center of the earth:

> "Furthermore, locomotion of the parts and the whole [of the earth] is, according to nature, to the center of the All (ἡ φορὰ τῶν μωρίων καὶ ὅλης αὐτῆς ἡ κατὰ φύσιν ἐπὶ τὸ μέσων τοῦ παντός ἐστιν) . . .But, since both have the same center, some might be perplexed to know toward which of these two, things which have heaviness (βάρος) and parts of the earth move according to nature -- whether [it be] toward it as the <u>center of the All</u> or <u>of the earth</u>. But it is necessarily toward the center of the All [that they move] , for light things as well as fire, which are borne in the [direction] opposite to heavy things, move toward the extremity of the place which surrounds the center. And it is <u>accidental</u> that the earth and the All have the same center."

(<u>On the Heavens</u>, III, 296 b 7-17)

For Galileo, on the other hand, the common center is the bodily center of the earth toward which heavy objects move as toward "their whole, their universal mother" (<u>Dialogue</u>, p. 37). Furthermore, the case is the same for all other planetary bodies (which are, contrary to Aristotle's opinion, made of corruptible, heavy matter), and if the world has a true common center it is the <u>sun</u> (<u>ibid</u>, p. 33).

That there is such a center of the planetary <u>system</u> and that it is close to the sun becomes respectively Hypothesis I and Proposition XII of Newton's <u>System of the World</u>, (<u>Principia</u>, Bk. III).

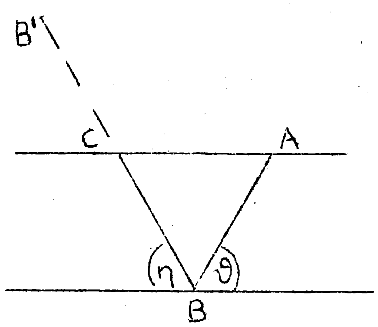
22. In Sagredo's cosmopoeia in the Fourth Day (Dover, p. 261; cf. the fourth day of <u>Genesis</u> I, 14-19) the orbits of the planets are such inertail lines about the sun as common center.

The Newtonian notion of inertia (Principia, Law I) has two facets:

1. There is no change of motion without a force at work, so that bodies neither expend nor accumulate their velocities of themselves -- a most extraordinary assertion from the traditional point of view, which, in accordance with common sense, supposes that motion itself, i.e., mere velocity, and not only change of motion, requires a cause. (See under definition of Impetus in text.)

   This facet of inertia is present in Galileo; in fact, he thinks of such inertial motion as the only truly natural kind (cf. Note 23, no. 6).

2. Inertial motion is in a right line of any direction. This is evidently not present in Galileo, unless it be in one curious and hidden case: In the scholium following Problem IX a most ingenious construction shows why bodies on inclined planes rise to the same height from which they have fallen (Dover, p. 216, Fig. 83).



Implicit Diagram

The tacit object of the construction is to show that even a rising motion is based on the tendency toward the common center and to avoid the vague notion of expendable impetus yet present in the Dialogue, p. 23).

For by pp. 214-215 the final velocity attained in fall from A to B, will be such that the body, when diverted at B to uniform motion along the horizontal (i.e., to inertial motion) will cover the distance of double AB in the same time that it took to fall through AB. If now the plane BC is constructed with $\measuredangle \eta = \measuredangle \vartheta$ (though these need not be equal) and extended to B', so that B'B = 2BC = 2AB, the body will be carried to B' by its inertial motion in the same time that it fell over AB, provided such a line of motion, being a straight line, and not everywhere equidistant from the common center may also be thought of as a line of inertial motion. If then this inertial motion, from B to B', requiring no new impetus is compounded with a falling motion from B' to C, the body will arrive at C, having in fact risen the distance BC in the same time as it descended over AB, but having in thought fallen, i.e., gathered impetus only over B'C.

23. "Correction" is perhaps the wrong word. All his life, from the early essay On Motion to the Dialogue Concerning the Two Chief World Systems, in which the Ptolemaic hypotheses based on Aristotelian physics are confronted with the

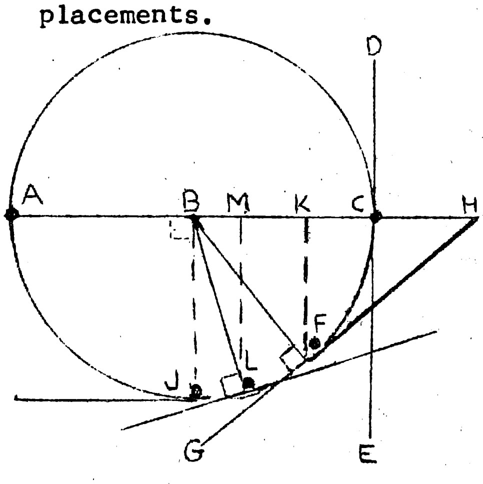
Copernican system interpreted according to Galilean physics, Galileo wrestled with Aristotle, to whom he is bound in two ways: 1. fundamentally insofar as he belongs to those who "put both sensibles and non-sensibles among the things that are" and who therefore "clearly conduct investigations about both kinds" (*Metaphysics* 989 b 25-27), i.e., who engage in the φυσικὴ πραγματεία. 2. in this particular--that he too considers bodies to have "natural" motions, that is, tendencies toward particular ends (though as no. 6 shows, he finally rejects such motions as really disorderly). But aside from these two points of agreement, he makes the following chief criticisms of Aristotle (based mostly on the text of *On the Heavens* which is actually read during the *Dialogue*), of which it might be useful to give a conspectus;

