

THE MUQARNAS PLATE FOUND AT TAKHT-I SULAYMAN: A NEW INTERPRETATION

In 1968 the German excavation team at Takht-i Sulayman, Iran, found a 50 cm gypsum plate amid the palace ruins. This thirteenth-century plate was spontaneously recognized to be a muqarnas design. Ulrich Harb published the details of the plate and also suggested an interpretation of its design;¹ Mohammad Yaghan has more recently published other readings.² In this paper we propose yet a different interpretation, which is more in accordance with surviving buildings of the period and the description of muqarnas by the famous mathematician al-Kashi (ca. 1430).

WHAT IS MUQARNAS?

The roots of the word “muqarnas” are unknown,³ as is the origin of the complex but ubiquitous architectural structure to which the word refers, which apparently developed around the tenth century in Iran and Iraq and almost simultaneously in North Africa. The relation between these two developments is not known. From the late eleventh century onwards, all Muslim lands adopted and developed the muqarnas, which became one of the most common features of Islamic architecture. One of the main characteristics of the muqarnas is its form as a three-dimensional unit that can be rendered in a two-dimensional outline.⁴

The muqarnas was, and still is, used in domes, niches, arches, and almost flat decorative friezes.⁵ In each instance the module as well as the depth of the composition is different and adapts to the size of the area involved or to the required purpose. In ceilings it serves a clear architectonic aim or at the very least provides the structural illusion of ascending movement culminating in a small cupola. The muqarnas is at the same time a linear system and an organization of masses. The earliest known example of a construction plan is the above-mentioned gypsum plate found at Takht-i Sulayman, showing the plane projection of one quarter of a muqarnas vault.

The earliest extant definition⁶ of the term “muqar-

nas” is given by the Timurid astronomer and mathematician Ghiyath al-Din al-Kashi, who ranks among the greatest mathematicians and astronomers in the Islamic world. Al-Kashi, a native of Kashan, Iran, went at some point in his life to Samarqand, now in Uzbekistan, where he became the founding director of the Ulugh Beg observatory, and where he died in 1429. Two years earlier he had finished one of his major works, *Key to Arithmetic* (*Miftāḥ al-ḥisāb*), which he intended for everyday use. As al-Kashi remarks: “I redacted this book and collected in it everything that is needed for him who calculates carefully, avoiding tedious length and annoying brevity.” By far the most extensive part of the work is Book IV, *On Measurements*, in the last chapter of which al-Kashi approximates the surface area of a muqarnas and gives the following definition:⁷

The muqarnas is a roofed (*musaqqaḥ*) [vault] like a staircase (*madraj*) with facets (*dil'*) and a flat roof (*sath*). Every facet intersects the adjacent one at either a right angle, or half a right angle, or their sum, or another combination of these two. The two facets can be thought of as standing on a plane parallel to the horizon. Above them is built either a flat surface not parallel to the horizon, or two surfaces, either flat or curved, that constitute their roof. Both facets together with their roof are called one cell (*bayt*). Adjacent cells, which have their bases on one and the same surface parallel to the horizon, are called one tier (*tabaqa*). The measure of the base of the largest facet is called the module (*miqyās*) of the muqarnas.

Al-Kashi distinguishes four types of muqarnas. The simple muqarnas and the clay-plastered muqarnas both have plane facets and roofs; the clay-plastered muqarnas is similar to the simple muqarnas, except that the height of its tiers might differ and a few tiers might have only a roof and no facets. The other two types are the curved muqarnas (fig. 1) and the Shirazi muqarnas. Al-Kashi explains:

The curved muqarnas is like a simple muqarnas in which the roofs of its cells are curved. Between the roofs of

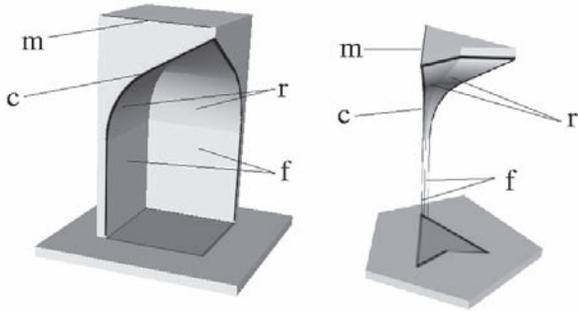


Fig. 1. Cell of a curved muqarnas standing on a square, and an intermediate element standing on a biped. (The square and the biped are horizontal plane projections, hence empty spaces.)

two adjacent cells a curved surface can be located in the form of either a triangle or two triangles.

The Shirazi muqarnas is like a curved muqarnas but has a greater variety of elements.

Hence the elements of a muqarnas consist of cells and of intermediate elements connecting the roofs of two adjacent cells. For a better understanding of the construction of a cell, we see in fig. 1:

- An element, which can be a cell (left) or an intermediate element (right).
- A cell, which consists of two facets (f) plus their roof (r).
- A facet (f) of a cell, which is its vertical side.
- A roof (r) of a cell, which is a surface not parallel to the horizon, or two joined surfaces either flat or curved.
- The curve (c) on the sides of the elements, which is the place where the elements are put together.
- The module (m), which is defined as the measure of the base of the largest facet, that being the side of the square. It is the unit of measurement of the muqarnas.
- An intermediate element, which is a surface, or two joint surfaces, connecting the roofs of two adjacent cells.

