Multiscale Neural PDE Surrogates for Prediction and Downscaling: Application to Ocean Currents

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Motivation

- High-resolution ($\leq$2 km) surface currents are critical for spill drift, coastal planning, and autonomous navigation.
- Goal: a **single neural surrogate** that
  1. solves spatio-temporal PDEs, and
  2. zero-shot downscales coarse solutions.

Problem Setup

- Low-resolution sample $\mathbf{a} \in R^{d_a}$, high-resolution sample $\mathbf{b} \in R^{d_b}$ with $d_b \gg d_a$.
- Classical super-resolution learns a fixed-size map $f : R^{d_a} \to R^{d_b}$.
- **Our formulation:** learn a map from vectors to *functions*

$$G^\star : R^{d_a \times T_{in}} \longrightarrow \mathcal{U}(D; R^c),$$

  where $\mathcal{U}(D; R^c)$ is a Banach space of c-channel fields on continuous spacio-temporal domain $D \subset R^d \times R^+$.
- A discrete high-res image is a sampling of $u = G^\star(\mathbf{a})$, hence we can evaluate u on (theoretically) *any* grid after training.

Models

DFNO (Downscaling Fourier Neural Operator): **Baseline** model combining low-res input with Fourier neural operator to produce resolution-flexible outputs.
DUNO (U-shaped Neural Operator): Variant replacing FNO with a u-shaped neural operator for increased expressiveness across PDE types.
SpecDFNO: Adds a spectral residual head to DFNO. A second neural operator estimates high-frequency residuals, improving fine detail accuracy.
Temp_DFNO and **Temp_SpecDFNO**: Temporal versions of DFNO/SpecDFNO. Extend convolutions along the time axis to capture spatio-temporal PDE dynamics.

References

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[3] P. Harder, A. Hernandez-Garcia, V. Ramesh, Q. Yang, P. Sattigeri, D. Szwarcman, C. D. Watson, D. Rolnick: *Hard-Constrained Deep Learning for Climate Downscaling*, arXiv:2208.05424 (2024)

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XST Xpert Solutions Technologiques

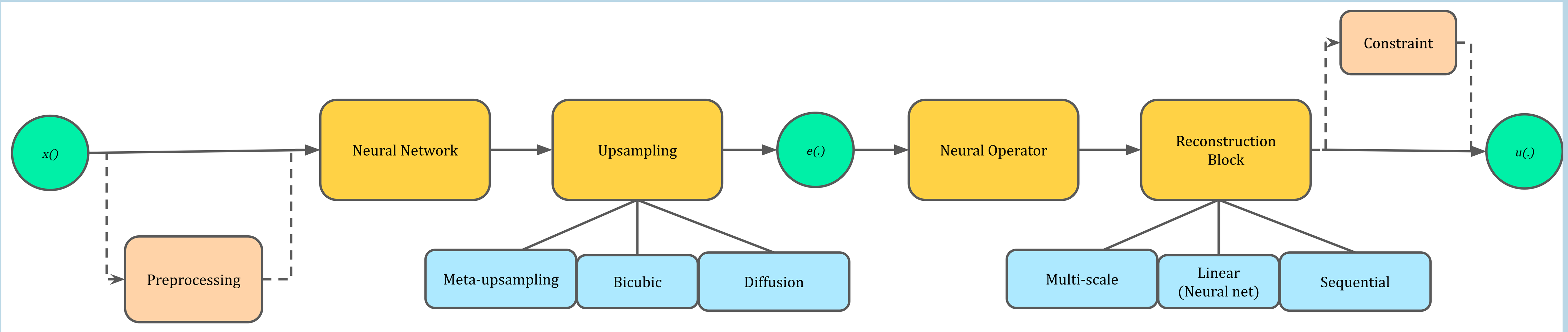
Mitacs

Framework

The approximator $G^\star_\theta$ is constructed as follows:

$$G^\star_\theta(\mathbf{a}) = P_{post}\big(F_{\theta_2}\big(T^{-1}\big(f_{\theta_1}\big(P_{pre}(\mathbf{a})\big)\big)\big)\big)$$

- $P_{pre} : R^{d_a \times T_{in}} \to R^{d_a}$ — optional normalisation, gradient extraction, channel stacking.
- $f_{\theta_1} : R^{d_a} \to R^d$ — neural network.
- $T^{-1} : R^d \to \mathcal{E}(D; R^c)$ — discretization inversion operator (mapping from a vector to a function)
- $F_{\theta_2} : \mathcal{E} \to \mathcal{U}$ — neural operator refining the field into function.
- $P_{post} : \mathcal{U} \to \mathcal{U}$ — reconstruction and soft-constraint layer enforcing physical laws.



The model is **resolution-agnostic** — when trained on 2× downscaling, it can do zero-shot 4×, 8× ... downscaling.

