
An Inversion Algorithm of Ice Thickness and InSAR Data for the State of Friction at the Base of the Greenland Ice Sheet

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Abstract

With the advent of climate change and global warming, the Greenland Ice Sheet (GrIS) has been melting at an alarming rate, losing over 215 Gt per yr, and accounting for 10% of mean global sea level rise since the 1990s. It is imperative to understand what dynamics are causing ice loss and influencing ice flow in order to successfully project mass changes of ice sheets and associated sea level rise. This work applies machine learning, ice thickness data, and horizontal ice velocity measurements from satellite radar data to quantify the magnitudes and distributions of the basal traction forces that are holding the GrIS back from flowing into the ocean. Our approach uses a hybrid model: InSAR velocity data trains a linear regression model, and these model coefficients are fed into a geophysical algorithm to estimate basal tractions that capture relationships between the ice motion and physical variables. Results indicate promising model performance and reveal significant presence of large basal traction forces around the coastline of the GrIS.

1 Introduction

With the advent of climate change and global warming, ice sheets worldwide have been melting at an alarming rate. The rate of ice mass loss has increased sixfold from 81 billion tons in the 1990's to 475 billion tons in the 2010s (10). The largest contributor to global ice loss is the Greenland Ice Sheet (GrIS), losing over 215 Gt of ice per yr, and accounting for 10% of mean global sea level rise since the 1990s (9). Rising sea levels have a wide array of disastrous impacts, including coastal erosion, storm surges, flooding, spread of disease, and habitat loss that will only continue to worsen in a warming climate (8). It is imperative to understand what dynamics are influencing ice loss and ice flow in order to successfully project mass changes of the GrIS and associated sea level rise.

Recent advances in satellite remote sensing systems have produced high-resolution maps of the Earth, making them an ideal tool for studying motion across large ice sheets. Two-pass Interferometric Synthetic Aperture Radar (InSAR) satellites use radar observations from multiple trips over an area of interest to determine surface motion (12). In this work, we utilize high-resolution InSAR ice velocity measurements of the GrIS derived from satellite imagery captured by the ESA's Sentinel-1 fleet (7). By inverting the data using a linear regression model, we quantify previously poorly-characterized forces and distributions of basal tractions that are holding the GrIS back from flowing into the ocean. Initial results reveal significant presence of large basal tractions forces around the coastline of the ice sheet.

2 Previous Work

Prior researchers at Stony Brook University set out to uncover a relationship between gravitational potential energy (GPE) and the velocity of viscous ice flow of the GrIs. They used GrIS bedrock and ice elevations, derived from topographical data provided by the NOAA’s ETOPO1 dataset, to map a vertically integrated gravitational potential energy (GPE) of the GrIS and associated ice velocity rates. However, the GPE velocity calculations vastly overestimated the ground truth InSAR ice velocities, and this difference was attributed to researchers assuming the ice was moving along a frictionless base. These results reinforce the notion that the basal tractions between the ice and the bedrock have a major influence over ice motion and ice velocity (5). However, these forces have been poorly characterized as they remain buried beneath thousands of meters of ice (5). Our work extends prior research to bridge the discrepancies between the GPE and InSAR ice velocities caused by the basal tractions, providing us a deeper understanding of the dynamics of the forces holding the GrIS back from flowing into the ocean. By employing machine learning and regression to perform an inversion, we are able to use InSAR and GPE velocity data to infer basal tractions that could not have been directly observed. Our novel approach uses a hybrid model: our velocity data is used to train a linear regression model, and these model coefficients can be fed into a geophysical algorithm to estimate distributions of basal traction forces that capture relationships between the ice motion and physical variables. To our knowledge, this is the first AI-driven work separating GPE and basal traction forces to understand ice sheet dynamics.

3 Methods

3.0.1 Dataset

Our data comes from two sources: ETOPO1 and Sentinel-1 radar satellite imagery. ETOPO1 provides topographical ice and bedrock elevation measurements which are used to calculate the thickness of the ice, and then generate gravitational potential energies (GPE) across the entire ice sheet (2). Roughly 1800 Sentinel-1 scenes were used with InSAR feature tracking techniques to derive surface horizontal ice velocity measurements of the GrIS (7). Both global-level datasets were parsed using the geopandas library to focus on the GrIS from 2016-2017 (3).