In his treatise al-Kashi shows the plane projection (fig. 2) of common elements consisting of simple geometric forms. These are, from left to right, a rhombus and a square and, underneath them, a barley kernel, a biped, and an almond—the almond and biped being complementary figures that together form a rhombus. Other elements like the half square (cut along

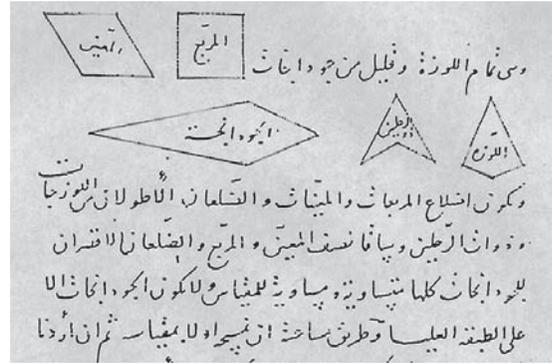


Fig. 2. Plane projections of the cells, as illustrated by al-Kashi in his *Key to Arithmetic*, Malek Library, Tehran, ms. 3180/1.

the diagonal), the half rhombus (an isosceles triangle with the shorter diagonal of the rhombus as its base), the rectangle, and the jug with its complement to a square, the large biped, are described by al-Kashi but not illustrated by figures.

As al-Kashi explains, the plane (horizontal) projection of an element, or the view from underneath, consists of simple geometric forms:

- A square, with sides equal to the module.
- A rhombus, which is a parallelogram with all sides equal to the module and with the acute angles equal to 45° .
- A half rhombus, which is a rhombus cut along the small diagonal.
- An almond (deltoid), which is a quadrilateral with two opposite right angles, an acute angle of 45° , and the two sides adjoining at the acute angle equal to the module.
- A small biped, which is the complement of an almond to a rhombus.
- A jug, which is a quarter octagon with the radius of the circumscribed circle equal to the module.
- A large biped, which is the complement of a jug to a square.
- A barley kernel (fig. 2, second row left), which is a quadrilateral with two opposite equal obtuse angles, and the two shorter sides equal to the module. Barley-kernels do not occur except on the upper tier where they can be used to fill the last and upper part of the vault.

A tier is a row of cells with their bases on the same surface, parallel to the horizon. In fig. 3 we see a part

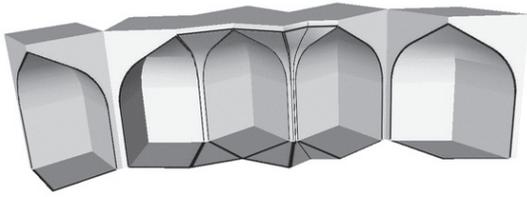


Fig. 3. A row of elements, or part of one tier. (As in fig. 1, the horizontal plane projections are drawn, but are in reality empty spaces.)

of a tier consisting of (from left to right): a cell on a square, a second cell on a square, an intermediate element on a half rhombus, a cell on a square, an intermediate element on a half rhombus, an intermediate element on a biped, a cell on a square, and a cell on a rhombus.

In this paper we deal with the curved muqarnas, for which only four possible measures for the bases of the facets occur. In the Shirazi⁸ muqarnas, in contrast, the possibilities are innumerable, as al-Kashi remarks—besides the curved roofs of the cells with intermediary triangles and bipeds, one finds triangles, squares, pentagons, hexagons, star polygons, etc. that are flat as well as curved. We emphasize that, with the exception of the Shirazi muqarnas, the two-dimensional horizontal projection of a muqarnas vault consists of a small number of simple geometrical elements.

In order to fit together, the elements have to be constructed according to the same unit of measure (module) and the curve on the sides of each element has to be the same. This curve (fig. 4) is described by al-Kashi as according with the “method of the masons,” indicating that it is taken from practice. The curve is carefully constructed in a rectangle (fig. 4, inner rectangle) whose height is twice its width, this relation being in agreement with the elements found at Takht-i Sulayman. Likewise, the design of the curve on the sides of the elements there follows approximately the same curve as al-Kashi describes. He constructs the curve with precise measurements: an angle of 30° between the upper line and the oblique line intersecting the opposite vertical. Then he divides the oblique line into five equal parts and rotates two-fifths down to meet the vertical line. Al-Kashi calculates the length of the vertical line between its base and the beginning of the curve with these precise measurements. This length is called the “factor” and is used in calculating the muqarnas surface.⁹

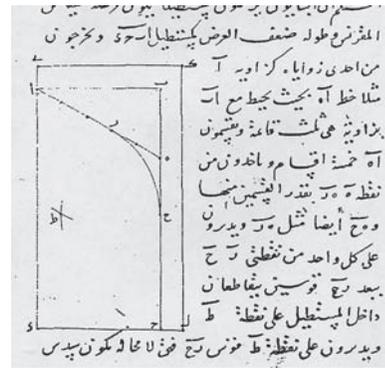


Fig. 4. Curve described by al-Kashi in his *Key to Arithmetic*, Malek Library, Tehran, ms. 3180/1.

