W. W. Schieffelin Claytor

PLANS FOR WORK

The research which I propose to undertake concerns the solution of the following problems:

Problem I - If $\mathcal{G}$ is a given two-dimensional manifold, what conditions characterize the point-sets $\mathcal{M}$ which are topologically imbeddable in $\mathcal{G}$?

That this problem is of wide interest, and of considerable importance, is indicated by the attention that a variety of its special cases have received at the hands of a number of investigators. Through the efforts of Ayers, Kuratowski, Whitney, and myself, it is now known precisely when a peanian continuum is homeomorphic with a subset of the surface of a sphere. Furthermore, Vaughan has characterized those point-sets obtained by omitting a finite number of open 2-cells from a 2-dimensional manifold; and more recently, MacLane has stated conditions under which a graph is imbeddable in a torus.

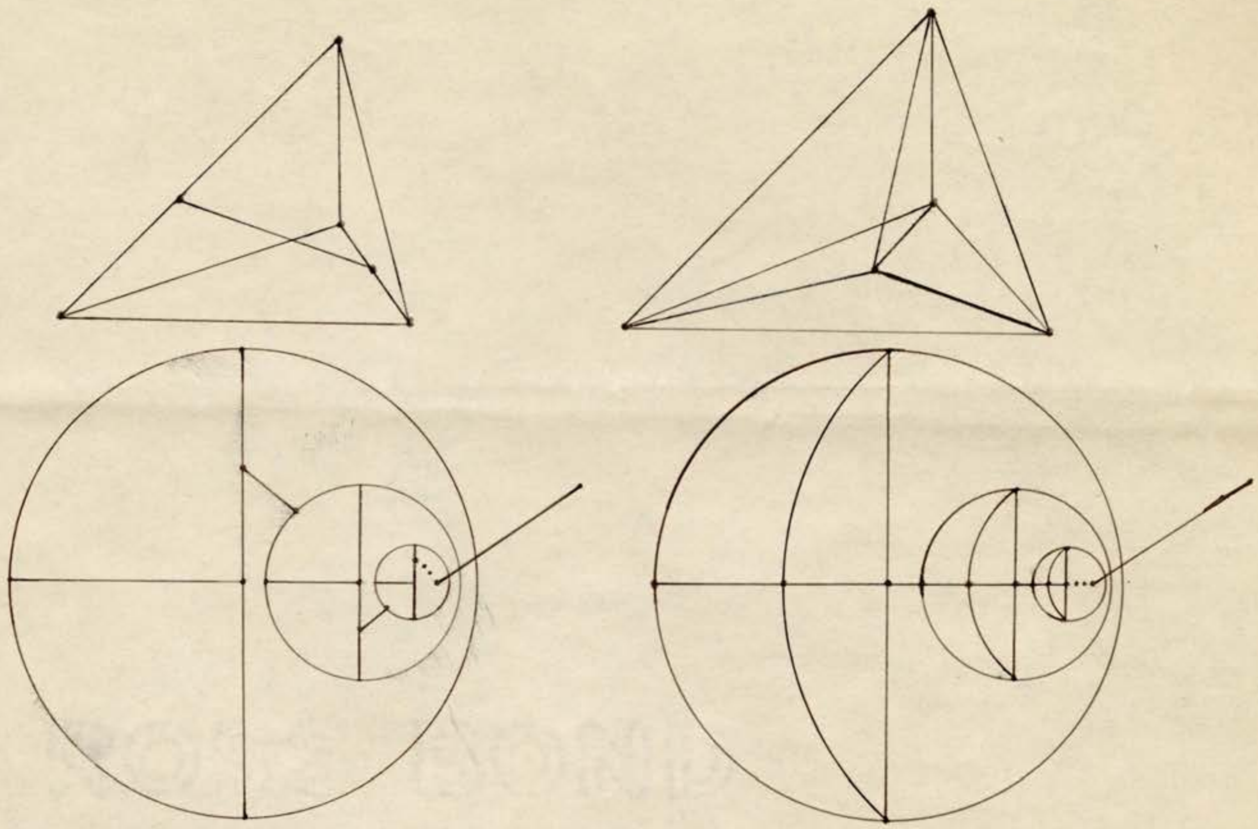
These results naturally suggest an inquiry into that situation which arises when the point-set $\mathcal{M}$ is not assumed to be <u>locally-connected,</u> and when the manifold $\mathcal{G}$ is allowed to be of arbitrary genus and orientability. It is my belief that a very careful synthesis and extension of the various novel ideas that have been contributed by the above-mentioned authors will finally yield valuable and attractive answers to these more general questions.

