Sensitivity Analysis for Climate Science with Generative Flow Models

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1. Introduction

- **Sensitivity analysis** is vital in climate science but computationally prohibitive with traditional physical models.
- We apply the **adjoint state method**, a technique from Neural ODEs [1], to efficiently compute these gradients in generative flow models.
- We demonstrate this method on **cBottle** [2], a generative atmospheric model, reducing the computational cost of sensitivity analysis from weeks on a supercomputer to hours on a GPU.

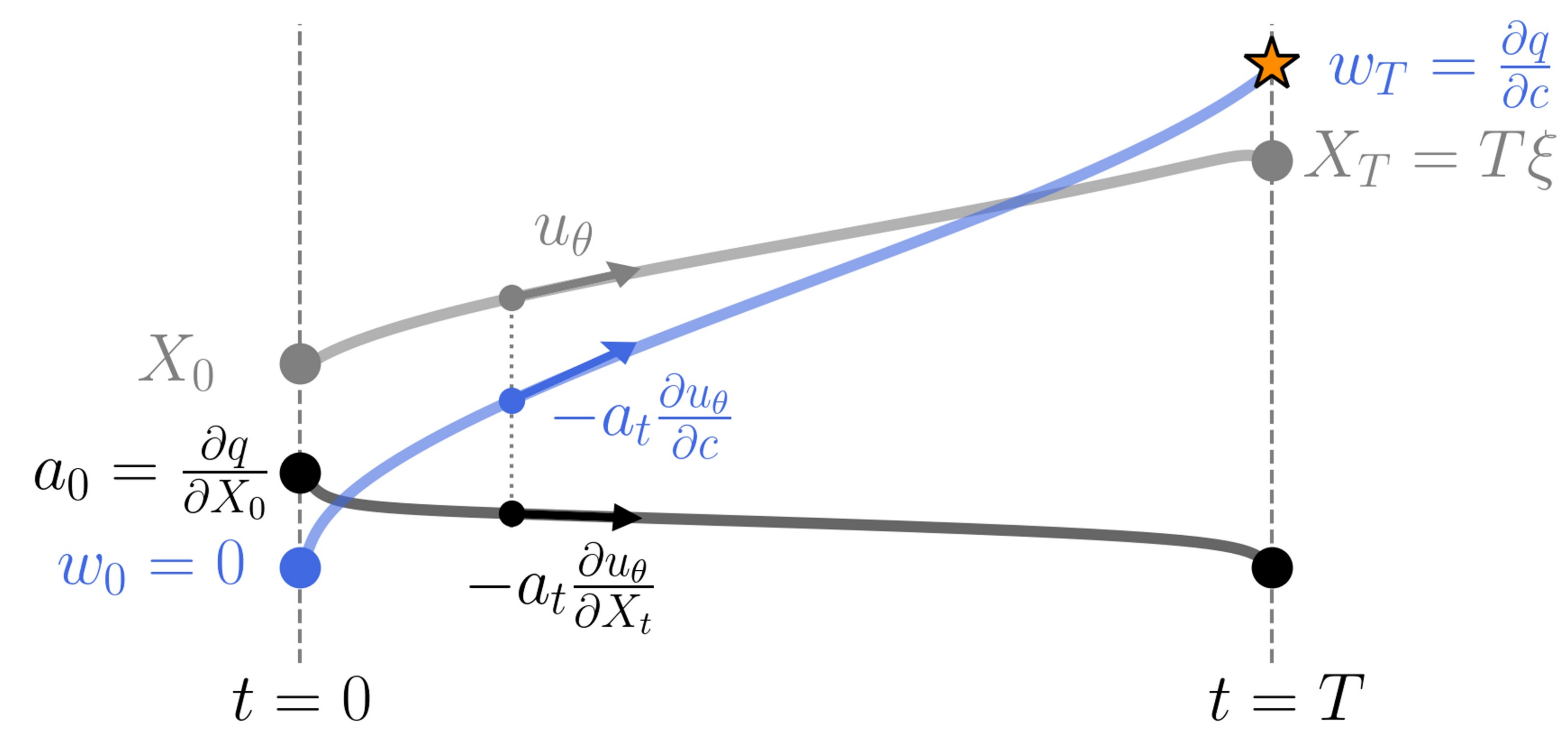

Figure 1. An illustration of the adjoint method. The gradient accumulator w_t aggregates the derivative at all noise levels.