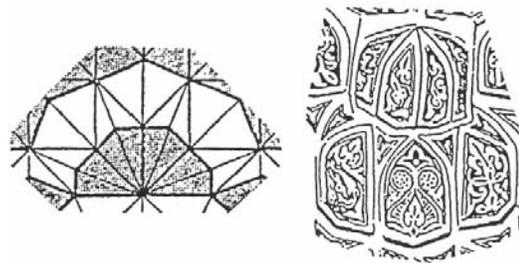


Fig. 5: Muqarnas in the Shrine of Bayazid at Bastam. (Left: after Harb, *Ilkhandische Stalaktitengewölbe*; right: after Pope, *Survey of Persian Art*, 1102, fig. 396)

Every cell consists of two walls, or facets, with a roof, consisting of two elements, above them. Al-Kashi’s observation that the two facets of a cell intersect in most cases at 90° , 45° , or 135° is again in agreement with the findings at Takht-i Sulayman.

The correlation between a muqarnas structure and its horizontal plane projection is shown in fig. 5. Here we see a part of the entrance portal of the Shrine of Bayazid at Bastam, Iran, from the front (at the right) and from underneath (left). The lower tier on the right side corresponds with the white row on the left. Similarly, the upper tier on the right side corresponds with the grayish row on the left. We see on the lower tier three cells standing on a jug, i.e., each of their plane projections is a jug. The two intermediate elements between these three cells, as well as the two intermediate elements connecting these cells and their neighbors, are all small bipeds. In the plane projection the side of the biped fits the longer side of the jug, which equals the module. Next to these elements a biped can partly be discerned—in this case a large biped, the complement to a jug. On the tier

above, four cells are constructed standing on almonds, which means that their horizontal plane projections are almonds. On the right and left of the outer cells the sides are visible. As they are vertical, their horizontal plane projections are straight lines.

In the section above we have explained the correlation between the two-dimensional muqarnas design, or ground plan, and the three-dimensional muqarnas structure. Every muqarnas can be projected on the horizontal plane in only one way. Is the reverse also true?—is there only one way to interpret this ground plan? To answer this question we have to take into account the following:

In the first place, if the height of the muqarnas elements remains the same throughout a whole structure, the structure will be steeper, like the Seljuk muqarnas in Anatolia. Hence, when a muqarnas structure has to be inserted into an existing vault, we have to adapt the height of the facets of the elements. In other words, when we want to construct a muqarnas into a not-very-pointed vault, the height of the facets of the elements has to decrease on the higher tiers in such a way that they will fit into the vault. When the height of the facets approaches zero, the remaining part of the vault, i.e., the part above the last tier, can then be finished in several ways. In some vaults the original brickwork is left visible; in others the ceiling is plastered and ornamented by painting, or by applying barley-kernels (fig. 2, second row left), or by using a combination of these two.

Second, some designs are sketchy and not worked out in much detail. Gülru Necipoğlu describes a late-fifteenth- or early-sixteenth-century scroll now preserved at the Topkapı Museum, Istanbul.¹⁰ This scroll, a pattern book from the workshop of a master builder, was probably compiled somewhere in western or central Iran, possibly in Tabriz. In the scroll we find patterns for ornamentation and patterns to be used as designs for muqarnas; it is a high-level design book for architects, builders, and artisans,¹¹ the best-preserved example of its kind, with far-reaching implications for the theory and praxis of geometric design in Islamic architecture and ornament. The Topkapı scroll includes several rough designs of which the artist has shown a small part worked out in detail, probably to avoid confusion.¹² While the scroll ranks among the oldest extant designs for muqarnas, even in modern Moroccan muqarnas designs, still to be seen in Fez, the artist tends to help the artisans by designating the required elements.¹³

Based upon the evidence cited above, we think that artists and artisans, even while using standard designs from a pattern book, probably had some freedom, however small, during the construction process. Such freedom is necessary when difficulties arise due to irregularities in the building. The artisans repeated endless variations based on old geometric formulas, slightly modifying them by trial and error. For these artists, muqarnas was, and remains, part of their daily life and culture. For those of us who are outsiders, muqarnas is beautiful geometric art to be admired and studied. We can try to understand its composition and discover its intriguing details, but muqarnas forms no part of our cultural identity.

THE PLATE FOUND AT TAKHT-I SULAYMAN

In this section we describe the earliest known example of a muqarnas design, the 50 cm stucco plate found at Takht-i Sulayman, Iran. The Ilkhanid seasonal palace of Takht-i Sulayman was a walled enclosure of oval shape, about a third of a mile long and a quarter mile wide, which contained a small lake. It was built over a Sasanian fire temple that seems to have been in use from the late fifth to the early seventh century and continued to serve as a Zoroastrian sanctuary for two more centuries after Persia had been conquered by Islam.¹⁴ The archeological site is in the Azerbaijan region of Iran, two hundred miles south of Tabriz and southeast of Lake Urmia. The Ilkhanid structures at Takht-i Sulayman, especially the north iwan palace hall complex and the west iwan complex, were lavishly decorated with tiles and stucco. The most striking use of stucco was for muqarnas. Construction of the palace must date back at least to the reign of the Ilkhanid ruler Abaqa (1265–82), because luster-painted tiles excavated at the site carry dates of 670, 671, and 674 (1271–73 and 1275–76). The palace included several polygonal structures, the north and south octagons in the west iwan complex and the dodecagon in the western arcade.