3.0.2 Inversion Set Up

Our inversion equation is given by:

$$\vec{d} = \overline{\overline{G}}m = v_{\text{InSAR}} - v_{\text{GPE}}$$

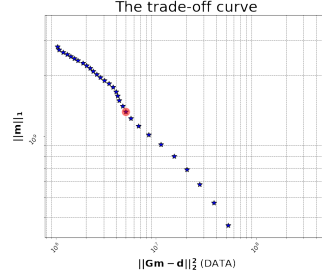
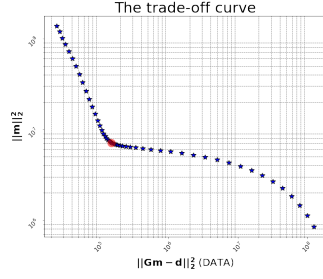
\vec{d} is our velocity field representing the difference between the InSAR and GPE velocities. To create our design matrix, $\overline{\overline{G}}$, we partitioned the GrIS into 1000 grid cells (each with size $2^\circ \times 2^\circ$) and generated 3 basis functions (ε_{xx} - Horizontal East and West effective body forces, ε_{yy} - Horizontal North and South effective body-forces, and ε_{xy} - Shear effective body-forces) representing the ice’s viscous thin-sheet responses for each cell. m is our linear-regression inversion model. Our goal to find best linear combination $\overline{\overline{G}}m$ that predicts \vec{d} .

3.0.3 Model

This is a linear inversion task, where the effective body-forces in one grid cell can have effects on its surrounding cells and beyond. Thus, it requires a regression model. We use the least squares regression method, shown to perform well in inversion tasks, and also employed Ridge (Tikhonov) and LASSO regularization with loss functions defined as $\|\overline{\overline{G}}m - \vec{d}\|_2 + a^2\|m\|_1$ and $\|\overline{\overline{G}}m - \vec{d}\|_2 + a^2\|m\|_2^2$ respectively (4; 6; 11). Trade-off (L-curve) criterion determined our optimal smoothing parameter, shown in Figures 1 and 2 with Figure 3 displaying performance metrics.

4 Results

Both the Ridge and LASSO regression models achieved a near identical fit to the velocity field, achieving R^2 values of 0.999 and 0.985 respectively. The Ridge model predictions (green vectors)



Metric	Ridge	LASSO
Best α	0.1520	0.0324
R^2	0.9994	0.9852
RMSE	6.3651	27.6796
MAE	4.3038	24.3230

Figure 3: Optimal parameter and model performance

Figure 1: Ridge trade-off curve Figure 2: LASSO trade-off curve

have been plotted against the ground truth velocity field (red vectors) in Figure 4, highlighting the model's accuracy. Given the unusually high R^2 score, we are planning on testing this model on larger datasets to verify model performance. We can now take the model's coefficients and convert our strain rate basis functions from \bar{G} to basal tractions through the geophysical algorithm described in Finzel et al. (1). The Ridge basal traction predictions are plotted in Figure 5, and indicate that the largest basal tractions responsible for holding the GrIS together lie on the coastline of the ice sheet.

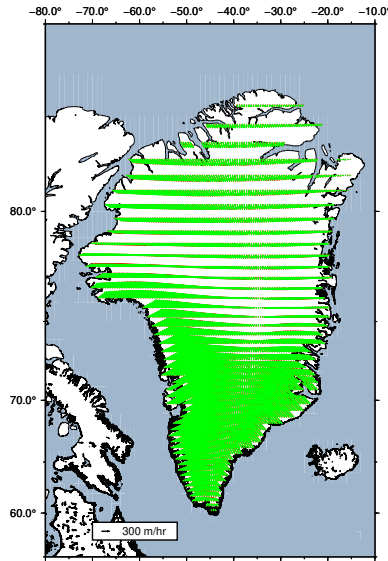


Figure 4: Ridge model velocity predictions

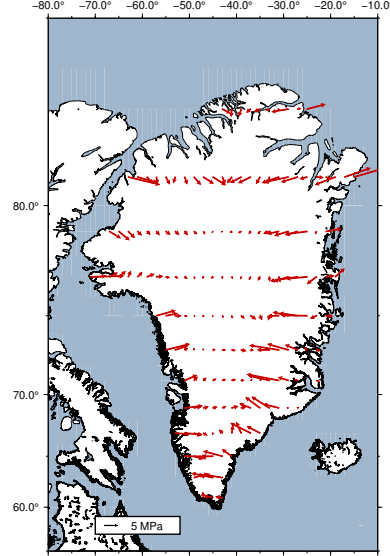


Figure 5: Ridge model basal traction predictions

5 Conclusion and Future Work

In this work, we developed an inversion algorithm for quantifying the magnitudes and distributions of basal tractions of the Greenland Ice Sheet. This is achieved via a hybrid approach, where our velocity data trains a linear regression model, and these model coefficients are fed into a geophysical algorithm to estimate basal tractions. This work has large implications on the ability to quantify basal tractions and how they are keeping the GrIS together, and serves as a step towards modeling ice loss and flux in relation to seawater intrusion, friction, and other forces. In the future, we hope to make this model more accurate and generalizable to other ice sheets so that it can become a helpful tool for climate scientists modelling rising sea levels. Our approach demonstrates the promise of applying AI to gain a deeper understanding of ice sheets, giving us valuable insight towards rising sea levels needed in the fight against climate change.

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