

Reference Guide & Formula Sheet for Physics

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Chapter 01. – Units, Unit Conversion, Symbols

Units are important. You will learn the names of lots of units this year. Each unit has a standard one or two-letter abbreviation. You must learn the abbreviations and use them. Start with these few.

(The unit of time is the second = s)

(The unit of distance or displacement is the meter = m)

(The unit of velocity is meters per second = m/s)

(The unit of acceleration is meters per second squared = m/s²)

Symbols are important. They stand in for known and unknown quantities in the equations you will derive to solve Physics problems. Many of these symbols are considered so standard that everyone everywhere uses the same symbol. This actually simplifies matters in many cases. We will use the standard symbols whenever possible. You will have to learn these as we move along. Start with these few.

(The symbol of time is t)

(The symbol of distance is d)

(The symbol of speed is v or s)

(The symbol of displacement is **d** or **r** or **s**)

(The symbol of velocity is **v**)

(The symbol of acceleration is **a**)

Subscripts are important. Subscripts add essential information that must be taken into account. For example, **v₀** usually indicates the velocity at time zero, while **v_i** and **v_f** usually indicate the initial and final velocities, and **v_{AVE}** is the average velocity. These are all different velocities. Always read the subscripts.

Time is important. Time started 14 billion years ago when the universe appeared. We will not be studying processes that began at the beginning of time. All times that we measure are therefore time intervals, or time differences, if you like. For us time zero always means that time when the clock started. And time **t** always means the time interval since the clock started. In light of this fact, is it never wrong to replace **t** with a **Δt** in any equation. If the time interval does not start at time zero on the clock, then the time must be written as **Δt**.

Standard Units: The standard units we use are known as SI units. For now, learn these first few.

Measure of length	meters = m
Measure of area	meters ² = m ² = m×m
Measure of volume	meters ³ = m ³ = m×m×m
Measure of time	seconds = s
Measure of velocity	meters per second = m/s
Measure of acceleration	meters per second ² = m/s ²

Chapter 01. – continued

Error and Precision are not the same thing. Error tells us how far the measurement is from the true answer. We will usually report error as **Percent Error**.

$$\% \text{Error} = 100\% \times |\text{Measured} - \text{True}| / \text{True}$$

Precision tells us only how consistently a given measuring device can measure values **ACCORDING TO ITS MANUFACTURER'S SPECIFICATION**. Precision is a measure of the reproducibility and consistency of the results. The measurements can be very consistent and still be consistently wrong, however.

Typically, the specification for the precision of a device might be reported something like one of the following,

$$\pm 0.002$$

$$\pm 1\%$$

$$3\frac{1}{2} \text{ digits}$$

Fine precision is no guarantee of high accuracy, however. Usually the two go together, but sometimes, probably by mistake, they do not. (Hubble Telescope!)

Unit Conversion Factors – Remember that all unit conversion factors only change the numerical answer because they change the units in which it is reported. These defined relationships always have very high accuracy, and practically an unlimited number of significant digits (even if the terminal zeroes are not written out).

$$100 \text{ cm} = 1 \text{ meter}$$

Example:

Suppose you want to convert 35.0 miles per hour to meters per second. You would need conversion factors based on the following equalities.

$$1 \text{ mile} = 5,280 \text{ feet}$$

$$1 \text{ foot} = 12 \text{ inches}$$

$$1 \text{ inch} = 2.54 \text{ centimeters}$$

$$100 \text{ centimeters} = 1 \text{ meter}$$

$$1 \text{ hour} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

$$35.0 \frac{\text{mi}}{\text{hr}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{1 \text{ m}}{100 \text{ cm}} \\ \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 15.7 \text{ m/s}$$

From each equality, choose the ratio that eliminates an unwanted unit and adds a unit that moves the answer in the desired direction.

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Chapter 02. – Motion Along one Axis

Physical quantities for which the direction of their motion or action is an important characteristic must be treated mathematically using vectors, not simple numerical values. Quantities that do not require directional information are called scalars. We begin our study of vectors by studying motion in one direction. This type of vector behaves much like a scalar quantity; only the notation is a little different at this point.

Vectors are symbolized with very **bold** letters. The most important quantities in this section are the instantaneous quantities listed here.

Instantaneous position: **$\mathbf{x}$**

Instantaneous position at time zero: **$\mathbf{x}_0$**

Instantaneous velocity: **$\mathbf{v}$**

Instantaneous velocity at time zero: **$\mathbf{v}_0$**

Instantaneous acceleration: **$\mathbf{a}$**

(for now, acceleration is assumed to be constant.)

Before we can rigorously define what we mean by instantaneous, we need to define some simpler quantities. The first of these are distance and displacement. In one dimension these might have the same numeric value.

d = distance = odometer reading

If the object starts at **$\mathbf{x}_0$** and moves back and forth

before settling at its final position, **$\mathbf{x}$** , then the distance could be much longer than the shortest path between the starting and ending points. On the other hand, the minimum distance is closely related to the displacement.

$$d_{\text{MIN}} = |\mathbf{x} - \mathbf{x}_0| = |\mathbf{x}_0 - \mathbf{x}| = |\mathbf{d}|$$

The minimum distance does not include information about the direction of travel; that is the meaning of those absolute value markers. The starting and ending

positions are **$\mathbf{x}_0$** and **$\mathbf{x}$** , but the scalar quantity called “minimum distance” does not care which is which. Subtraction in either order is permitted.

$$\mathbf{d} = \text{displacement} = \mathbf{x} - \mathbf{x}_0 = \mathbf{d}$$

The vector quantity called “displacement”, on the other hand, must be calculated as the final position vector minus the initial position vector. That result always gives us the minimum distance and the direction of the motion. For motion along the x-axis, as one example, a positive displacement indicates motion to the right. A negative displacement indicates motion to the left.