2. The adjoint sensitivity method

- Velocity network $u_\theta(X_t, t, c)$, clean sample $X_0 \sim p_{data}(\cdot\,|c)$, pure noise at T, $X_T = T\xi$, $\xi \sim \mathcal{N}(0, \mathrm{Id})$ and conditioning c. Sampling ODE:
$$\mathrm{d}X_t = u_\theta(X_t, t, c)\mathrm{d}t$$
- The sensitivity of a quantity $q(X_0)$ with respect to the conditioning to the flow model is $\frac{\partial q}{\partial c}(X_0)$.
- Define the adjoint state $a_t = \frac{\partial q}{\partial X_0}\frac{\partial X_0}{\partial X_t}$ and the gradient accumulator $w_{T-t} = a_t \frac{\partial X_t}{\partial c}$. The system of ODEs

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} X_t \\ a_t \\ w_t \end{bmatrix} = \begin{bmatrix} u_\theta \\ -a_t \frac{\partial u_\theta}{\partial X_t} \\ -a_t \frac{\partial u_\theta}{\partial c} \end{bmatrix}, \text{ with } \begin{bmatrix} X_0 \\ a_0 \\ w_0 \end{bmatrix} = \begin{bmatrix} X_0 \\ \frac{\partial q}{\partial X_0} \\ 0 \end{bmatrix},$$

when integrated from 0 to T yields $w_T = \frac{\partial q}{\partial c}(X_0)$.

3. Gradient self-consistency check

- We first check the gradients against finite differences. Let a finite difference in SST be δc. For small enough δc and smooth q, we have
$$\frac{1}{2}\left(\frac{\partial q}{\partial c}(c) + \frac{\partial q}{\partial c}(c + \delta c)\right) \cdot \delta c \approx q(c + \delta c) - q(c).$$
- We test this in cBottle on the outgoing shortwave radiation in a patch in the North Atlantic. The extracted gradients are consistent with the function.