Aristotle	Galileo (Refs. to Dialogue)
1. The world has a geometric center and a periphery which are mutually determined.	1. The geometric structure follows on the location of the bodies (p. 33-37).
2. Near or at this center is the earth, at rest.	2. The sun probably determines the common center of gravity (p. 33). The earth moves (p. 124 ff.).
3. Toward the periphery, bodies, i.e., heavenly bodies, are incorruptible.	3. The world is homogeneous throughout; all bodies are corruptible (pp. 38 ff.).
4. Toward the center, i.e., in the sublunary region, there are two natural motions, up and down.	4. There is only one true natural motion, down, toward the center of gravity (p. 33).
5. Consequently bodies are either absolutely light or absolutely heavy, for this means only that they tend naturally up or down.	5. Bodies are only heavy, in varying degrees, and are so independently of their natural place (p. 245).
6. Such (terrestrial) motions are in a straight line, and are comprehended by and understandable within the scheme of being.	6. It is dubious whether motion is a straight line ever ever really occurs. When it does it is either as a prelude to creation or in correction of the world's temporal imperfections (pp. 20-21, 28 ff.) Only circular motion is motion according to the order of the world (p. 32), i.e., truly orderly motion is not motion to a proper end, but motion with a proper trajectory.

Now the question is, is it possible to "correct" Aristotle in this way? According to Galileo's own metaphor (Dialogue p. 16) it scarcely is. He says that Aristotle tries to shape the rules of architecture according to the building, rather than the building according to the rules. That is to say, Aristotle understands the cosmos as the unique intelligible whole, so that its "architecture" is really only its pattern set out in terms of the nature of being. Galileo, on the other hand, demands rules for the construction of the universe, imagining it to be homogeneous enough to conclude from local observations to larger parts, and expecting these rules to tell how things move rather than why they are.

Therefore, by Galileo's own description, it is scarcely possible to tamper with Aristotle's world in a partial way. That he does so results in a curious kind of senselessness in his use of Aristotelian terms (e.g., his use of the word "natural", cf. Dialogue, pp. 16, 134-135, 234-235), and a repeatedly admitted ἀπορία concerning the nature of the world. Certainly he knows himself to know -- as yet -- far less of the world than Aristotle thought he knew (Dialogue, p. 101 ff., cf. Note 2). This seems to me to be the best, and, indeed, the only, sense in which Galileo can be said to be a "Platonist" (Koyré, III, p. 120 ff.).

24. The proof given in On Mechanics (pp. 169-175) is well worth a summary, since it contains anticipations bearing on the pendulum, centrifugal force and infinitesimal displacements.



1. Let the diameter AC of the circle be a lever with arms movable about the center B as fulcrum, and equilibrated by equal weights at A and C. Now bend AB to position AF and the moment of weight F is diminished, "the distance of point F from the line BJ, which goes from support B to the center of the earth, being diminished". And thus for any positions BL etc.

2. But the weight can also be thought of as hanging from K, M, etc. And so the moment of such "inclined weights" diminishes as BK, BM, etc. diminish in relation to arm AB = BF, etc.

3. And also the weight A may be thought of as losing its moment because it is more and more supported, i.e., pulled

up, in its descent through the quadrant CJ by the arms BJ and BL.

4. And also CJ may be thought of as a curved supporting plane, in which case the moment of the weight will be total at C where it is least supported, and none at J where it is, as it were, on a horizontal.

5. But such motion is as motion on a series of differently inclined planes tangent to the arms at all points in the quadrant.

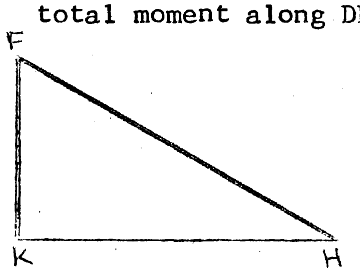
Now it was (informally) shown in 2. that

moment of weight at C : moment of weight at F :: BF : KB, etc., and, since $\triangle$ BFK $\sim$ $\triangle$ FHK (Euclid, VI, 8),

FH : KF :: BF : KB

so that

total moment along DE : partial moment on FH :: FH : FK.



6. And thus for the inclined plane FHK total moment down FK : partial moment along FH :: length FH : height FK.

7. And since "the force to move the weight need only insensibly exceed that which sustains it"; force : weight :: length : height.

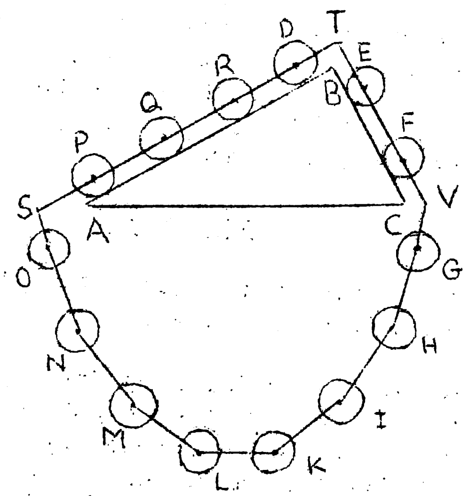
24. A most ingenious anticipation of Salviati's lemma, which used to be available in the old Junior Mechanics Manual, comes from the fascinating Fleming, Simon Stevin's Elements of the Art of Weighing (published 1586); the following is a condensation of the original as given in The Principal Works of Simon Stevin, ed. E. J. Dijksterhuis, Amsterdan, 1955, Vol. I, pp. 175-179 (facing Dutch and English):

The Wreath of Spheres

"Up to this point the properties of vertical weight have been explained; in the following pages the properties of oblique weights will be described, the common principle of which is contained in the following theorem.

Theorem XI. Proposition XIX

Given a triangle whose plane is at right angles to the horizon, with its base parallel thereto, while on each of the other sides there shall be a rolling sphere, of the same weight: as the right side of the triangle is to the left side, so is the apparent weight of the sphere on the left side to the apparent weight of the sphere on the right side.