Chapter 02. – continued

With the definition of distance in hand we can define the scalar quantity known as the average speed.

$$V_{\text{AVG}} = \text{average speed} = \text{distance}/\Delta\text{time} = d/\Delta t$$

Where Δt is the time interval between the moment when the object was at the initial position and the moment when it was at the final position. The time interval is often writing simply as t , but this is only true if the time interval begins at the moment when the clock reads zero. (Think of the clock as a stopwatch.)

With the definition of the displacement in hand we can define the vector quantity known as the average velocity

$$\mathbf{V}_{\text{AVG}} = \text{average velocity} = \text{displacement}/\Delta\text{time} = \mathbf{d}/\Delta t$$

Instantaneous Position – $\mathbf{x}$ – the position of a moving object at one moment in time; also known as an instant of time. Position is always assumed to be instantaneous.

Instantaneous Velocity – $\mathbf{v}$ – is average velocity over an infinitesimal displacement in an infinitesimal time interval. It is the velocity at one instant. As a practical matter it is usually good enough to measure the average velocity over a short displacement in a brief time interval and then take the ratio of those two measurements to estimate the instantaneous velocity.

Constant-Acceleration Linear Motion

Once our final definitions for instantaneous position and instantaneous velocity are completed, the following equations of motion apply to all systems that have a constant acceleration. (When the acceleration is constant, the instantaneous and average accelerations have the same magnitude and direction.)

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a} \cdot t \quad \text{no } \mathbf{x}$$

$$(\mathbf{x} - \mathbf{x}_0) = \mathbf{v}_0 \cdot t + \frac{1}{2} \cdot \mathbf{a} \cdot t^2 \quad \text{no } \mathbf{v}$$

$$\mathbf{v}^2 = \mathbf{v}_0^2 + 2 \cdot \mathbf{a} \cdot (\mathbf{x} - \mathbf{x}_0) \quad \text{no } t$$

$$(\mathbf{x} - \mathbf{x}_0) = \frac{1}{2} \cdot (\mathbf{v}_0 + \mathbf{v}) \cdot t \quad \text{no } \mathbf{a}$$

$$(\mathbf{x} - \mathbf{x}_0) = \mathbf{v} \cdot t - \frac{1}{2} \cdot \mathbf{a} \cdot t^2 \quad \text{no } \mathbf{v}_0$$

Average velocity can be obtained from the initial and final instantaneous velocities, if and only if the acceleration is constant.

$$\mathbf{V}_{\text{AVE}} = \frac{\mathbf{v} + \mathbf{v}_0}{2} = \text{average velocity}$$

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Chapter 02. – continued

Constant Acceleration is rare in nature but common in the problems we will be working. Constant acceleration gives the equations of motion their simplest form and makes them easier to solve. Gravity provides a ready source of objects moving with constant acceleration.

Strictly speaking, we cannot define average acceleration until we have a definition for instantaneous velocity. Then the average acceleration is

$$\mathbf{a}_{\text{AVG}} = \text{average acceleration} \\ = \text{velocity change}/\Delta\text{time} = \Delta\mathbf{v}/\Delta t$$

When the acceleration is constant, the instantaneous and average acceleration have the same magnitude and direction. Since the instantaneous acceleration is the same at all moments, the average acceleration must have the same magnitude and direction.

One dimensional vectors

To this point, we've used only vectors that behave exactly like signed numerical values, where the sign indicates the direction along the axis of motion.

Vectors can also be thought of as arrows with pointed ends showing the direction of the motion. We could even use these arrows to describe the one-dimensional vectors discussed in this chapter.

In the next chapter, where objects are free to move in two dimensions, we will use the arrow representation first. There is also a method that allows us to reuse the vector concepts from this chapter; the signed numbers. We will separate the vectors into what are called their **components**. Components are independent one-dimensional sub-sets of the motion.

Chapter 03. –

Components of a Vector and Vector Addition

$$\mathbf{V} = v \angle \theta = 34.0 \text{ m/s} \angle 48.0^\circ$$

$$v_x = v \cos \theta = 34 \text{ m/s} \cdot (\cos 48^\circ) = 22.8 \text{ m/s}$$

$$v_y = v \sin \theta = 34 \text{ m/s} \cdot (\sin 48^\circ) = 25.3 \text{ m/s}$$

$$\mathbf{V} = v_x \mathbf{i} + v_y \mathbf{j} = 22.8 \mathbf{i} + 25.3 \mathbf{j} \text{ m/s}$$

$$\mathbf{W} = w \angle \theta = 52.0 \text{ m/s} \angle 113.0^\circ$$

$$w_x = w \cos \theta = 52 \text{ m/s} \cdot (\cos 113^\circ) = -20.3 \text{ m/s}$$

$$w_y = w \sin \theta = 52 \text{ m/s} \cdot (\sin 113^\circ) = 47.9 \text{ m/s}$$

$$\mathbf{W} = w_x \mathbf{i} + w_y \mathbf{j} = -20.3 \mathbf{i} + 47.9 \mathbf{j} \text{ m/s}$$

