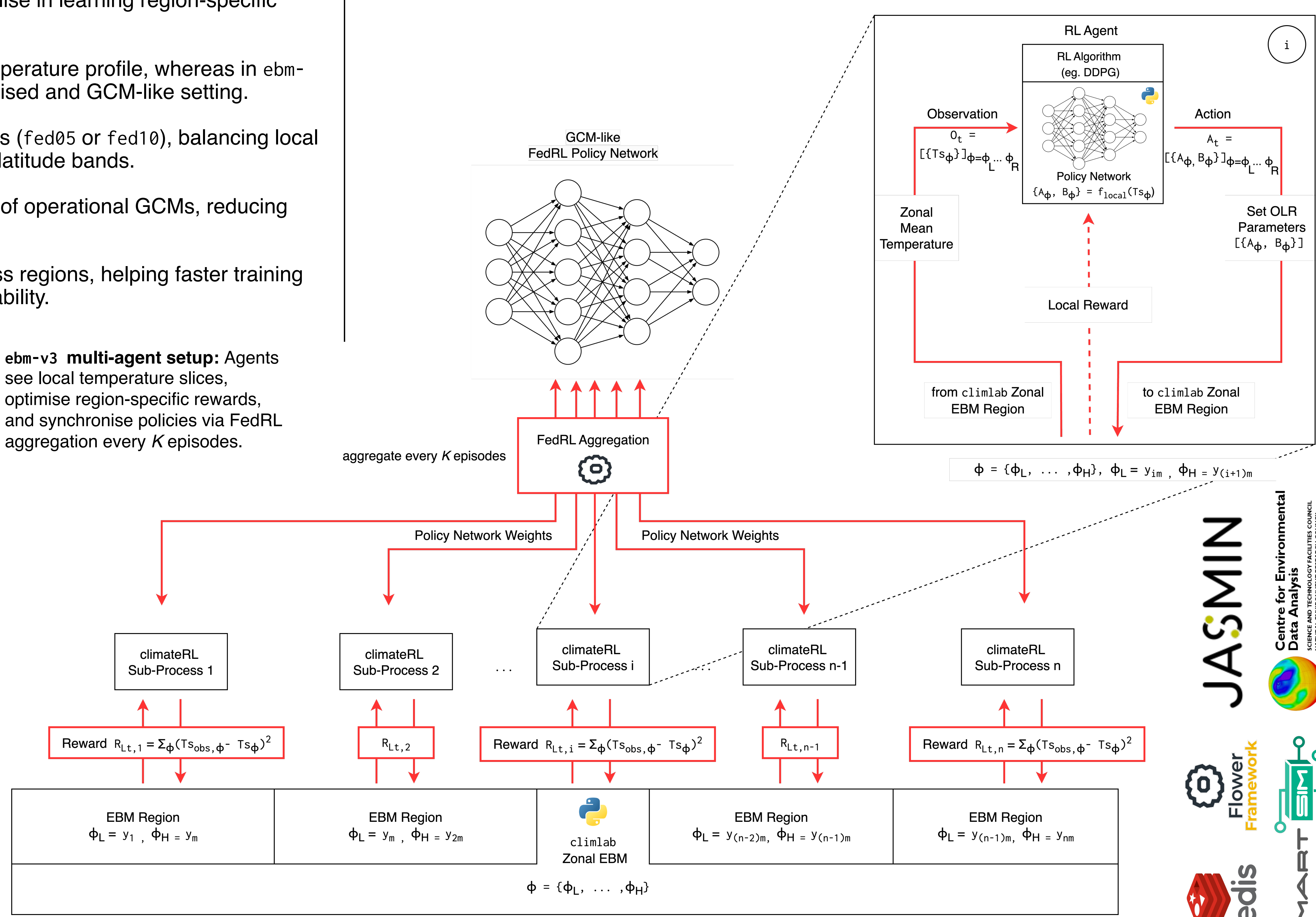
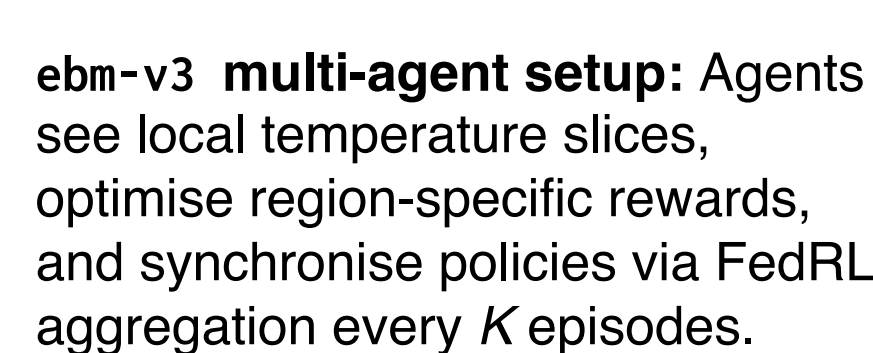