The German Archeological Institute excavated the palace remains between 1959 and 1978. When investigating the remains of a farm on top of the second fire temple, excavators found a gypsum plate in a hollow of a pantry floor. The plate had a geometric pattern incised on it, which was spontaneously recognized by the leader of that excavation, Dietrich Huff, as being a muqarnas design.¹⁵ The plate is now kept in the Islamic Department of the Iran Bastan Museum in

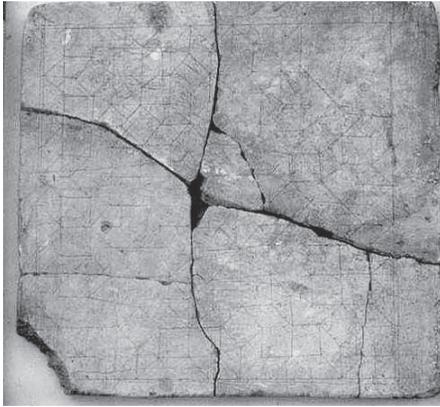


Fig. 6. Plan of a muqarnas quarter vault found at Takht-i Sulayman, Iran. (Photo: courtesy of Ulrich Harb)

Tehran. It was studied by Ulrich Harb, and his results are the point of departure for our research.

The plate (fig. 6), 47 cm high, 50 cm wide, and 3.5 to 4 cm thick, contains a quadratic field 42 cm in length, covered with a geometric grid. A small part of the bottom left corner is broken off and does not survive. The remaining plate is broken into seven parts, which fit together except near the middle, where there is a small hole. The design consists mainly of squares and rhombi, with isosceles right triangles along the frame of the field. The sides of the squares and rhombi, as well as the legs of the triangles, are all 3.5 cm in length. Their areas have been symmetrically arranged around a diagonal axis. The construction is completed in the upper right corner by an irregular quarter octagon. The angles of the various figures are all multiples of 45° , with the exception of some semi-regular quadrangles and isosceles triangles along the diagonal. Under the clearly drawn grid are visible poorly erased lines that at some points may have been auxiliary lines but at others seem to have no direct connection with the actual design. The artisan seems to have reworked his design, and it is not clear whether the present design was ever used, or whether it was altogether abandoned.

At the excavation site many prefabricated muqarnas elements were found buried under the ruins of the palace. These could have been used to construct several different vaults within the palace, but where such vaults might have been located can no longer be determined. As it remains an open question whether the design on the plate was ever executed, it is not clear whether the prefabricated elements correspond to the plate.

The following may be noted concerning the excavated muqarnas elements, described in detail by Harb. They have been found in three different sizes, namely, with measure units of 21, 26, and 42 cm. Only one element of 42 cm (twice 21) has been found, and its height is undetermined, as it is an intermediate element. The height of the cells with a unit of measurement of 21 is 42, and the height of the cells with a unit of measurement of 26 is 52: hence the height of the cells is twice their unit of measurement. The elements with a unit of measurement of 21 might originate from the south octagon.

THE MUQARNAS DESIGN

The stucco plate found at Takht-i Sulayman, showing the projection of a quarter of a muqarnas-vault ground plan, is the earliest known example of a muqarnas design. Despite occasional textual references to plans, there are no known working drawings for Islamic architecture from the pre-Mongol era. Rag paper was introduced to Samarqand by Chinese prisoners of war in 751, and because it was much cheaper than papyrus and parchment, its use spread throughout the Islamic world after the tenth century. Not until the Mongols arrived in the 1220s, however, did an extensive paper industry develop in Tabriz and other Iranian towns, where the use of paper became essential due to the increasing elaboration of geometric design.

Fourteenth-century sources frequently mention architectural drawings produced either on clay tablets or on paper. Until Necipoğlu's discovery of the Topkapı scroll, the earliest known examples of such architectural drawings were a collection of fragmentary post-Timurid design scrolls on sixteenth-century Samarqand paper housed at the Uzbek Academy of Sciences in Tashkent. These scrolls almost certainly reflect the sophisticated Timurid drafting methods of the fifteenth century. The Timurid and post-Timurid scrolls show a decisive switch to the far more complex radial muqarnas with an increasing variety of polygons and star polygons.

A continuous tradition from the thirteenth-century Takht-i Sulayman plate to muqarnas designs still in use in the present-day Islamic world is evident: a few years ago we visited a workshop in Fez, Morocco, where the artisans used a construction plan for a muqarnas on a 1:1 scale. The pieces cut out for constructing the muqarnas could actually be put on the plan such that the cross-section of the element, in

this case of a wooden beam, exactly matched the figure on the plan.¹⁶ As in the Ilkhanid period 700 years earlier, the plane projection of the elements in the Moroccan plan consists of simple geometric figures: squares, half squares, rhombi, half rhombi, rectangles, almonds, and bipeds. The standard patterns compiled in modern Moroccan sketchbooks indicate that the master who drew them repeated inherited formulas rather than inventing new ones.