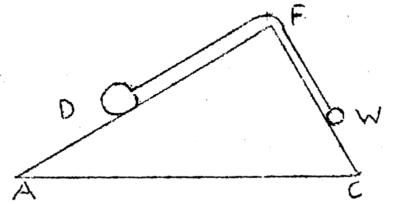
Given: Let there be the triangle ABC as described, with side AB equal to twice side BC. On AB let there be a sphere D and on BC a sphere E equal to each other in weight and size.
Required to prove:

AB(2):BC(L)::Apparent wgt.of E:Apparent weight of D.

Preliminary: About ABC make a wreath of 14 spheres equal in weight and size, D, E, F,...R, strung on a string through their centers in such a way that they can roll about them, and so that 4 spheres lie on AB and 2 on BC. Let there be fixed points S, T, V, such that the string can slide over them so as to allow the spheres to roll on sides AB and BC.

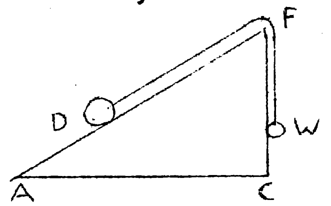
Proof: If the apparent weight of D+R+Q+P were not equal to the apparent weight of E+F, either the spheres on the left or those on the right must be heavier. Let it be the four spheres on the left. And the spheres G+H+I+K are equal in weight to the spheres L+M+N+O. Therefore, the side of the eight spheres (L+...+R)+D has a greater apparent weight than the side of the six spheres E+...+K. "But be= cause that which is heavier always preponderates over that which is lighter, the eight spheres will roll downwards and the other six will rise". Let this have happened and let D have fallen where O was and let E, F, G, H be where P, Q, R, D were. Now the wreath of spheres will have the same appearance as before, so that the same thing will happen "since the reason is the same, and the spheres will perform by themselves a perpetual motion, wich is wrong". Therefore, the apparent weight of spheres D+(L+...+R) is equal to that of spheres E+...+K. Subtract the equal weight L+M+N+O and G+H+I+K respectively from the left and right sides. The remainders, D+R+Q+P and E+F, will be of equal apparent weight, so that E will be double the weight of D. So that

BA(2) : BC(1) :: App. w. of E. : App. w. of D.



Corollary I

Now use only spheres D and W = ½D. Their apparent weight will be the same, for as P+Q+R+D were to E+F, so is D to W, i.e., as 2 to 1. Therefore, as AB(2) : BC(1) :: sphere D : sphere W.



Corollary II

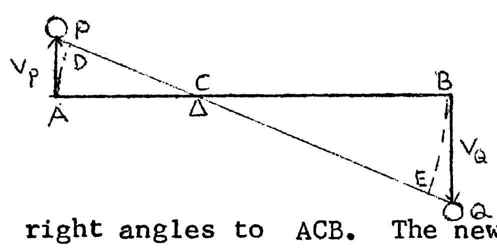
And similarly if ABC be a right triangle with AB = 2BC.

Stevin's "apparent weight" (stallwicht - rendered in Snell's Latin version by sacoma from Greek σήκωμα - weight) is Salviati's partial momentum, the active downward tendency "oblique weights" show along the supporting inclined plane.

Stevin's proof relies on a thought experiment which seems to show that if a physical situation is precisely identical before and after every motion, the bodies must have moved "of themselves", which is impossible (Aristotle, Physics 255 a 7 ff.). Therefore, when the spheres, taken as units of weight, have been made to correspond to the units of length by being laid along them, the rectilinear part of the wreath will be in equilibrium.

In Cor. I, the four spheres on the left and the counter-balancing spheres on the right are collected into two counterbalancing spheres, measuring each others' apparent weight. Cor. II finally gives the special case of the single inclined plane, the case with which Galileo begins. This is what distinguishes Galileo's proof and allows him to use it as a basis for the science of motion: that he discerns it as identical with the case of free fall and thinks of using as a measure the case of maximum or total momentum.

25. There is implicit here the so-called "principle of virtual velocities or displacements" first ennunciated by J. Bernoulli in 1717, and formulated much later as the "principle of virtual work":



If two weights P and Q disturb the equilibrium of lever ACB, the arms respectively suffer some displacements AD and BE, which for small angles may be regarded as straight lines at right angles to ACB. The new equilibrium will occur when

P : Q :: BE : AD

or when the forces are inversely as the displacements. But since the adjustment takes place in a single time, the displacements are as the average velocities, so that the new equilibrium can be thought of as achieved by compounding weights and their *virtual* velocities in the first position.

("Virtus" is a School translation of δύναμις, so "virtual" means "potential".)

Newton refers to this principle when he says in the scholium on the Third Law (*Principia*, p. 27) that "the power and use of machines consists only in this, that by diminishing the velocity we may augment the force, and the contrary". See Galileo, *On Mechanics*, p. 155 ff.; Mach, p. 59 ff.; A. Wolf, *A History of Science Technology and Philosophy*, I, pp. 46-47.)

26. The fact that the image of an inclined plane is also its time diagram indicates that the plane is a *time-scaling* device, i.e., it slows down motion without changing its rule.

27. This beautiful proposition is presented in no less than four ways (Dover, pp. 183, 189, 190, 191) and an extension of it, which allows the spherical starry container of the world to have come into being upon the single initial act of a creator seated at the point A on its periphery, and over a definite time Δt_1 (pp. 193-194), is said by Sagredo to hide a "mystery related to the creation of the universe", particularly to the "residence of the prime cause".