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3. Results

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- Figure 10 displays the performance of DDPG in ebm-v2 and ebm-v3. The figure is organized into two rows and three columns.
- Top Row: Training Curves**
- Left Plot (ebm-v1):** Shows the negative episodic return (Y-axis, ranging from 2×10^4 to 10^5) versus the number of steps (X-axis, 0 to 20K). The legend indicates four series: 1. TQC (blue line), 2. DDPG (orange line), 3. TD3 (green line), and a dashed black line for the threshold (THRESH) at approximately 2.8×10^4 .
 - Middle Plot (ebm-v2):** Shows the negative episodic return (Y-axis, ranging from 10^4 to 10^5) versus the number of steps (X-axis, 0 to 20K). The legend indicates four series: 1. TQC (blue line), 2. DDPG (orange line), 3. TD3 (green line), and a dashed black line for the threshold (THRESH) at approximately 2.8×10^4 .
 - Right Plot (ebm-v3):** Shows the negative episodic return (Y-axis, ranging from 10^4 to 10^5) versus the number of steps (X-axis, 0 to 20K). The legend indicates four series: 1. TQC (blue line), 2. DDPG (orange line), 3. TD3 (green line), and a dashed black line for the threshold (THRESH) at approximately 2.8×10^4 .
- Bottom Row: Zonal Skill (areaWRMSE)**
- Left Plot (ebm-v2):** Shows the area weighted RMSE (Y-axis, ranging from 0 to 30) versus latitude (X-axis, 90°S-60°S to 60°N-90°N). The legend indicates five series: CLIMLAB (grey bars), FED05 (blue bars), FED10 (orange bars), NOFED (green bars), and EBM-v1 (red bars).
 - Right Plot (ebm-v3):** Shows the area weighted RMSE (Y-axis, ranging from 0 to 25) versus latitude (X-axis, 90°S-60°S to 60°N-90°N). The legend indicates five series: CLIMLAB (grey bars), FED05 (blue bars), FED10 (orange bars), NOFED (green bars), and EBM-v1 (red bars).

1. Multiple RL agents are assigned to latitude bands, allowing each to specialise in learning region-specific correction policies that adapt to local climate dynamics.
2. In `ebm-v2`, agents optimise local rewards while observing the full zonal temperature profile, whereas in `ebm-v3`, agents receive only local temperature slices, creating a more decentralised and GCM-like setting.
3. Periodic global aggregation synchronises policy networks every K episodes (`fed05` or `fed10`), balancing local adaptation with global stability and enabling coherent coordination across latitude bands.
4. Decentralised design mirrors the modular, spatially decomposed structure of operational GCMs, reducing optimisation complexity and supporting geographically adaptive skill.
5. Federated coordination (FedRL) enables agents to share knowledge across regions, helping faster training and generalisation, while preserving the flexibility to adapt to regional variability.



1. Combining RL with federated learning and spatial decomposition enables geographically adaptive, state-dependent parametrisations that evolve with model states.
2. FedRAIN-Lite aligns with the modular design of GCMs, allowing region-specific corrections while preserving global coordination.
3. DDPG emerges as a robust and efficient baseline, consistently achieving stable convergence and low zonal errors across all setups.
4. FedRAIN provides a scalable pathway from idealised EBMs to operational GCMs, supporting future data-driven and physically aligned climate modelling.

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