Wilber¹⁷ relates how in Isfahan he watched an elderly workman who had been charged with repairing a badly damaged stalactite half dome of the Safavid period. On the floor below the damaged elements the workman had prepared a bed of white plaster and on this surface was engaged in incising a half-plan of the original stalactite system.

GENERAL DISCUSSION OF THE PLATE

The starting point for our reconstruction will be the design as given in fig. 7, which consists of the lines from the plate as recognized by Harb.¹⁸ We adopt Harb's suggestion that the quarter octagon at the upper right corner of the plate corresponds to the center of the muqarnas.

The design is based on squares and rhombi. It is impossible to build a muqarnas corresponding to the plate by using only elements whose plane projections are squares and rhombi, as Harb¹⁹ has already remarked. This means that some figures must be split: a square can also be interpreted as a jug combined with a large biped, and a rhombus can be interpreted as an almond with a small biped. On the diagonal are found hexagons consisting of irregular figures: each hexagon has two figures that are neither squares nor jugs, but something in between, and two isosceles triangles. Because these figures are not described by al-Kashi, we have changed this division of the hexagon to a combination of known elements—square, jug, and two half rhombi—by changing the inner lines in the hexagon (fig. 8). It is clearly easier to draw the inner lines like diagonals of a hexagon, as in the left figure. It could be that there was no need for the designer to draw very exactly, because the craftsmen knew from experience how to interpret such a hexagon.

Another justification for changing these lines is that the suggested combination of elements in the right figure also appears in other Ilkhanid structures, for example in the basement vault of the north iwan of the Friday Mosque in Natanz. We are aware that the

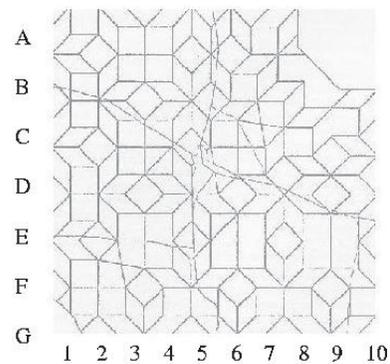


Fig. 7. Design of the plate as read by Harb.

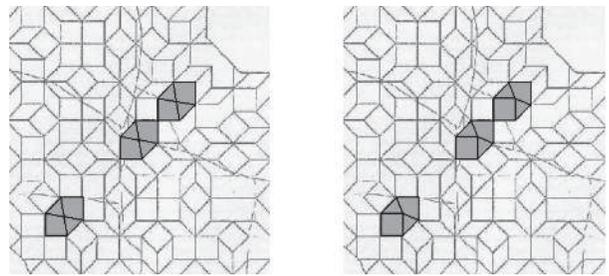


Fig. 8. Left: original design on the plate. Right: our interpretation after changing inner lines of irregular hexagons.

above two arguments are only speculative. The alternative would be to use elements not according to al-Kashi's rules: elements with curved sides different in length from the module.

NEW INTERPRETATION

When a craftsman has to build a muqarnas directly from the design, he will do this by using his experience. Some combinations of elements appear more often in plans, and it will thus be clear to him how to handle these. To imitate this experience, we compare element combinations in the plan with the same combinations in projection plans of muqarnas from the same period. In fig. 9 we show side-by-side the design of the plate as read by Harb and our interpretation, in which the different tiers are indicated by two alternating colors. As said before, the ground plan on the plate only shows a quarter of a vault. A three-dimensional computer reconstruction of the whole vault according to our interpretation is found in fig. 10.

A muqarnas is built from the outside in, and hence we have to read a muqarnas design in this direction. We read the Takht-i Sulayman plate in this way, by

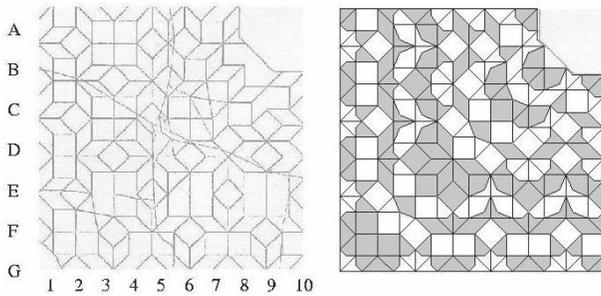


Fig. 9. Left: design of the plate, as read by Harb. Right: our interpretation.

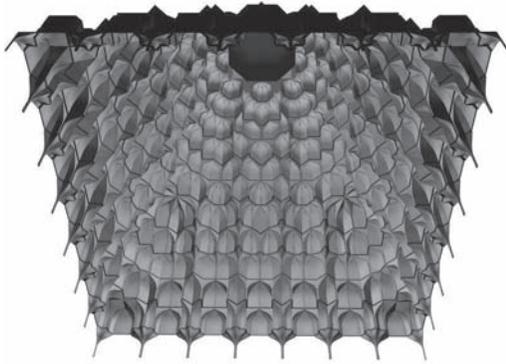


Fig. 10. Three-dimensional computer reconstruction of our interpretation (whole vault).

starting at the outer boundary. There we find six half eight-pointed stars (fig. 9, left: A1, C1, E1, G3, G6, G9), which also appear, for example, in the projection plan of the basement vault of the north iwan of the Friday Mosque in Natanz. We interpret these half eight-pointed stars in the same way as they appear in this basement vault. This means that each rhombus of the half eight-pointed stars consists of an almond and a biped.