Chapter 03. – continued

$$\mathbf{V} + \mathbf{W} = (22.8-20.3) \mathbf{i} + (25.3+47.9) \mathbf{j} \text{ m/s}$$

$$\mathbf{V} + \mathbf{W} = 2.5 \mathbf{i} + 73.2 \mathbf{j} \text{ m/s} = \mathbf{W} + \mathbf{V}$$

$$\mathbf{V} - \mathbf{W} = (22.8-(-20.3)) \mathbf{i} + (25.3-47.9) \mathbf{j} \text{ m/s} \\ = 43.1 \mathbf{i} - 22.6 \mathbf{j} \text{ m/s}$$

$$\mathbf{W} - \mathbf{V} = (-20.3-22.8) \mathbf{i} + (47.9-25.3) \mathbf{j} \text{ m/s} \\ = -43.1 \mathbf{i} + 22.6 \mathbf{j} \text{ m/s}$$

$\mathbf{V} - \mathbf{W}$ and $\mathbf{W} - \mathbf{V}$, point in opposite directions and both are perpendicular to $\mathbf{V} + \mathbf{W} = \mathbf{W} + \mathbf{V}$.

Projectile Motion – working with components

$$\text{Horizontal position: } x - x_0 = v_x \cdot t$$

$$\text{Vertical position: } y - y_0 = v_{y0} \cdot t - \frac{1}{2} g \cdot t^2$$

$$\text{Horizontal velocity: } v_x = v_0 \cos \theta$$

$$\text{Vertical velocity: } v_y = v_0 \sin \theta - g t^2$$

$$\text{Horizontal acceleration: } a_x = 0$$

$$\text{Vertical acceleration: } a_y = -g = \text{constant}$$

Chapter 04. –

Newton's First Law – Law of Inertia. Forces make objects move. No force means no change in the motion.

Newton's Second Law – Forces cause acceleration.

$$\mathbf{F}_{\text{net}} = \Sigma \mathbf{F}_{\text{Ext}} = m_{\text{sys}} \cdot \mathbf{a}_{\text{sys}}$$

Newton's Third Law – Forces are created in pairs.

$$\text{Weight} = \mathbf{W} = m \cdot \mathbf{g}$$

$$\mathbf{g} = 9.80 \text{ m/s}^2 \text{ near the surface of the Earth} \\ = 9.795 \text{ m/s}^2 \text{ in Fort Worth, TX}$$

$$\text{Friction Force} = \mathbf{F}_F = \mu \cdot \mathbf{F}_N$$

If the object is not moving, you are dealing with static friction and it can have any value from zero up to $\mu_s F_N$

If the object is sliding, then you are dealing with kinetic friction and it will be constant and equal to $\mu_k F_N$

Free-Body Diagram – Show all the forces acting on and object. Components must be dashed to distinguish them from forces. You cannot use a force and its components in the same problem.

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Chapter 05. –

Uniform Circular Motion - Centripetal Acceleration

$$a_R = \frac{v^2}{r}$$

Uniform Circular Motion – Period, Frequency and V

$$T = \frac{1}{f} \text{ and } f = \frac{1}{T} \text{ and } v = \frac{2\pi r}{T}$$

Centripetal Force

$$F_c = \frac{mv^2}{r} = m\omega^2 r$$

Minimum Speed at the top of a Vertical Loop

$$v = \sqrt{rg}$$

Circular Unbanked Track – Car Rounding a Curve.

$$F_c = \mu mg = \frac{mv^2}{r} = ma_R$$

Banked Circular Track

$$v^2 = r \cdot g \cdot \tan \theta, \text{ without friction}$$

Universal Gravitation – Conservative Force

$$F = G \frac{m_1 m_2}{r^2}$$

where $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2 = 6.67 \text{ E-11 N} \cdot \text{m}^2 / \text{kg}^2$

Chapter 06. –

Work done by a constant force = $\mathbf{F} \cdot \mathbf{D} \cdot \cos \theta$

Where $\mathbf{D}$ is the displacement of the mass and

θ is the angle between $\mathbf{F}$ and $\mathbf{D}$. unit : $\text{N} \cdot \text{m} = \text{J}$

Work done by a varying force

On a graph of Force vs displacement the work is the area between the curve and the x-axis. Later on we will evaluate this by taking the anti-derivative of the function that describes the force in terms of position.

Mechanical Energy – Kinetic Energy

$$\text{KE}_{\text{Linear}} = K = \frac{1}{2} \cdot m \cdot v^2$$

Chapter 06 – continued

Mechanical Energy – The Work-Energy Theorem

The net work done on a body equals the change in the kinetic energy of the body.

$$W_{\text{net}} = \Delta KE = KE_f - KE_i \\ = \frac{1}{2} \cdot m \cdot v_f^2 - \frac{1}{2} \cdot m \cdot v_i^2$$

Mechanical Energy – Gravitational Potential Energy

$$\text{PE}_{\text{Grav}} = \text{PE}_g = m \cdot g \cdot h \\ = m \cdot g \cdot y$$

Hooke's Law – a non-constant force

$$\mathbf{F} = -k \cdot \mathbf{x}$$

$\mathbf{x}$ = displacement from equilibrium

k = the spring constant

= proportionality constant between the restoring force and the displacement.

