Letters to the Editor.

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The Motion of the Spinning Electron.

IN a letter published in NATURE of February 20, p. 264, Messrs. Uhlenbeck and Goudsmit have shown how great difficulties which atomic theory had met in the attempt to explain spectral structure and Zeeman effects, can be avoided by using the idea of the spinning electron. Although their theory is in complete qualitative agreement with observation, it involved an apparent quantitative discrepancy. The value of the precession of the spin axis in an external magnetic field required to account for Zeeman effects seemed to lead to doublet separations twice those which are observed. This discrepancy, however, disappears when the kinematical problem concerned is examined more closely from the point of view of the theory of relativity.

As usual, letters in heavy type will denote vectors. The anomalous Zeeman effect seems to require that the spin axis of the electron precesses about an external magnetic field **H** with angular velocity

$$\frac{e}{mc}\mathbf{H}, \quad . \quad . \quad . \quad . \quad . \quad (A)$$

where c is the velocity of light and $-e$, m are the electronic charge and mass. Suppose such a spinning electron moves with velocity **v** through electric field **E**. At first sight it would seem that, being subject to magnetic field

$$\mathbf{H} = \frac{1}{c}[\mathbf{E} \times \mathbf{v}],$$

the spin axis will precess about the instantaneous normal to the orbital plane with angular velocity

$$\frac{e}{mc^2}[\mathbf{E} \times \mathbf{v}]. \quad . \quad . \quad . \quad . \quad . \quad (B)$$

As the mean value of this expression is just twice the angular velocity with which the perihelion of the orbit rotates on account of the variation of mass of the electron, this would lead to twice the observed doublet separation.

There is, however, an error in the above reasoning; the precession of the spin axis so calculated is its precession in a system of co-ordinates (2) in which the centre of the electron is momentarily at rest. System (2) is obtained from system (1), in which the electron is moving and the nucleus at rest, by a Lorentz transformation with velocity **v**. If the acceleration of the electron is **f**, and system (3) is obtained from system (1) by a Lorentz transformation with velocity $\mathbf{v} + \mathbf{f}dt$, then the precession which an observer at rest with respect to the nucleus would observe, and which should be summed to give the secular precession, is that precession which would turn the direction of the spin axis at time t in (2) into its direction at time $t + dt$ in (3) if both directions were regarded as directions in (1). To a first approximation system (3) is obtained from system (2) by a Lorentz transformation with velocity $\mathbf{f}dt$ together with a rotation $(1/2c^2)[\mathbf{v} \times \mathbf{f}]dt$. Thus the observed rate of precession will be, to a first approximation,

$$\frac{e}{mc^2}[\mathbf{E} \times \mathbf{v}] - \frac{1}{2c^2}[\mathbf{v} \times \mathbf{f}].$$

To a first approximation

$$\mathbf{f} = -\frac{e}{m}\mathbf{E},$$

so the rate of precession is

$$\frac{e}{2mc^2}[\mathbf{E} \times \mathbf{v}], \quad . \quad . \quad . \quad . \quad . \quad (C)$$

just half the expression (B).

The interpretation of the fine structure of the hydrogen lines proposed by Messrs. Uhlenbeck and Goudsmit now no longer involves any discrepancy. In fact, as Dr. Pauli and Dr. Heisenberg have kindly communicated in letters to Prof. Bohr, it seems possible to treat the doublet separation as well as the anomalous Zeeman effect rigorously on the basis of the new quantum mechanics. The result seems to be full agreement with experiment when the calculation is based on formulæ (A) and (C).

I hope in a later paper to develop the above kinematical argument in greater detail.

In conclusion, I wish to express my appreciation of the encouragement and help of Prof. Bohr and Dr. Kramers.

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