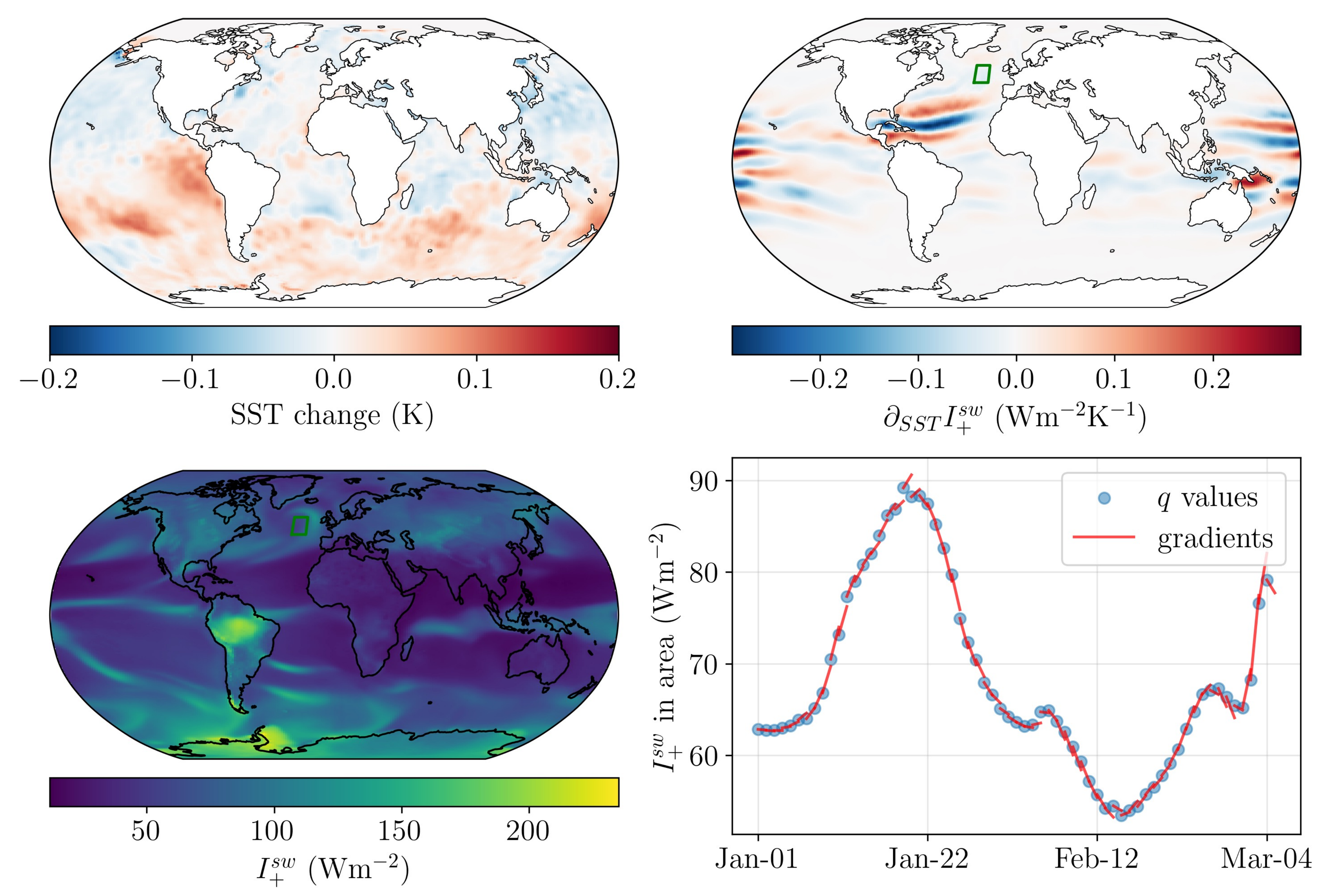

Figure 2. Clockwise from top-left: typical SST change in 2021 Jan; sensitivity map of shortwave radiation in the green patch; visual comparison between gradients extracted and the function; typical shortwave radiation map.

4. Application: the SST pattern effect

- Ocean surface temperatures (SSTs) strongly shape atmospheric behavior and radiation balance.
- Traditionally, quantifying this requires years of climate simulations, taking **weeks on a supercomputer**.
- With the **adjoint sensitivity method** applied to cBottle, we obtain the sensitivity of net global radiation balance $q = \bar{N}$ with respect to SST **in less than 10 A100-GPU-hours**.