Potential Energy of a spring – Conservative Force

$$\text{Work done on a spring} = \text{PE} = W = \frac{1}{2} \cdot k \cdot x^2$$

Power = rate of work done, unit = J/s = W = watts

$$\text{average power} = \bar{P} = \frac{\text{Work}}{\Delta \text{time}} = \frac{Fd}{\Delta t} = Fv$$

Chapter 07. –

Linear Momentum

$$\text{momentum} = \mathbf{p} = m \cdot \mathbf{v} = \text{mass} \cdot \text{velocity}$$

Newton's Second Law

$$F_{\text{net}} = \Sigma F_{\text{Ext}} = \frac{\Delta p}{\Delta t} = \frac{mv - mv_0}{\Delta t} = \frac{m(v - v_0)}{\Delta t} = ma$$

Impulse = Change in Momentum

$$\mathbf{F} \cdot \Delta t = \Delta \mathbf{p} = \Delta(m \cdot \mathbf{v})$$

Conservation of Momentum in Collisions

$$m_A \cdot v_A + m_B \cdot v_B = m_A \cdot v_A' + m_B \cdot v_B'$$

Sum of Momenta before the Collision

Sum of Momenta after the Collision

Center of Mass – point masses on a line (x only), on a plane (x and y only), or filling space (x, y, and z)

$$x_{\text{cm}} = \Sigma(m_i \cdot x_i) / M_{\text{total}}$$

$$y_{\text{cm}} = \Sigma(m_i \cdot y_i) / M_{\text{total}}$$

$$z_{\text{cm}} = \Sigma(m_i \cdot z_i) / M_{\text{total}}$$

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Chapter 08. –

Angular Distance – Radian measure

$$\theta = \text{arc length} / \text{radius} = \ell / r = s / r$$

$$360^\circ = 2\pi \text{ radians}$$

Angular Speed vs. Linear Speed

$$\text{Linear speed} = v = r \cdot \omega = \text{radius} \cdot \text{angular speed}$$

Constant Angular-Acceleration in Circular Motion

$$\begin{aligned} \omega &= \omega_0 + \alpha \cdot t && \text{no } \theta \\ \theta - \theta_0 &= \omega_0 \cdot t + \frac{1}{2} \cdot \alpha \cdot t^2 && \text{no } \omega \\ \omega^2 &= \omega_0^2 + 2 \cdot \alpha \cdot (\theta - \theta_0) && \text{no } t \\ \theta - \theta_0 &= \frac{1}{2} \cdot (\omega_0 + \omega) \cdot t && \text{no } \alpha \\ \theta - \theta_0 &= \omega \cdot t - \frac{1}{2} \cdot \alpha \cdot t^2 && \text{no } \omega_0 \end{aligned}$$

$$\text{Torque} = \tau = \mathbf{F} \cdot \mathbf{L} \cdot \sin \theta$$

Where θ is the angle between $\mathbf{F}$ and $\mathbf{L}$; unit: N•m

Newton's Second Law for Rotation

$$\text{torque} = \tau = \mathbf{I} \cdot \alpha$$

$$\text{moment of inertia} = I_{CM} = m \cdot r^2 \text{ (for a point mass)}$$

Rotational Kinetic Energy (See LEM on last page)

$$\begin{aligned} KE_{\text{rotational}} &= \frac{1}{2} \cdot \mathbf{I} \cdot \omega^2 = \frac{1}{2} \cdot \mathbf{I} \cdot (v / r)^2 \\ KE_{\text{rolling w/o slipping}} &= \frac{1}{2} \cdot m \cdot v^2 + \frac{1}{2} \cdot \mathbf{I} \cdot \omega^2 \end{aligned}$$

Moment of Inertia – I_{CM}

point mass	$I_{CM} = m \cdot r^2$
cylindrical hoop	$I_{CM} = m \cdot r^2$
solid cylinder or disk	$I_{CM} = \frac{1}{2} m \cdot r^2$
solid sphere	$I_{CM} = \frac{2}{5} m \cdot r^2$
hollow sphere	$I_{CM} = \frac{2}{3} m \cdot r^2$
thin rod (center)	$I_{CM} = \frac{1}{12} m \cdot L^2$

When the thin rod is rotated about its end rather than about its center of mass, the moment of inertia becomes

$$\text{thin rod (end)} \quad I_{\text{End}} = \frac{1}{3} m \cdot L^2$$

$$\text{Angular Momentum} = L = \mathbf{I} \cdot \omega = m \cdot v \cdot r \cdot \sin \theta$$

Angular Impulse equals CHANGE IN Angular Momentum

$$\Delta L = \tau_{\text{Average}} \cdot \Delta t = \Delta(\mathbf{I} \cdot \omega)$$

Chapter 09. –

Elasticity; Stress and Strain

(Assumes objects stretch according to Hooke's Law as long as they are not stretched past the proportional limit.)

For tensile stress

$$\mathbf{F} = k \cdot \Delta L = (EA/L_0) \cdot \Delta L$$

F = applied force
 E = elastic (or Young's) modulus
 A = cross-sectional area – perpendicular to the force
 L_0 = the original length
 ΔL = change in the length

$$\Delta L / L_0 = (1/E) \cdot (F/A)$$

$$\text{strain} = (1/E) \cdot (\text{stress})$$

$$E = (\text{stress} / \text{strain})$$

Compressive stress is the exact opposite of tensile stress. Objects are compressed rather than stretched. As for springs the equations are the same for both tensile and compressive stress and the same elastic modulus is used for both calculations.

$$\Delta L / L_0 = -(1/E) \cdot (F/A)$$

$$\text{strain} = -(1/E) \cdot (\text{stress})$$

$$E = -(\text{stress} / \text{strain})$$

There is a negative sign because the length decreases as the force increases (ΔL is negative).