Another noteworthy part of the plate is the star in the middle of it (fig. 9, left: D5). Similar stars are used in the famous muqarnas in the interior of the tomb of Shaykh ‘Abd al-Samad al-Isfahani of Natanz,²⁰ where they consist of intermediate rhombus elements of which the long diagonal determines the direction of the element. Such intermediate rhombus elements are not found at the excavation site of Takht-i Sulayman, but it is not certain that this design was ever built there. We feel that it is justified to use these elements in our interpretation: we have a concrete example from the same period that shows that such elements exist.

Between the first two tiers (which principally consist of almond-biped combinations) and the star in the middle of the muqarnas, we find element combinations consisting of four squares. This structure appears four times at equal distance from the boundary of the plate (fig. 9, left: A3, C3, E6, E9). The rhombus star determines the direction of the two upper squares of the square combinations at C3 and E6. If we use this direction there is no way to fit the lower two squares to these upper two. We can solve this problem by interpreting each square as a jug combined with a large biped. It seems reasonable to expect that these four square combinations are situated in the same tiers and consist of the same combination of elements.

If we use the rules described above, we can build our muqarnas from the outside to the inside, keeping in mind that each non-intermediate rhombus element in the direction of the long diagonal has to be interpreted as a combination of an almond and a small biped element. We also have to decide for each of the remaining squares whether we want to interpret it as a jug-biped combination or not.

The muqarnas of the vault over the east portal in the shrine of Bastam has the same style: the first tiers consist mainly of almond-biped combinations.²¹ In this muqarnas the squares on the lower tiers are split into jugs and large bipeds. The example of the niche in Natanz also starts with almond-biped combinations, but here the squares are not split. So both interpretations of the squares appear.

At Takht-i Sulayman intermediate elements standing on large bipeds only occur in pairs.²² However, not many of these element combinations were found. Thus we can argue that if the plate was conceived by the same artist who designed the muqarnas actually built in the palace of Takht-i Sulayman, this artist did not use many cells standing on jugs and intermediate elements standing on large bipeds in his muqarnas vaults. For this reason, we decided in our interpretation to split squares into jugs and large bipeds only when necessary, as is the case in the four square combinations discussed above.

HARB'S INTERPRETATION

The main difference between the interpretation given by Harb (fig. 11) and our interpretation (figs. 9, right, and 10) is the global geometric form of the muqarnas. In Harb's interpretation the muqarnas starts in the four corners. The first tier consists only of two

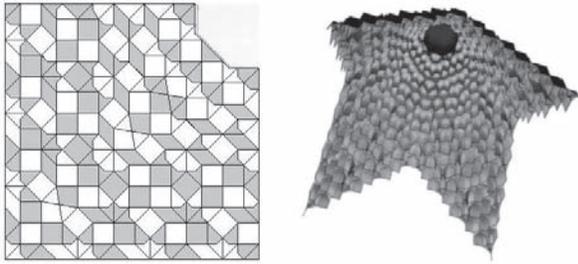


Fig. 11. Left: projection plan of Harb's interpretation (quarter vault). Right: three-dimensional computer reconstruction of this interpretation (whole vault).

elements in these corners. The number of elements in subsequent tiers increases, which means that the length of the subsequent tiers grows until the elements span the whole circumference. This happens for the first time at the eighth tier.

In our interpretation, and also, as we shall see, in that of Yaghan, the elements span the whole circumference in each tier. Comparing our interpretation with extant muqarnas from the same period, we see that in almost all muqarnas the elements of the first tier span the whole circumference. For example, the only instance described by Harb in which this is not the case is the muqarnas in the interior of the tomb of Shaykh 'Abd al-Samad al-Isfahani of Natanz. But for this muqarnas the projection plan is not a rectangle: extra space is used for the corners.²³

In his interpretation, Harb mainly uses elements found at Takht-i Sulayman. Exceptions are the elements used for the hexagons on the diagonal of the plate (fig. 8), and the element used for the upper triangle. For the hexagons he uses two cells standing on something in between a jug and a square, and two intermediate elements each standing on an isosceles triangle. In this way, he uses elements that directly correspond to the figures drawn on the plate. However, the elements he uses are not described by al-Kashi, nor are they found at Takht-i Sulayman. For the upper triangle he uses a cell standing on a triangle, with the long side of the triangle corresponding to the front of this cell.

Harb interprets every square of the plate as cells standing on a square and doesn't split any square into a jug and a large biped, as we did. He interprets certain rhombi as combinations of almonds and small bipeds—that is, those rhombi whose long

diagonal is used for direction, pointing to the front of the element.