Simplicio

THE SOPHIST AND A HUNTER'S ART

Let us not be deceived, it was a great moment in history when long ago the world agreed that to treat of great things with the greatness they deserve, lesser and easier instances of them must be studied first.[1] Certainly if there was such a time it must have been when men discovered that the greater things underlying and running through man's experience are available to him always in terms of single experiences. It is at that time that the world must have seen how in the instance of a single experience there can be a sign of the principle behind it, or, rather that in a single experience there lies the instance of a principle, the sign of a principle. For example, as with one who suffers from a dullness of vision, magnification of the instance of justice to the level of its instance in a state can make it possible to see more clearly the instance of justice in an individual soul.[2] Then in following the Sophist as a hunter or a debater or as a maker of fantastic images it seems the Stranger must expect to find something significant of sophistry, the greater thing underlying these instances of hunter, debater, and image maker.

That sign, he discovers, is deception; which is to say that what in each case makes this hunter or debater or image-maker a sophist is the exercise of a certain art of deception. However, says the Sophist at that point, 'how a thing can appear and seem and yet not be, and how a man can say that which is not true is all very puzzling.' In short, what could deception possibly be? And so it seems the great thing of which we expect to treat is now deception, but what in the end we claim to have caught is the Sophist himself. How are we to distinguish in all of this what is sought above all; that which underlies and in some way is signified by all of this?

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1. Sophist, 218C5.
2. Republic., 368D. In answer to those who wonder whether this isn't just the reverse; that this is an instance not lesser but greater, it might be said, with a good deal of faith in the example, that the author of those lines sees the instance of justice in the soul to be a greater thing than its instance in a state.

If there is hope of understanding these things it lies in taking to heart the world's age-old agreement about beginning with lesser instances. Like the Stranger, we have the name Philosopher; a name without, or as he says, minus a definition. Setting the Philosopher up then as a great thing for which instances are sought, we too might discover that what we have been seeking after all is a thing in a sense greater than the philosopher, yet so linked with him as to be available only through him. Just as an art of deception characterized the Sophist, yet deception was through him made most wonderfully apparent, perhaps an art of distinction is a characteristic of the Philosopher, yet only through him does distinction become clear.

There are two men who share the acquisitive art of hunting. Both by a designed approach which is their art hope in some way to corner or expose their beast. The difference is that while one may want the pelt, the tusk, or the meat, the other wants knowledge of the creature's life and habits. While one seeks the thrill of conquering the beast, the other desires to understand its significance in the animal world, its relation to other animals, and, in general, all the aspects of <u>what it is</u>. What they do not share is what in the end they are out to get, for it seems quite clear that the one, in conquering, grabs hold of an instance, the other, in seeing, of a greater thing underlying.

The greater art is certainly that one which looks to embrace the greater thing, and it is not without meaning to say that, even with the simple materials of our search so far, the hunter striving by his art to see is the greater artist. The force of his being the greater artist is, possibly, one of the most important things to understand, and it may be more fully understood by carefully examining a hunter's art.

A hunter exploits his art in bringing the beast into a position where he can get at it. To arrange the beast's position in this way and then to descend upon it seem to be the two <u>parts of a hunt</u>: pursuit and confrontation. Further, how could we consider hunting to be an art if no decision was made about when, where, and how to look for the prey? Therefore, the hunter as an artist, will plan beforehand and base his plan upon what he thinks he knows along with the assumptions he is aware of making about his prey. Long experience will have taught him to regard the pursuit as a source of discovery in the light of which the original plan is reappraised. That is, what seemed to be knowledge is sometimes later seen as an assumption, and, vice-versa, what could only be assumed beforehand, is sometimes confirmed as fact. He does all this so that he may separate what pertains to the thing he is after from what does not, to get the beast into a position where what it is is distinct from what it is not. Such matters of skill or craft as keeping up-wind, maintaining a strict silence, hiding, and all the other traditions of hunting are never questioned in themselves. Art emerges in the selection of a certain combination of these elements suitable to the conditions as they present themselves.

We said that of the two kinds of hunters the one striving by his art to see was the greater artist, and of the art of hunting, what seems essential is its continual review of plan, (which itself is ultimately a fabric of facts and assumptions), in the hope of making the beast stand out. Without a doubt, both hunters look here and there and try to make out this and that in the exercise of a hunter's art. The one after a beast's hide has, in a strict sense, to see every bit as well as the other during the pursuit, <u>for sight is that sense above and before all others which makes clear to men the many differences between things</u>. Again, the difference between these hunters is in the confrontation where what they are after is made manifest. One, in laying hold of the object has at that point relinquished his sight for the sake of having the lesser thing, while the other, seeing now for the sake of sight itself, to discern what this could possibly be, makes his play for what is beyond the instance. Then what, in combining hunt and hunter, design and the interest directing it, shall be produced but a man who, by examining what he knows and assumes in the light of experience, strives in the end to see, and having seen, knows?

Yet remember there is the beast, Without it no picture of hunting could be complete. For the <u>parts of hunting</u> are necessarily three: the hunter, the hunt, and what is hunted. The hunter after hides must take into account that what he is hunting little welcomes the intrusion of being hunted.

He and the beast are worlds apart. All he has from the other world of the beast are signs of where in that world the beast might be, and they often mislead. Hunters of the longest experience, having the greatest reputation for acuteness, think sometimes that the prey is here when it is there, or that it is this when it is that. Also by the beast's design the signs may purposefully deceive. So guileful are some beasts that they will follow the one who for hours remains convinced they are straight ahead. For the hunter after great things there is, as well, this distance between the worlds of the hunter and what is hunted. He too has only the signs. Though not in the active sense of the former hunter's prey, the mysteries of order and relation seem to defy penetration. What must somehow impress both hunters is exactly how a sign can lie: what in it does not pertain to what they are after. Perhaps for the hunter after hides it is the design of the beast that makes the sign lie, but always for the hunter after knowledge it is the design of things.