Shear stress is the application of two forces that distort an object (like deforming a rectangle into a parallelogram). The forces are equal and opposite (parallel, but not oriented to directly oppose each other). (A second pair of matched forces is also required to maintain equilibrium while the stress is applied.)

$$\Delta L / L_0 = (1/G) \cdot (F/A)$$

$$\text{strain} = (1/G) \cdot (\text{stress})$$

$$G = (\text{stress} / \text{strain})$$

G = shear modulus
 A = area – Parallel to the force.
 L_0 = original length of object
 ΔL = change in length due to force

Note that ΔL is perpendicular to L_0 .

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Chapter 09. – continued

Bulk Stress - When the force is applied uniformly all over an object, we expect its volume to shrink. When applied this way, the ratio of force to area is called “pressure”, and ΔP is the change in pressure that induces a corresponding change in the volume.

$$\begin{aligned}\Delta V/V_0 &= -(1/B) \cdot (\Delta P) \\ \text{strain} &= -(1/B) \cdot (\text{stress}) \\ B &= -(\text{stress} / \text{strain})\end{aligned}$$

B = bulk modulus
 V_0 = initial volume of the material
 ΔV = Change in the volume
 ΔP = change in pressure

The minus sign indicates that the volume decreases when the pressure increases. When ΔP is positive, ΔV is negative, and vice versa. One of the two is always negative.

Chapter 10. –

Pressure under Water (or immersed in any liquid)

$$P = \rho \cdot g \cdot h$$

P = Pressure at depth
 h = depth below the surface
 ρ = density of the fluid

Density = mass / volume

$$\rho = \frac{m}{V} \left(\text{unit : } kg / m^3 \right)$$

Buoyant Force - Buoyancy

$$\begin{aligned}F_B &= \rho \cdot V \cdot g \\ &= m_{\text{Displaced fluid}} \cdot g \\ &= \text{weight}_{\text{Displaced fluid}} \\ \rho &= \text{density of the fluid} \\ V &= \text{volume of fluid displaced}\end{aligned}$$

Continuity of Fluid Flow

$$\begin{aligned}Q_{\text{Volume Flow Rate}} &= A_{\text{in}} \cdot v_{\text{in}} = A_{\text{out}} \cdot v_{\text{out}} \\ A &= \text{Cross-sectional Area} \\ v &= \text{velocity of the fluid}\end{aligned}$$

Chapter 10. – continued

Poiseuille's Equation (Laminar flow in horizontal tubular pipes.)

Q = volume flow rate of fluid = m^3/s

$$Q = (\pi \cdot r^4) \Delta P / (8 \cdot \eta \cdot L)$$

r = inside radius of pipe = m
 $\Delta P = P_1 - P_2$ = Pressure change = Pa
 η = coefficient of viscosity = $Pa \cdot s$
 L = length of pipe = m

Bernoulli's Equation

$$P + \rho \cdot g \cdot h + \frac{1}{2} \cdot \rho \cdot v^2 = \text{constant}$$

$$Q_{\text{Volume Flow Rate}} = A_1 \cdot v_1 = A_2 \cdot v_2 = \text{constant}$$

Chapter 11. –

Period of Simple Harmonic Motion – Ideal Spring

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{also } f = 1/T$$

where k = spring constant, and m is the mass.

Simple Pendulum

$$T = 2\pi \sqrt{\frac{L}{g}} \quad \text{also } f = 1/T$$

where L is the length of the pendulum and g is the local acceleration due to gravity.

Velocity of Periodic Waves

$$f = 1/T$$

$$v = f \cdot \lambda = \lambda/T$$

where T = the period of the wave

Speed of a Wave on a String

$$T = \frac{mV^2}{L}; \text{ therefore, } v = \sqrt{\frac{T}{m/L}}$$

T = tension in string
 m = mass of string
 L = length of string

Sinusoidal motion

$$x = A \cdot \cos(\omega \cdot t) = A \cdot \cos(2 \cdot \pi \cdot f \cdot t)$$

ω = angular frequency
 f = frequency

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Chapter 12. – Doppler Effect

$$f' = f \frac{343 \pm \begin{matrix} \text{Toward} \\ \text{Away} \end{matrix} v_o}{343 \mp \begin{matrix} \text{Toward} \\ \text{Away} \end{matrix} v_s}$$

v_o = velocity of observer:

v_s = velocity of source

$v_{\text{SOUND}} = 343 \text{ m/s}$

Decibel Scale

$$\text{dB (Decibel level)} = 10 \log (I / I_o)$$

I = intensity of sound

I_o = intensity of softest audible sound

Chapter 13. –

Ideal Gas Law

$$P \cdot V = n \cdot R \cdot T$$

n = # of moles of gas

R = gas law constant

= 8.31 J/mol•K.

Thermal Expansion of Solids

$$\text{Linear: } \Delta L = L_o \cdot \alpha \cdot \Delta T$$

$$\text{Volume: } \Delta V = V_o \cdot \beta \cdot \Delta T$$

Chapter 14. –

Heating a Solid, Liquid or Gas

$$Q = m \cdot c \cdot \Delta T \quad (\text{no phase changes!})$$

Q = the heat added

c = specific heat.