YAGHAN'S INTERPRETATION

Mohammad Yaghan interprets the plate differently,²⁴ albeit basing his alternatives on Harb's reconstruction. Yaghan splits the plate into two parts—central and outer. The central part provides the transition from a square to an irregular octagon at the top. The outer part stretches from the boundary of the muqarnas to the edges of the central part. When the plate is split into these two parts, the star in the middle of the plan (fig. 7: D5) is situated on the dividing-line between the two different parts, and therefore the rhombi are also split. They are thus interpreted as almond-and-biped combinations, rather than as a combination of intermediate elements standing on a rhombus. Another difference between our interpretation and Yaghan's concerns the use of squares. In the alternatives he presents, Yaghan seems to avoid the use of square cells and splits almost every square into a jug and a large biped.

Yaghan gives two possible alternatives for the central part and two for the outer part of the plate. The main difference between the two alternatives for the outer part is in how the square combinations (fig. 7: A3, C3, E6, E9) are interpreted.

In the first alternative for the outer part, the square combinations (fig. 7: A3, C3, E6, E9) are all interpreted as combinations of jugs and bipeds, as can be seen in the left part of fig. 12. The hexagon E-F, 2–3 on the diagonal is interpreted as a combination of two intermediate elements standing on an irregular rhombus, and a cell standing on an irregular jug. Yaghan uses elements that are not found as plane projections in his design. In his interpretation, not all the inner lines of the hexagon are used as sides of elements; some of them instead are interpreted as diagonals, even though all other lines drawn in the design correspond to sides of elements.

As the second alternative (fig. 12, right), Yaghan tries interpreting the square combinations as niches. For this reason he splits the squares into jugs and bipeds, such that the jugs join each other. The four cells standing on jugs then form the niche. In a three-dimensional reconstruction it is not possible to connect the next tier, which contains the intermediate elements standing on large bipeds, to these jug cells. Yaghan solves this problem in his three-dimensional

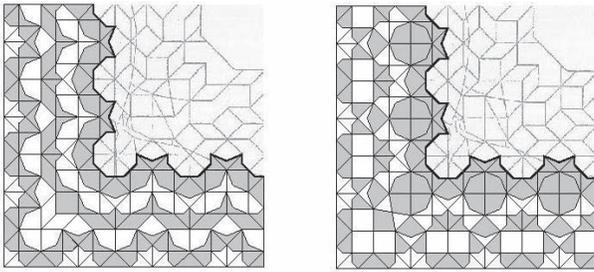


Fig. 12: Yaghan's alternative interpretations for the outer part of the plate.

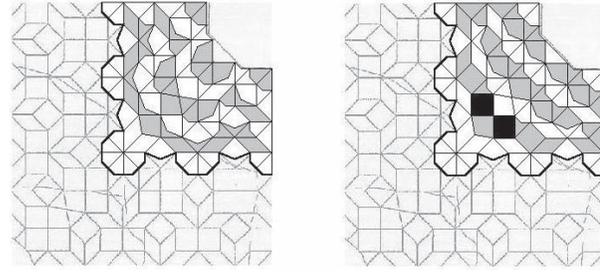


Fig. 13: Yaghan's alternative interpretations for the central part of the plate.

reconstruction by interpreting the large bipeds as cells instead of intermediate elements. By doing this he creates a situation that conflicts with the historical context: he needs to connect the backs of his newly created muqarnas elements to the jug cells on the same tier—a situation not found in existing buildings. Moreover, examples of cells standing on large bipeds are not known.

In both of his proposed alternatives for the central part of the plate, as in his first alternative for the outer part, Yaghan uses lines in the design as diagonals, rather than sides, of elements. He interprets the irregular hexagon (fig. 7: D6) adjacent to the central star as two intermediate elements standing on an irregular rhombus and a cell standing on an irregular jug in the manner described before. This makes it possible to fit the split rhombus in the center to the central part. The main difference between his two alternatives for the central part is the way he interprets the two squares above the star (fig. 13, right: black). In his first alternative (fig. 13, left) the squares are split, which makes it possible to spread them over three tiers. This influences the form of the next tiers. The first interpretation ends in the same way as does ours. The difference between the two interpretations—Yaghan's and ours—depends on the choice of squares to be split.

In the second alternative for the central part (fig. 13, right), Yaghan does not split the squares (fig. 13, right: black) and spreads them over only two tiers. This makes it possible to end this part in the same way as did Harb in his interpretation. Here the difference between Yaghan's interpretation and Harb's is again the choice of which squares have to be split.

CONCLUSIONS

The design incised on the plate found at Takht-i Sulayman is an important document for our understanding of the ubiquitous architectural ornament, muqarnas. Although the plate was first described and interpreted by Ulrich Harb,²⁵ the global geometric form of the muqarnas corresponding with Harb's interpretation is not in accordance with the historical context: the corners run out in a point while the design is a square. In his subsequent reading,²⁶ Mohammad Yaghan interprets some of the lines in the design as diagonals of elements instead of their sides, which is against the usual rules. Furthermore, for one of his interpretations he uses elements that do not appear in actual muqarnas of the same period.

In this paper we have presented a new interpretation of this muqarnas design by reconsidering Harb's understanding of it. Our interpretation fits well in the historical context since, first, we only use combinations of elements that appear in muqarnas of surviving buildings of the same period and, second, it is consistent with the description of al-Kashi.