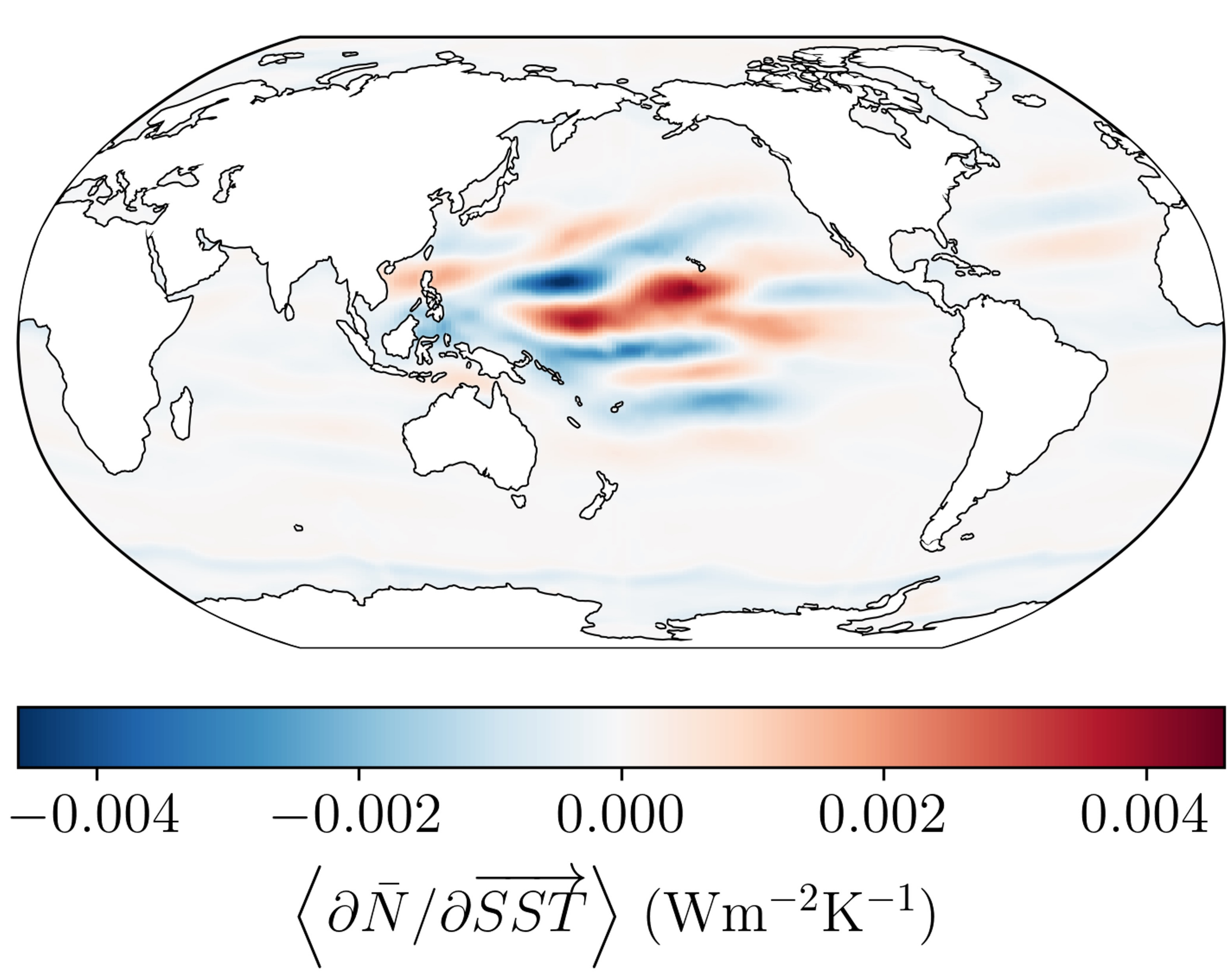

Figure 3. The sensitivity of global net radiation balance with respect to SST, averaged over historical AMIP SSTs from 1971-2020.

References

[1] Ricky T. Q. Chen, Yulia Rubanova, Jesse Bettencourt, and David Duvenaud. Neural ordinary differential equations. NIPS'18

[2] Noah D. Brenowitz, Tao Ge, Akshay Subramaniam, Peter Manshausen, Aayush Gupta, David M. Hall, Morteza Mardani, Arash Vahdat, Karthik Kashinath, and Michael S. Pritchard. Climate in a bottle: Towards a generative foundation model for the kilometer-scale global atmosphere, 2025.

[3] Jonah Bloch-Johnson, Maria AA Rugenstein, Marc J Alessi, Cristian Proistosescu, Ming Zhao, Bosong Zhang, Andrew IL Williams, Jonathan M Gregory, Jason Cole, Yue Dong, et al. The green's function model intercomparison project (gfmip) protocol. Journal of Advances in Modeling Earth Systems, 16(2):e2023MS003700, 2024.