ΔT = temperature change, K or °C

Heat required for a Phase Change

$$Q = m \cdot L$$

m = mass of material

L = Latent Heat of phase change

Flow of Heat through a Solid

$$\Delta Q / \Delta t = k \cdot A \cdot \Delta T / L$$

k = thermal conductivity

A = area of solid

ΔT = Temperature difference

L = thickness of solid

Chapter 15. –

First Law of Thermodynamics

$$\Delta U = Q_{\text{Net}} + W_{\text{Net}}$$

Change in Internal Energy of a system =

+Net Heat added to the system

+Net Work done on the system

Work done on a gas or by a gas

$$W = P \cdot \Delta V$$

2nd Law of Thermodynamics

The change in internal energy of a system is

$$\Delta U = Q_{\text{Added}} + W_{\text{DoneOn}} - Q_{\text{lost}} - W_{\text{DoneBy}}$$

Maximum Efficiency of a Heat Engine (Carnot Cycle)

(Temperatures in Kelvin)

$$\%Eff = (1 - \frac{T_c}{T_h}) \cdot 100\%$$

$$\text{Efficiency} = \text{Work}_{\text{out}} / \text{Energy}_{\text{in}}$$

Mechanical Advantage = force out / force in

$$M.A. = F_{\text{out}} / F_{\text{in}}$$

Entropy change at constant T

$$\Delta S = Q / T$$

(Applies to phase changes only: melting, boiling, freezing, etc)

Chapter 16. –

Coulomb's Law

$$F = k \frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_o} = 8.85 \times 10^{+9} \frac{N \cdot m^2}{C^2} \approx 9E9 \frac{N \cdot m^2}{C^2}$$

Electric Field around a point charge

$$E = k \frac{q}{r^2}$$

$$k = \frac{1}{4\pi\epsilon_o} = 8.85 \times 10^{+9} \frac{N \cdot m^2}{C^2} \approx 9E9 \frac{N \cdot m^2}{C^2}$$

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Chapter 17. –

Capacitors and Capacitance

$$Q_c = C \cdot V_c$$

Q_c = charge on capacitor

(The unit of electric charge is the coulomb = C)

C = capacitance of the capacitor

(The unit of capacitance is the farad = F = C/V)

V_c = voltage across the plates

(The unit of voltage is the volt = V = J/C)

Potential Energy stored in a Capacitor

$$P = \frac{1}{2} \cdot C \cdot V^2$$

Chapter 18. –

Ohm's Law

$$V = I \cdot R$$

V = voltage across the resistor

(The unit of voltage is the volt = V = J/C)

I = current through the resistor

(The unit of current is the ampere = A = C/s)

R = resistance of the resistor

(The unit of resistance is the ohm = Ω = V/A)

Resistance of a resistor (or any resistive material)

$$R = \rho \cdot L / A_x$$

ρ = resistivity of the material

(The unit of resistivity is the ohm•meter = $\Omega \cdot m$)

L = length of the material

(The unit of length is the meter = m)

A_x = cross-sectional area of the Material or resistor

(The unit of cross-sectional area is meter² = m²)

Electric Power (The unit of power is the watt = W = J/s)

$$P = I^2 \cdot R$$

$$P = V^2 / R$$

$$P = I \cdot V$$

Chapter 19. –

Resistor Combinations

SERIES

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

PARALLEL

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n} = \sum_{i=1}^n \frac{1}{R_i}$$

Capacitor Combinations

SERIES

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n} = \sum_{i=1}^n \frac{1}{C_i}$$

PARALLEL

$$C_{eq} = C_1 + C_2 + C_3 + \dots$$

Kirchhoff's Laws

$$\text{Node Rule: } \sum_{\text{node}} I_i = 0$$

$$\text{Loop Rule: } \sum_{\text{loop}} \Delta V_i = 0$$

Capacitance, C, of a Capacitor

$$C = \kappa \cdot \epsilon_0 \cdot A / d$$

κ = dielectric constant

A = area of plates

d = distance between plates

$$\epsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$$

(Capital "C" is also used as the abbreviation for the unit of electric charge; the coulomb = C. Do not confuse the two uses of capital "C". The unit of capacitance is the farad = F. The capital "F" is frequently used as the symbol for force. Do not confuse the two uses of the capital "F".)

RC Circuit formula (Charging with one battery, one resistor and one capacitor)

$$V_{\text{Battery}} - V_{\text{capacitor}} - I \cdot R = 0$$

RC Circuits (Charging) - $R \cdot C = \tau$ = time constant

$$V_c = V_{\text{MAX}} \cdot [1 - e^{-t/RC}]$$

But $V_c - I \cdot R = 0$ (from Ohm's Law), therefore,

$$I = (V_{\text{MAX}} / R) \cdot e^{-t/RC}$$

$$= I_{\text{MAX}} \cdot e^{-t/RC}$$

And $Q_c = C V_c$ (from the definition of capacitance), so

$$Q_c = C \cdot V_{\text{MAX}} \cdot [1 - e^{-t/RC}]$$

$$= Q_{\text{MAX}} \cdot [1 - e^{-t/RC}]$$

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Chapter 19. – continued

RC Circuits (Discharging) - $R \cdot C = \tau$ = time constant

$$V_c = V_{MAX} \cdot e^{-t/RC}$$

But $V_c - I \cdot R = 0$ (from Ohm's Law), therefore,

$$I = (V_{MAX} / R) \cdot e^{-t/RC}$$

$$= I_{MAX} \cdot e^{-t/RC}$$

And $Q_c = CV_c$ (from the definition of capacitance), so

$$Q_c = C \cdot V_{MAX} \cdot e^{-t/RC}$$

$$= Q_{MAX} \cdot e^{-t/RC}$$

Chapter 20. –

Magnetic Field around a wire

$$B = \frac{\mu_o I}{2\pi r}$$

Magnetic Flux

$$\Phi = B \cdot A \cdot \cos \theta$$

Force caused by a magnetic field acting on a moving charge

$$F = q \cdot v \cdot B \cdot \sin \theta$$

Chapter 21. –

Induced Voltage

$$Emf = N \frac{\Delta \Phi}{\Delta t} \quad N = \# \text{ of loops}$$

Lenz's Law – induced current flows to create a B-field opposing the change in magnetic flux.