Experiments & Results

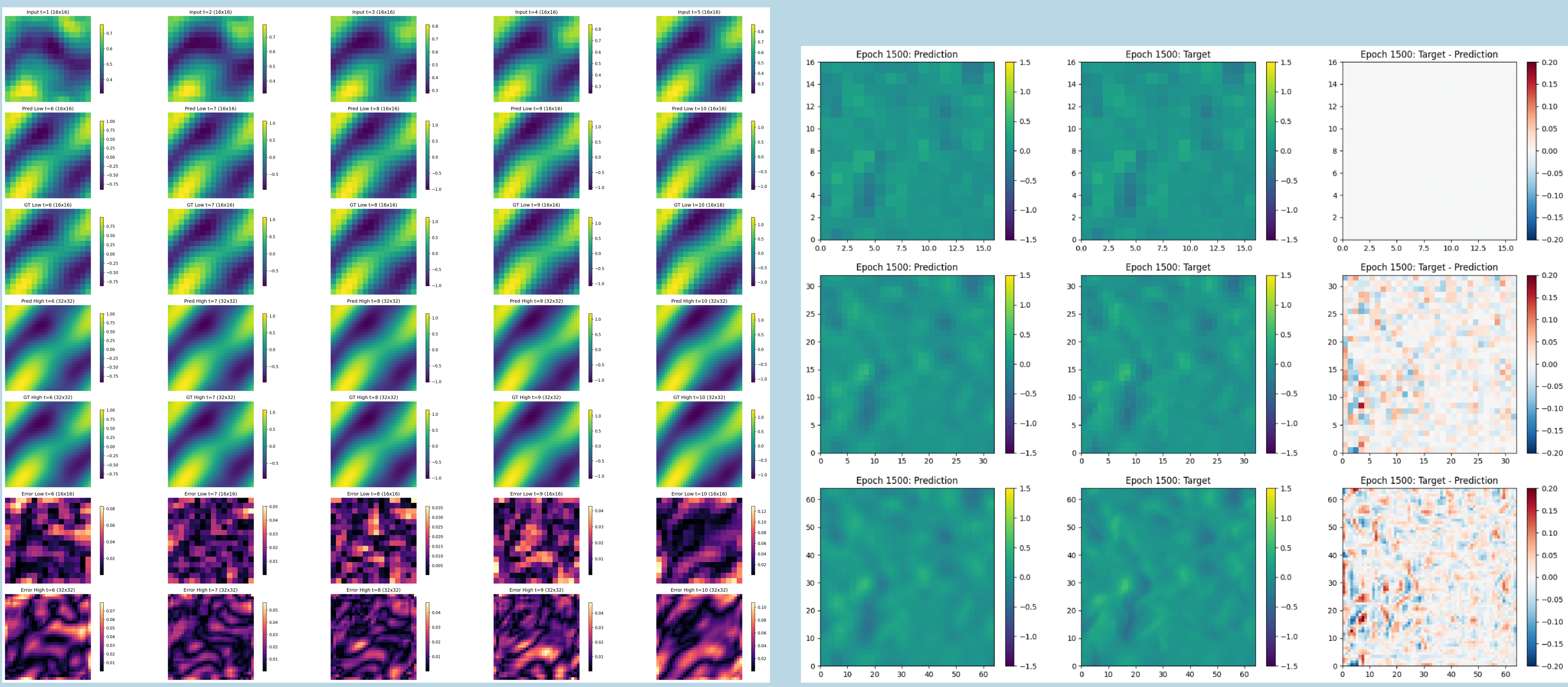
- **Surrogate PDE task** – Temporal DFNO variants predict five future Navier–Stokes frames; evaluated on 32^2 and 64^2 unseen grids.
- **Static down-scaling task** – Zero-shot 2×/4× on CMEMS surface currents; baselines are CNN-U-Nets trained at 2× and 4×.

Navier–Stokes surrogate

Model	Resolution	MAE	MSE
Temp_DFNO	16×16	0.017941	0.000603
	32×32	0.017426	0.000573
	64×64	0.019775	0.000722
Temp_SpecDFNO	16×16	0.017806	0.000599
	32×32	0.017372	0.000568
	64×64	0.019736	0.000712
DFNO-2	32×32	0.0124	0.0004
	64×64	0.0246	0.0018
DFNO-4	32×32	0.0208	0.0012
	64×64	0.0168	0.0007

CMEMS 4× down-scaling

Model	Loss	MAE	SSIM
CNN-U-Net (4×)	L2	0.395	0.116
DFNO	L2	0.042	0.840
DUNO	L2	0.043	0.822
SpecDFNO	L2	**0.037**	**0.874**



Left: Temporal DFNO – low-res input, $16^2/32^2$ predictions, ground truth. *Right:* SpecDFNO – 4× reconstruction of a CMEMS patch.

Future Work

- **Uncertainty quantification.** Incorporating uncertainty quantification through probabilistic neural operators and ensemble-based diffusion strategies to express confidence in the high-resolution outputs.
- **Theoretical analysis.** Characterize the limits of these models, particularly as the governing physical behavior and physics equations change with increasing resolution.