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NOTES

1. Ulrich Harb, *Ilkhandische Stalaktitengewölbe: Beiträge zu Entwurf und Bautechnik*, Archäologische Mitteilungen aus Iran, vol. 4 (Berlin: Dietrich Reimer, 1978).
2. Mohammad-Ali Jalal Yaghan, "Decoding the Two-Dimensional Pattern Found at Takht-i Sulayman into Three-Dimensional Muqarnas Forms," *Iran* 38 (2000): 77–95.
3. On the etymology of the word "muqarnas," see Wolfhart Heinrichs, "The Etymology of Muqarnas: Some Observations," in Asma Afsaruddin and A. H. Mathias Zahniser, eds., *Human-*

- ism, Culture, and Language in the Near East: Studies in Honor of Georg Krotkoff (Winona Lake: Eisenbrauns, 1997), 175–84.
4. For a quick general introduction, see Yvonne Dold-Samplonius, “Calculating Surface Areas and Volumes in Islamic Architecture,” in Jan P. Hogendijk and Abdelhamid I. Sabra, eds., *The Enterprise of Science in Islam: New Perspectives*, Dibner Institute for the History of Science and Technology (Cambridge, MA and London: MIT Press, 2003), 235–65.
 5. For a visual representation, see Yvonne Dold-Samplonius, Silvia L. Harmsen, Susanne Krömker, and Michael J. Winckler, *Magic of Muqarnas*, a video about muqarnas in the Islamic World (University of Heidelberg, IWR, 2005).
 6. A discussion of different definitions of “muqarnas” is found in Mohammad-Ali Jalal Yaghan, *The Islamic Architectural Element “Muqarnas”* (Vienna: Phoibos Verlag, 2001), 11–18.
 7. Yvonne Dold-Samplonius, “Practical Arabic Mathematics: Measuring the Muqarnas by al-Kashi,” *Centaurus* 35 (1992–93): 193–242, see 226–27.
 8. In Timur’s time, when building activity vastly expanded, local constructors could manage the simpler buildings, but for the special and more artistic monuments architects and artisans were imported from the conquered lands, first Khwarazm, then Tabriz and Shiraz, and finally India and Syria. It is known that Timur brought in architects from Shiraz in 1388 and 1393, and that many migrated of their own free will. The names of several Shirazi architects have been transmitted, the most famous being Qawam al-Din b. Zayn al-Din al-Shirazi, the only active builder whose surviving structures display a distinctive architectural style. This might well be the reason why the type of muqarnas constructed with many variations—“innumerable possibilities” as al-Kashi explains—was called Shirazi.
 9. For a more extensive explanation, see Dold-Samplonius, “Practical Arabic Mathematics,” 221–22.
 10. Gülru Necipoğlu, *The Topkapı Scroll: Geometry and Ornament in Islamic Architecture*, *Topkapı Palace Library MS H. 1956* (Santa Monica: The Getty Center, 1995).
 11. In Necipoğlu’s publication of this scroll, an interesting computer reconstruction of a Timurid muqarnas is described by Mohammad Al-Asad; see his chapter, “The Muqarnas: A Geometric Analysis,” in Necipoğlu, *Topkapı Scroll*, 349–59.
 12. Necipoğlu, *Topkapı Scroll*, 338–42, designs 96–104.
 13. Yvonne Dold-Samplonius, “How al-Kashi Measures the Muqarnas: A Second Look,” in *Mathematische Probleme im Mittelalter: Der lateinische und arabische Sprachbereich*, ed. M. Folkerts, Wolfenbütteler Mittelalter-Studien, vol. 10 (1996), 56–90.
 14. Linda Kamaroff and Stefano Carboni, eds., *The Legacy of Genghis Khan: Courtly Art and Culture in Western Asia, 1256–1353* (New York: Metropolitan Museum of Art and New Haven and London: Yale University Press, 2002), 84–103.
 15. Harb, *Ilkhandische Stalaktitengewölbe*, 9.
 16. Such a plan, used to construct a muqarnas in present-day Fez, is illustrated in Dold-Samplonius, “How al-Kashi Measures the Muqarnas,” 71.
 17. Donald N. Wilber, *The Architecture of Islamic Iran: The Ilkhanid Period* (New York: Greenwood Press, 1955), 73.
 18. Harb, *Ilkhandische Stalaktitengewölbe*, 60.
 19. *Ibid.*, 61–62.
 20. *Ibid.*, 56.
 21. *Ibid.*, 46–47; Arthur Upham Pope, *A Survey of Persian Art from Prehistoric Times to the Present*, 6 vols. (London and New York: Oxford University Press, 1939), vol. 2, 1102.
 22. Harb, *Ilkhandische Stalaktitengewölbe*, 41–42.
 23. On the shape of the muqarnas, see Yaghan, “Decoding the Two-Dimensional Pattern,” 84.
 24. *Ibid.*
 25. Harb, *Ilkhandische Stalaktitengewölbe*, 41–42.
 26. Yaghan, “Decoding the Two-Dimensional Pattern,” 84.