Inductors during an increase in current

$$V_L = V_{cell} \cdot e^{-t / (L / R)}$$

$$I = (V_{cell} / R) \cdot [1 - e^{-t / (L / R)}]$$

L / R = time constant

Transformers

$$N_1 / N_2 = V_1 / V_2$$

$$I_1 \cdot V_1 = I_2 \cdot V_2$$

Chapter 22. –

Energy of a Photon or a Particle

$$E = h \cdot f = m \cdot c^2$$

h = Planck's constant

$$= 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$$

f = frequency of the photon

Chapter 23. –

Snell's Law

$$n_1 \cdot \sin \theta_1 = n_2 \cdot \sin \theta_2$$

Index of Refraction - definition

$$n = c / v$$

c = speed of light in a vacuum

$$= 3 \times 10^8 \text{ m/s} = 3 \text{ E}+8 \text{ m/s}$$

v = speed of light in the medium

$$= \text{less than } 3 \times 10^8 \text{ m/s}$$

Thin Lens Equation

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{o} + \frac{1}{i}$$

f = focal length
 i = image distance
 o = object distance

Magnification Equation

$$M = -d_i / d_o = -i / o = H_i / H_o$$

Helpful reminders for mirrors and lenses

Focal Length of:	positive	negative
mirror	concave	convex
lens	converging	diverging
Object distance = o	all objects	
Object height = H_o	all objects	
Image distance = i	real	virtual
Image height = H_i	virtual, upright	real, inverted
Magnification	virtual, upright	real, inverted

Chapter 24. –

Chapter 25. –

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Chapter 26. –

The Lorentz transformation factor, β , is given by

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}$$

Relativistic Time Dilation

$$\Delta t = \Delta t_0 / \beta$$

Relativistic Length Contraction

$$\Delta x = \beta \Delta x_0$$

Relativistic Mass Increase

It is usually expressed in terms of the momentum of the object

$$\mathbf{p} = \mathbf{m}_v \cdot \mathbf{v} = \mathbf{m}_0 \cdot \mathbf{v} / \beta$$

where β is the Lorentz transformation factor.

Mass-Energy Equivalence

$$\mathbf{m}_v = \mathbf{m}_0 / \beta$$

$$\text{Total Energy} = \mathbf{KE} + \mathbf{m}_0 \mathbf{c}^2 = \mathbf{m}_0 \mathbf{c}^2 / \beta$$

$$\text{Usually written as } \mathbf{E} = \mathbf{m} \mathbf{c}^2$$

Postulates of Special Relativity

1. **The laws of Physics have the same form in all inertial reference frames.**

(Absolute, uniform motion cannot be detected by examining the equations of motion.)

2. **Light propagates through empty space with a definite speed, c , independent of the speed of the source or the observer.**

(No energy or mass transfer can occur at speeds faster than the speed of light in a vacuum.)

Chapter 27. –

Blackbody Radiation and the Photoelectric Effect

$$\mathbf{E} = \mathbf{h} \cdot \mathbf{f}$$

$\mathbf{h}$ = Planck's constant

Early Quantum Physics

Rutherford-Bohr Hydrogen-like Atoms

$$\frac{1}{\lambda} = R \cdot \left(\frac{1}{n_s^2} - \frac{1}{n^2} \right) \text{meters}^{-1}$$

or

$$f = \frac{c}{\lambda} = cR \left(\frac{1}{n_s^2} - \frac{1}{n^2} \right) \text{Hz}$$

R = Rydberg's Constant
= $1.097373143 \text{ E7 m}^{-1}$

n_s = series integer (2 = Balmer Series)

n = an integer $> n_s$

de Broglie Matter Waves

For light:

$$\mathbf{E}_p = \mathbf{h} \cdot \mathbf{f} = \mathbf{h} \cdot \mathbf{c} / \lambda = \mathbf{p} \cdot \mathbf{c}$$

Therefore:

$$\mathbf{p} = \mathbf{h} / \lambda$$

By analogy, for particles, we expect to find that

$$\mathbf{p} = \mathbf{m} \cdot \mathbf{v} = \mathbf{h} / \lambda,$$

Thus, matter's wavelength should be

$$\lambda = \mathbf{h} / \mathbf{m} \mathbf{v}$$

Chapter 28. –

Chapter 29. –

Energy Released or Consumed by a Nuclear Fission or Nuclear Fusion Reaction

$$\mathbf{E} = \Delta \mathbf{m} \cdot \mathbf{c}^2$$

Where Δm is the difference between the sum of the masses of all the reactants and the sum of the masses of all the products.