Of course the hunt, if it is successful, bridges the gap and makes the transition from world to world, and a hunter of any mettle always expects to get what he is after, or if he doesn't get it, to come again to the woods at a more opportune time. The beast is either plucked bodily from his world, or in the case of our hunter, is made to yield the secrets of his world's interior, so to speak. Some will say the latter is meagre fare to return with after such a mighty chase. What is there to lay hold of in habits and significance, order and relation? What hearty feast, what warm covering, what valuable trinkets can we make with merely what the eyes have seen and the mind considered? The hunter after knowledge might return that a gain in understanding is surely the greatest and most liberating of gains. Certainly those who first asked, presumably the Giants,[3] will be dissatisfied with this answer and ask again. But here, whatever the Giants have to say, we are interested in just such 'meagre' fare as understanding. The minute a man sets himself the task of finding even the least thread of relation between things he has packed off into the hunter's

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3. Sophist 246B. ". . .(who) literally grasp in their hands rocks and oaks; of these they lay hold, and obstinately maintain, that the things only which can be touched or handled have being or essence, because they define being and body as one, and if anyone else says that what is not a body exists, they altogether despise him, and will hear of nothing but body."

world where, like the hunter, he has committed himself to the examination of what he thinks he knows in the light of experience; where, in the face of signs that lie, he must bring his prey to light.

Since the point of hunting as an acquisitive art is the capture and the parts of a hunt pursuit and confrontation, it follows that a crisis occurs in confronting the prey. The question 'will we get it?' becomes crucial. This moment comes to the hunter after hides when he throws his spear or casts his net. If we try to imagine the worst possible issue of his long labor we might think of an enraged beast, rushing headlong upon its would-be conqueror and raining a fatal violence on him and his designs. But what of the case where a guileful beast simply follows its unwary pursuer? How could anyone say there is any longer a sense in which the beast is being pursued? It has ceased to exist as prey, while the hunter, thoroughly deceived, continues to regard it as such. What a mistake to imagine his former brave death as his worst fate when he has now lost the point of his art.

By however much greater a value he places on understanding, the hunter after knowledge shall lose that much greater a thing in losing the point of his art. Surely the one who holds understanding to be the greatest of gains, in the disappearance of the object he was to have seen, has sustained the greatest of losses. Clearly, though the hunt and all it implies is the life of these men, the hunter after hides could sustain himself in the meanwhile on vegetables or less wily animals. To what, however, is the hunter after knowledge to turn? There is unique quality in his plight as compared with that of the other hunter, in that there is no substitute for the perfectly unique thing he desires, knowledge.

Now what beast is this that can thwart the best efforts of a man to find it? It is clear enough with the hunter after hides how a clever beast might involve him in the predicament of being after nothing, yet exactly how could those in pursuit of understanding be driven to such extremes? The fact is, there is a certain wonderful creature capable of the most magnificent displays in this respect. His deceit has the great virtue of seeming unchallengeable, the acme of deception, discouraging a good number of his less-talented pursuers from ever returning to the wilderness with which he surrounds himself. This is the arch deceiver who can appear in any of six different places to confuse a hunter dull enough to imagine he has cornered him in one, and if some earnest man persists through all six to pursue him, who can in a remarkable seventh form seem to vanish completely. This is the Sophist.

Two men set out to know what the Sophist is. Because ". . .any who (likes) may ask him anything. . . (and he answers). . .in a grand and bold style. . .which becomes those who know"[4], they will exercise their art in discovering all the possible occupations of a man who claims in all things to know. In the beginning the Sophist is gracious enough to appear as a hunter, of all things, whose object is the pay he shall receive as a teacher of virtue to youth (and wealth). A little more searching and he turns up as a merchant, selling the goods of the soul between cities and, by changing his hat, in the third form is a retailer, selling the goods of the soul in his own city. Fourthly, with a smock and no hat, he himself produces the goods he sells. In the fifth place we find him as one who acquires by open force, ". . .a hero of debate, who professed the eristic art"[5], and, as a strange wolf among soul-purgers, the Sophist emerges unexpectedly in the sixth. No great distinction yet accrues to the two hunters for having come this far; though they have brought to light ". . .various (of his) forms unrecognized by the ignorance of men"[6], still "the multiplicity of names applied to him shows that the common principle to which all these arts of knowledge are tending is not understood"[7]. These are without a doubt the occupations of a man who claims in all things to know, but what is the common sign that relates them and will give us an understanding of the Sophist? Upon re-examing what they think they know they discover that no man can understand all things. Hence, they conclude, since the Sophist claims to do so he must be some sort of jester, and all six men uncovered previously, by the sign of their pre-

4. Meno, 71
5. Sophist, 231D
6. Sophist, 231 D
7. Ib. 232

tending to know, must be jesters also, And now, deep in the inmost glade of a hunter's wilderness, they see before them one who creates images of the truth, which, because of his sheer ignorance of it or his purposeful design, are bound to be distorted and fantastic. Here in his seventh appearance, is the Sophist himself, fully attired as the arch-deceiver; for an image of the truth, already being other than the truth in being a sign of it and not truth itself, when distorted by design becomes a sign of what is not, the acme of deceit. But before our brave hunters are out with even the first flush of victory, the creature asks of them ". . . how a thing can appear and seem, and not be, or how a man can say a thing which is not true?"[8] And, by the Gods, he disappears.

The kernel of the Sophist's challenge lies in the confusion we experience when we isolate from our conversation the statements that 'something is not' or that 'nothing is', statements which it seems are true only if they are not true. This is precisely the point where most hunters have left off in their search, deeming the challenge insuperable. The Sophist must be allowed in the confusion to fall back to the other side of what was discovered to be the image-making art, to being a maker of likenesses. In the words of the Stranger from Elea, "I was doubtful before in which of them (the branches of the image-making art) I should place the Sophist, nor am I even now able to see clearly; verily he is a wonderful and inscrutable creature. And now in the cleverest manner he has got into an impossible place"[9]. For, in addition, the Sophist asks what they mean by an <u>image</u>, and they must admit that they mean something which is what it is not. 'Oh?', says the Sophist, 'and, as I said before, exactly how can that be?' This is how the hunters after knowledge have lost their prey.

