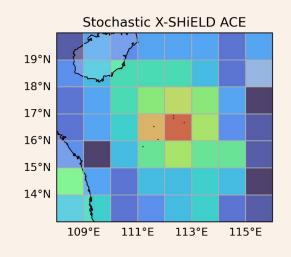
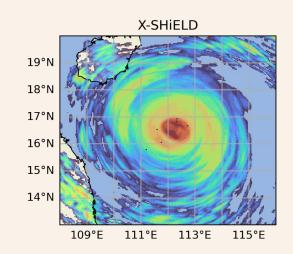
# Emulating Climate Across Scales with Conditional Spherical Fourier Neural Operators

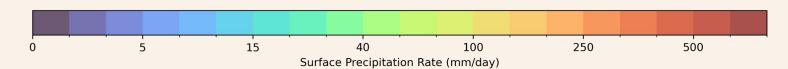
Oct 30, 2025



#### **Downstream Application: Downscaling**



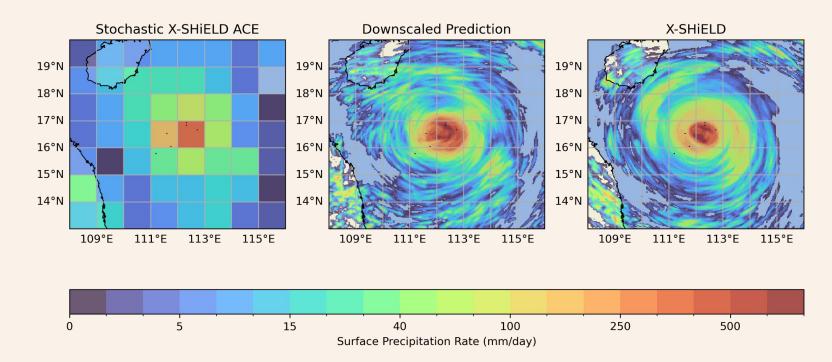






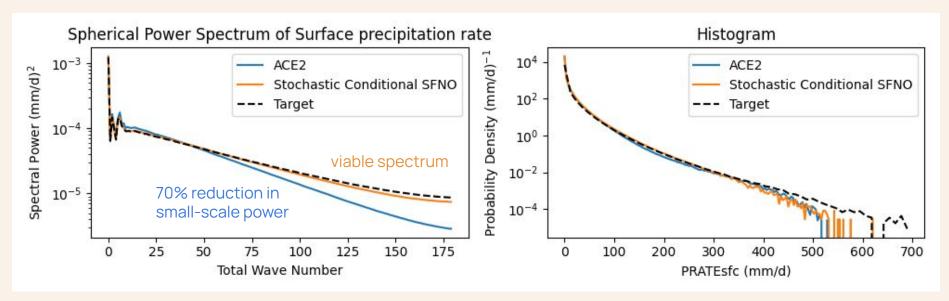
# Results

#### Downstream Application: Downscaling





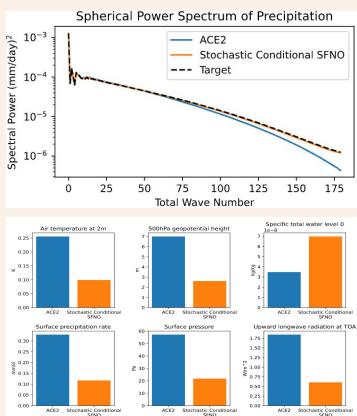
# Deterministic vs Stochastic power spectra



X-SHiELD training and evaluation (based on ERA5 pre-training for both)

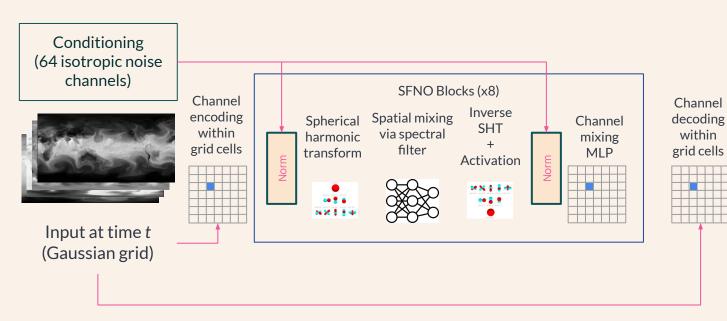
#### Promising climate results on c96-shield

- Our base configuration worked well off the bat for C96 - AMIP data
  - Used a heavy weighting of Energy Score (ES) with a
    90 % ES and 10% CRPS
  - Isotropic noise
  - Pre-training on 1 step then multi-step FTing with a specified distribution
- This dataset is fairly large, and has less small-scale power than other datasets



#### What did we do?

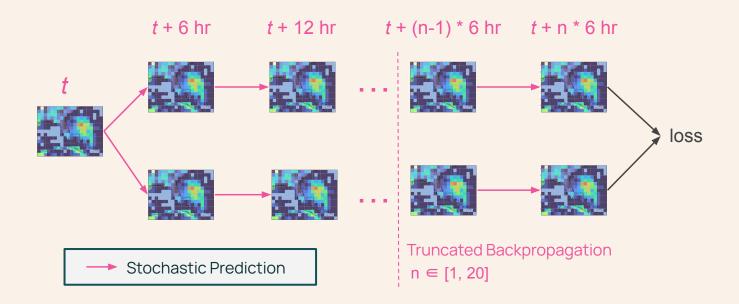
#### **Conditional SFNO Architecture**





Prediction of t + 6 hours (Gaussian grid)

#### **Training**



Initial training has n=1, then we fine-tune for up to 20 steps

#### **Loss Function**

"Almost fair" CRPS combined with reweighted energy score.

Energy score weighting set to give similar magnitude to CRPS, and independence from domain size.

$$L(F,y) = 0.1 \cdot \text{afCRPS}_{0.95,2}(F,y) + 0.9 \cdot \frac{2}{\sqrt{n_l n_m}} \text{ES(SHT} \circ F, \text{SHT}(y))$$

$$afCRPS_{\alpha,M}(F,y) = \mathbb{E}_{X \sim F}[|X - y|] - (1 - \frac{1 - \alpha}{M}) \frac{1}{2} \mathbb{E}_{X,X' \sim F}[|X - X'|]$$

$$\mathrm{ES}(F,\vec{y}) \; = \; \mathbb{E}_{\vec{X} \sim F} \left[ \; \|\vec{X} - \vec{y}\| \; \right] \; - \; \tfrac{1}{2} \, \mathbb{E}_{\vec{X},\vec{X}' \sim F} \left[ \; \|\vec{X} - \vec{X}'\| \; \right]$$

### Take-away points

- We have a successful strategy for fine-tuning a model usable for downscaling
- Optimizing over longer rollouts improves climate skill
- Optimizing spectral coefficients via energy score corrects small-scale spectral power