A successful treatment of a problem of the type under consideration usually pre-supposes that we are in possession of at least one set-theoretic characterization of the containing manifold For instance, Zippin's set-theoretic definition of the spherical surface was indispensible in the writing of my thesis.) Such basic work as this has virtually been accomplished, and whatever else of the sort that might be needed can readily be supplied.

Although closed point-sets are generally not as manageable as peanian continua, it is expected that a considerable portion of the extensive approximation theory may be utilized to advantage in the imbedding of these sets.

That I have quite a reasonable expectation of success in the present undertaking may be attributed to the fair amount of experience that I have had in this line of work. My latest result concerning the imbedding of continua in the surface of a sphere is the following:

A peanian continuum is topologically imbeddable in a spherical surface when and only when it fails to contain a homeomorphic image of each of the following figures:

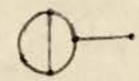


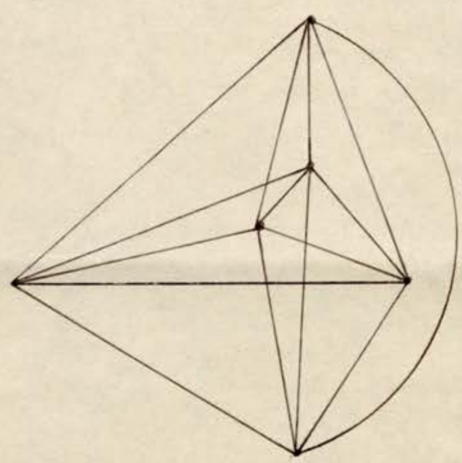
Problem II - We say that an r-dimensional complex K^r possesses the property $P(r,n)$ provided that <u>every</u> topological image of K^r in euclidean n-space has a pair of linked cycles, one of which is of dimension r .

Q.1: Do there exist complexes K^r which possess the property $P(r,n)$ for all values of $r < n$ and $\geq \left[\frac{n}{2}\right]$?

Q.2: What internal invariants characterize complexes having the property $P(r,n)$ <u>irreducibly</u>?

For $n = 1$ it is clear that a complex K^0 must merely contain at least <u>four</u> points in order to possess the property $P(0,1)$; and for

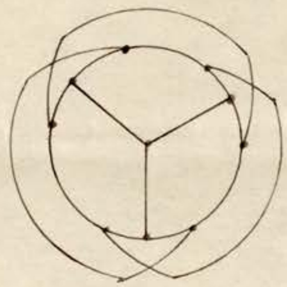
$n = 2$, it may be shown that a graph possesses the property $P(1,2)$ when and only when it contains one of the figures  and  . With reference to complexes having the property $P(n-1, n)$, it is quite easy to see that such exist for all dimensions, but no particular attention has been paid to their structure except when $n=2, r=1$. In consideration of the general question Q.1, we might say that it is not at all evident that there exist complexes having the property $P(r, n)$ for <u>all</u> admissable values of r and n . As a matter of fact, quite a proof is needed to show that there actually exists a graph having the property $P(1,3)$ (e.g., a graph which always contains a pair of <u>linked</u> simple closed curves <u>however</u> it is topologically imbedded in 3-space.) The following figure is an example of such a graph:

F: 

6 - vertices

15 - edges

I have made the investigation of the collection of graphs, which have the property $P(1,3)$ <u>irreducibly</u>, depend on a set of deformations through which any one of the graphs may be obtained from F , the only member of the collection which is <u>directly</u> shown to possess property $P(1,3)$. An instance of a graph which is thus "generated" from F is the following:



10 - vertices

15 - edges

It should now be a relatively easy matter to obtain an explicit statement of the internal invariants of the entire set of these graphs. I hope that this investigation may be carried out in a fashion that is capable of immediate extension to analogous considerations in higher dimensional spaces.

It is of interest to note that the existence of a complex having the property $P(n,2n)$ leads to the well-known corollary that there exists an n-dimensional complex that is not homeomorphic with a subset of euclidean $2n$-dimensional space.

Also, the problem of deciding when a two-dimensional complex is topologically imbeddable in 3-space leads directly to a consideration of complexes having the properties $P(1,3)$ and $P(2,3)$.

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On account of the extremely progressive mathematical atmosphere existing at the University of Moskau (where the recent

Topological Congress was held), I should very much appreciate an opportunity to continue my scientific studies there under the guidance of Professor Paul Alexandroff, the eminent authority on questions in topology.

Whatever work of importance that I might accomplish while studying in such a stimulating and scholarly environment would undoubtedly be received for early publication by some one of the many recognized journals covering mathematical subjects.

As a student of mathematics it is my ultimate aim and purpose, first, to cultivate to the highest possible degree that faculty of properly appreciating the worthy accomplishments of other scientists, and, then, to strive to the full extent of my ability to emulate their discoveries by way of my own contributions to knowledge.

Respectfully submitted,

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