What an impossible position! How can so few words have the power to constrain our mighty hunters? First, the possibility that signs could lie is questioned, then, that they could have any existence at all. What is more, this statement's power to destroy knows no master. Even the

8. Ib. 236E.
9. Ib. 236D.

opponent of not-being, when he says ". . .that not-being in itself can neither be spoken, uttered or thought, but that it is unthinkable, unutterable, unspeakable, (and) indescribable"[10], seems also to be in the absurd position of predicating being of not-being.

Yet as though in spite of what even before these discoveries seems to be an overwhelming predicament, the hunters now train their entire attention, their art, upon a resolution of these few words, but, for all their labor, their reward is to discover next that they stand bereft of art as well as object. They agree that as long as a man remains in a predicament about not-being he must forbid himself the statements 'nothing is', 'something is not'. At the same time they notice that if two things are not the same, such as, for instance, the Sophist and the Philosopher, there is no other way in which a man can express the relationship of Sophist to Philosopher than by using the forbidden statements. For Sophist being something like Philosopher, he would have to say '<u>something</u> Sophistical <u>is not</u> Philosophical', or Sophist being nothing like Philosopher, '<u>nothing</u> Sophistical <u>is</u> Philosophical'. We discover for the second time how mistaken we were in our previous estimation of a hunter's worst fate. Surely the loss of his object, the point of his art, is now far exceeded by his utter inability to draw distinctions. The Sophist would take away his eyes[11] if he could, by means of which one hunter hoped above all to observe distinctions. That signs would lie, he thought, was in the nature of things. The Sophist claims that is impossible. He expected to get the beast into a position where what it is would be distinct from what it is not. This too the Sophist claims is impossible. He finds that in one stroke he has been stripped of the entire foundation of his art.

We have seen that to hunt is to attempt to draw out the beast from its dark world to where it can be seen, to where it is distinct. Drawing it out, for one of our hunters, was the end of the process of seeing. For the other it was exactly the opposite. In confronting the beast he was just

10. Ib. 238C.
11. His sight (p. 4)

beginning to see, to see what it was. But we know now that that meant to see as well what it was not. (In a most important way the one who grabbed at that moment for the trinket, for the warm covering shut his eyes.) The hunter who wants always to see we hoped would be an instance in which there would be a sign of the Philosopher. That sign we discover is distinction. Because the Philosopher hopes to see in what is presented to him how things are the same and how they are not the same; he distinguishes for the sake of distinction. The challenge of the Sophist is a great one, great because it has helped us to see a greater thing in the instance of the hunter, the Philosopher. It has helped us to see what the Philosopher does.

However, the Sophist is not as great as his challenge. He is not so because, having presented the challenge, he too shuts his eyes. After all it is what he is after that makes the difference, and for some reason he has surrendered the philosophy that made the challenge, and is after money. Rightly or wrongly he draws distinctions, he works in the instance. He even goes through the instance but only so far as to be stopped by his own challenge. Only the hunter who wants always to see will go through with the challenge, only the Philosopher.

Persistence in the face of what seems to be a thorough defeat might be a sign of blindness in a man or, just the opposite, of profound insight into himself and the nature of the event. Men, perhaps in their ignorance, might recommend to the hunters that they retreat homeward, that is if they can any longer find their way: we wonder at the following statement:

> How, then, can anyone put faith in me? For now as always, I am unequal to the refutation of not-being. And therefore, as I was saying, do not look to me for the right way of speaking about not-being; but come, let us try the experiment with you.
>
> What do you mean?
>
> Make a noble effort, as becomes youth, and endeavor with all your might to speak of not-being

> in the right manner, without introducing into it either existence or unity or plurality.[12]

But then this:

> You see the handles for objection and the difficulties which will arise are very numerous and obvious.
>
> They are indeed.
>
> We have gone through a very small portion of them, and they are really infinite.
>
> If that is the case, we cannot possibly catch the Sophist.
>
> Shall we then be so faint-hearted as to give him up?
>
> Certainly not, I should say, if we can get the slightest hold on him.[13]

And especially this:

> And is it not shameless when we do not know what knowledge is, to be explaining the verb 'to know'? The truth is, Theaetetus, that we have long been infected with logical impurity. Thousands of times have we not repeated the words 'we know', and 'do not know', and 'we have or have not science and knowledge', as if we could understand what we are saying to one another', so long as we remain ignorant about knowledge; and at this moment we are using the words 'we understand', 'we are ignorant' as though we could still employ them when deprived of knowledge or science.
>
> But if you avoid these expressions, Socrates, how will you ever argue at all?
>
> I could not, being the man I am. The case would be different if I were a true hero of dialectic: . . . But, seeing that we are no great wits, shall

12. Sophist, 239c.
13. Ib. 241C

> I venture to say what knowing is? For I think that the attempt may be worth making.
>
> Then by all means venture, and no one shall find fault with you for using the forbidden terms.[14]

Where lies the root of the persistence in these men? Is it blindness or insight which gives rise to it; is it merely insistence or is it courage? I think it is the latter. If so, have they seen, deep within themselves and in their language, a reason why the one should say that the refutation of not-being is the deceit of an arch-deceiver and the other that, being the man he is, it can make sense to speak of knowing, And can the Stranger say that, as the Phoenix from her ashes, so has his art been reborn? I think so.

Leon Rottner

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14. Theaetetus, 196C

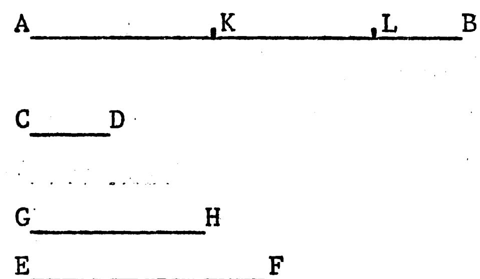
A QUESTION ABOUT SYMBOLS

On page six (mimeographed edition) of the Preface to Francois Viète's Introduction to the Analytical Art, the author thereof has occasion to observe that Viete's solution to the Diophantine problem of finding two numbers whose sum and difference is known "would have been meaningless for Diophantus and for the Greek mathematicians generally". The meaninglessness would, the author implies, have arisen from Viète's use of symbols which stand for "multiplicity in general . . . which is no particular multiplicity (and yet) is treated as if it were a particular multiplicity". It is the intent of this Note to raise the question whether the use of such symbols would have been meaningless to Euclid and to do so through a consideration 1) of how the same problem could be solved strictly within an Euclidean framework and 2) Euclid's treatment of number in Bks. VII - IX generally. If it can be shown that Euclid's method may be used to solve the problem, or that he himself has recourse to "general multiplicity, which is no particular multiplicity, (and yet) is treated as if it were a particular multiplicity", it would seem that the meaninglessness of Viète's enterprise to at least one Greek mathematician would be in doubt. Such a showing, however, would in no way bear on Viète's other contributions to a revolution in mathematics (eg., the translation of the above problem into one of finding "sides" instead of numbers; the uniting of arithmetical and geometrical analysis; both of which might well have been meaningless to Greek mathematicians).

An Euclidean solution to the problem might be as follows. (We will follow the three-fold method of analysis, indeterminate solution, and synthesis which Apollonius exhibits in Bk. II, 44-51, and which seems to bear a great similarity to what Viète calls Zetetic, Poristic, and Rhetic.)

Given the difference of two numbers and their sum. To find the numbers. (If, like Diophantus, we think of the unit as divisible, this enunciation of the problem will be identical with his. If, like Euclid, we think of the unit as indivisible, we shall, as will appear when we come to the indeterminate solution, need to add this διορισμός.) Thus it is necessary that the difference between the sum of the numbers and their difference be even.

Let the sum of the two numbers be AB and their difference CD. Then it is required to find two numbers whose sum is AB and and whose difference is CD.



1. Analysis

Let them have been found and let the smaller of them be GH. And let AK be made to contain as many units as there are in GH, and EF to contain as many as there are in GH and CD. Let BL be made to contain as many units as there are in CD. Then the greater of the numbers, EF, is equal to AK and LB. Since, then, the two numbers together make up AB, AK together with AK and LB are equal to AB, that is

$2AK + LB = AB$ Let LB be subtracted from each.

Then $2AK = AL.$

Hence $AK = \frac{1}{2}AL.$

And since $AL = AK + KL,$

Then $AK = KL.$

And since $GH = AK,$

$GH = \frac{1}{2}AL$

And $EF = KB$

2. Indeterminate Solution

If, then, the sum and difference of two numbers be given, the difference of their sum and difference is given. And hence half that difference is given, and therefore the smaller of the numbers is given. But also the difference between the numbers. Hence the greater is also given.

3. Synthesis a: Finding the numbers

Then they will be found thus. From the given sum AB subtract the given difference CD by making LB contain as many units as there are in CD. Let AL, the difference, be divided into two equal parts AK, KL, and as many units as there are in AK, so many let there be in GH. And as many as there are in KL and CD together, that is KB, so many let there be in EF. I say that the smaller of the numbers is GH and that the greater is EF, that is, that $EF + GH = AB$ and that $EF - GH = CD$.

b: Proof

For since	$AK + KB = AB$	
And	$AK = GH$,	
While	$KB = EF$,	
Therefore	$GH + EF = AB$	
Furthermore, since	$EF = LK + CD$,	if KL be subtracted from each,
Then	$EF - KL = CD$	
But	$KL = KA$	
And	$KA = GH$	
Therefore	$EF - GH = CD$	

Q. E. F.

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Even though Euclid nowhere in Bks. VII-IX specifically handles problems such as the above, it would appear that his method of proof of the number theorems would lend itself to their solution in the manner indicated. And hence his solutions would differ from Diophantus's solutions to the same sort of problems. For although the Diophantine enunciation of the problem is as general as the Euclidean, the Diophantine method is to let the sum and difference of the numbers be represented by marks which signify particular groups of monads, e.g., " $\bar{\rho}$ " or " $\bar{\mu}$ ", whereas the Euclidean would be to let the sum and difference of the numbers be represented by the marks "A__________B" and "C______D".

Hence while Euclid, like Diophantus, speaks in the enunciations of theorems and problems about any numbers, plane, prime, etc., and not, like Viete, about "sides", yet in the proofs themselves of the "number" books, he speaks about "letting be" the numbers "AB", "CD", etc. And if we ask what these things "AB", "CD", are, we learn that they are not definite groups of monads, but letters through which the reader may identify the black marks to the right of the printed text. And if we ask about those black marks, we learn that they are neither lines, for they have breadth, nor numbers, for they have position and continuity; that they are neither representations of lines, since nothing in the proofs demands that they be breadthless, intersecting at points, or boundaries of figures, nor representations of particular numbers, since they lack either that internal discreteness through which the difference of more and less distinguishing particular numbers from each other might be revealed, or the conventionally significant characteristics of the Diophantine signs of particular numbers. We must conclude, therefore, that they are nothing but marks which stand for indeterminate numbers, i.e., that they are symbols which, substituting for any multiplicity, do not stand for any particular one yet are treated as if they did. They could, in other words, serve their function just as well if they were smudges, since their only function seems to be to present to the reader visible things, distinct from one another, through which he can think pure patterns determining any determinate numbers in particular relations. We might say that they stand-under the indeterminate numbers that through them the reader might himself under-stand what determines the numbers. The only conditions, therefore, that these marks need to fulfill if they are to be images of indeterminate numbers is 1) visibility, 2) identity, and 3) distinctness.

It is true, that <u>as extended things</u>, they are also divisible and ordered. A smudge can be divided and any one is greater than, less than, or equal to any other one, and to that extent Euclid's indeterminate number images have some of the same properties as the imaged. But having these properties is accidental to these images <u>qua imaging</u>, although these properties are not accidental to the indeterminate numbers for which they stand. Hence, since "AB" is also visible, identical with itself, and distinct from "CD", there is no reason why in the proofs of Bks. VII-IX, "AB" has to identify "A____B" rather than itself simply stand for the indeterminate number. And if "AB" and "CD" can serve as well as "A____B" and "C___D", then so can "a" and "b". However, if we use "a" and "b" to stand for indeterminate numbers, when we want to image the indeterminate number for which "b" stands subtracted from the indeterminate number for which "a" stands, instead of marking "C____D" off on "A____,E_______________B" at, for example, "E", since our new marks are no longer extended and divisible in quite the same way that the old ones are, we need to adopt another convention and signify the result of subtraction by the mark "a-b" rather than by letting such a result be implied by placing "E" at approximately the same distance from "A" that "C" is from "D". But the necessity for this new convention only arises from an accidental aspect of the old marks, namely their divisibility, and not from any claim which the old marks might have had to being more truly images or less symbolic than the new.

Similarly, if we want to make an image of three indeterminate numbers two of which are separately greater than a third and two of which are separately less than a third, instead of writing

"AB" "A________________B"
"CD" "C__________D"
"EF" "E_____F"

we can write "$a > b > c$". The only important difference between the two sets of symbols is that the fact of one indeterminate number's being greater than another must in the second case be signified through the sign " $>$ ". This sign itself needs definition, before it is used, and its presence is required because "a" is not distinct from "b" except by position, shape, and alphabetical priority, none of which reveals relations of size among the indeterminate numbers which "a" and "b" symbolize. In the other case, the fact of one indeterminate number's being greater than another need not be explicitly signified at all, since smudge "A______B", distinguishable from smudge "C______D" not only by position and perhaps shape, but by size as well, can be used in juxtaposition with smudge "C_____D" to reveal relations of size among the symbolized numbers immediately, i.e., without an intermediate sign such as " $>$ ".

Hence the use of symbols to stand for multiplicity in general which is not any particular multiplicity (and yet) is treated as if it were a particular multiplicity would not seem to have been meaningless to at least one Greek mathematician.

This discussion of course leaves open the question about the marks in Bk. V, their symbolic status, their relation to the marks in Bk. VII, their relation to magnitude-in-general, and their relation to the "mathesis universalis".

Edward G. Sparrow, Jr.

PREFACE

It is said that tragedies no longer exist, that heroes are shining figures of the past, and that nobility has been forgotten, How wrong you are, you who wait for tall heroes in shining armor and cannot see their bare and wasted shadows on a dark street.

You were not there when the tragedy circled above their joyful and unconscious heads, preparing itself, waiting patiently for the first winter. You did not see orestes and elektra play in the garden, or antigone wander alone by the lake; you did not see oedipus look at sky and smile, forgetting. Their faces were soft then and you did not see the darkness form within their eyes.

Nor were you there when they suddenly looked at each other in fear, apprehension and a coming despair. You did not hear them run at night, screaming, trying to escape what was already within them, you did not see them fall exhausted to the ground, nor did you listen for their silent tears. You were waiting for tall heroes and they, alone, cold, watched the tragedy begin, unfold itself, and looking back for the last time at laughter and their purety, let themselves be carried into their inevitable future.

In a second, an eternity, a sunless day, the crimes were committed, revealed and buried. But then did the past begin, and oedipus, invisible and alone, became the eternal actor and spectator of his own tragedy. Then did orestes return to elektra to see mirrored in her eyes the horror of his crime; and antigone began to witness the endless defile of undefeated creons. Only then was the tragedy being played, silent and unseen, swaying from sun mountains to dark tombs.

And in the now deserted street, an old man searches for the children he created, afraid of their existence, while the children search for one another, doubtful of their past. The city alone remembers them and the story they had not written,but were forced to live.

Have you not seen their eyes of blood, the hands they often hide? And when they walk at night, fighting with their faces the cold wind, fighting with their bodies the cold wind, struggling against this wind they cannot see, do you never look into those faces, white faces they have distorted into ugliness?

You dream of glorious heroes and they move around you, concealed crimes and hidden sorrow. They sit next to you and watch you dream of heroes, but you, blinded, cannot see their eyes become dark and a soft and bitter smile penetrate their faces. For you dream of glorious heroes, and they remember joyful children, you dream of distant lands, and they remember quiet streets. And you say that tragedies no longer exist, but they know that they never end.

Michèl

Dear Mr. Krimins,

I am glad that we are in such far-reaching agreement about the Forum. But I am sorry that you seem to think I run it. I don't and never did. For a time I was unofficial Faculty adviser to it. But that time is past.

If I may give the Polity a piece of advice, it is to put the Forum on a democratic or committee basis. That may not bring the Vice-President to St. John's, but it may provide a more balanced political diet (including vitamins from nearby Washington) and lead to livelier participation than we have had heretofore.

The Forum should be the concern of all who have an interest in politics.

Beate Ruhm von Oppen