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Chapter 30. –

Radioactive Decay Rate Law

$$N = N_0 \cdot e^{-k t} = (1/2^n) \cdot N_0$$

$$A = A_0 \cdot e^{-k t} = (1/2^n) \cdot A_0$$

$k = (\ln 2) / \text{half-life}$

N_0 = initial number of atoms

A_0 = initial activity

n = number of half-lives

Chapter 31. –

Chapter 32. –

Chapter 33. –

Appendix A –

Appendix B –

Appendix C –

Appendix D –

Appendix E –

Lorentz Transformation Factor

$$\beta = \sqrt{1 - \frac{v^2}{c^2}}$$

Appendix F –

Appendix G –

Appendix H –

Appendix I –

Appendix J –

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MISCELLANEOUS FORMULAS

Quadratic Formula

if $a \cdot x^2 + b \cdot x + c = 0$

then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric Definitions

$\sin \theta = \text{opposite} / \text{hypotenuse}$

$\cos \theta = \text{adjacent} / \text{hypotenuse}$

$\tan \theta = \text{opposite} / \text{adjacent}$

$\sec \theta = 1 / \cos \theta = \text{hyp} / \text{adj}$

$\csc \theta = 1 / \sin \theta = \text{hyp} / \text{opp}$

$\cot \theta = 1 / \tan \theta = \text{adj} / \text{opp}$

Inverse Trigonometric Definitions

$\theta = \sin^{-1}(\text{opp} / \text{hyp})$

$\theta = \cos^{-1}(\text{adj} / \text{hyp})$

$\theta = \tan^{-1}(\text{opp} / \text{adj})$

Law of Sines

$a / \sin A = b / \sin B = c / \sin C$

or

$\sin A / a = \sin B / b = \sin C / c$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = c^2 + a^2 - 2ca \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Trigonometric Identities

$\sin^2 \theta + \cos^2 \theta = 1$

$\sin 2\theta = 2 \sin \theta \cos \theta$

$\tan \theta = \sin \theta / \cos \theta$

$\sin(\theta + 180^\circ) = -\sin \theta$

$\cos(\theta + 180^\circ) = -\cos \theta$

$\sin(180^\circ - \theta) = \sin \theta$

$\cos(180^\circ - \theta) = -\cos \theta$

$\sin(90^\circ - \theta) = \cos \theta$

$\cos(90^\circ - \theta) = \sin \theta$

Fundamental SI Units

Unit	Base Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Luminous Intensity	candela	cd
Quantity of Substance	moles	mol
Plane Angle	radian	rad
Solid Angle	steradian	sr or str

Some Derived SI Units

Symbol/Unit	Quantity	Base Units
C coulomb	Electric Charge	A·s
F farad	Capacitance	A²·s⁴/(kg·m²)
H henry	Inductance	kg·m²/(A²·s²)
Hz hertz	Frequency	s⁻¹
J joule	Energy & Work	kg·m²/s² = N·m
N newton	Force	kg·m/s²
Ω ohm	Elec Resistance	kg·m²/(A²·s²)
Pa pascal	Pressure	kg/(m·s²)
T tesla	Magnetic Field	kg/(A·s²)
V volt	Elec Potential	kg·m²/(A·s²)
W watt	Power	kg·m²/s³

Non-SI Units

°C	degree Celsius	Temperature
eV	electron-volt	Energy, Work

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Latin Symbols for Quantities and Units

Aa acceleration, Area, A_x =Cross-sectional Area, Amperes, Amplitude of a Wave, Angle,
Bb Magnetic Field, Bel (sound intensity), Angle,
Cc specific heat, speed of light, Capacitance, Angle, Coulomb, $^{\circ}\text{C}$ =degrees Celsius, candela,
Dd displacement, differential change in a variable, Distance, Distance Moved, degrees, $^{\circ}\text{F}$, $^{\circ}\text{C}$,
Ee base of the natural logarithms, charge on the electron, eV=electron volt, Energy,
Ff Force, *frequency of a wave or periodic motion*, Farad, $^{\circ}\text{F}$ =degrees Fahrenheit,
Gg Universal Gravitational Constant, acceleration due to gravity, Gauss, grams, Giga-,
Hh depth of a fluid, height, vertical distance, Henry, Hz=Hertz,
Ii Current, Moment of Inertia, image distance, Intensity of Light or Sound,
Jj Joule,
Kk K or KE = Kinetic Energy, force constant of a spring, thermal conductivity, coulomb'slaw constant, kg=kilogram, Kelvin, kilo-, rate constant for Radioactive decay $=1/\tau=\ln 2 / \text{half-life}$,
Ll Length, Length of a wire, Latent Heat of Fusion or Vaporization, Angular Momentum, Thickness, Inductance,
Mm mass, Total Mass, meter, milli-, Mega-, m_0 =rest mass, mol=moles,
Nn index of refraction, moles of a gas, Newton, Number of Loops, nano-, Newton-meter,
Oo Ohm(Ω),
Pp Power, Pressure of a Gas or Fluid, Potential Energy, momentum, Pa=Pascal,
Qq Heat gained or lost, Charge on a capacitor, charge on a particle, object distance, Flow Rate,
Rr radius, Ideal Gas Law Constant, Resistance, magnitude or length of a vector, rad=radians,
Ss speed, second, Entropy, length along an arc,
Tt time, Temperature, Period of a Wave, Tension, Tesla, $t_{1/2}$ =half-life,
Uu Potential Energy,
Vv velocity, Velocity, Volume of a Gas, velocity of wave, Volume of Fluid Displaced, Voltage, Volt,
Ww weight, Work, Watt, Wb=Weber,
Xx distance, horizontal distance, x-coordinate east-and-west coordinate,
Yy vertical distance, y-coordinate, north-and-south coordinate,
Zz z-coordinate, up-and-down coordinate,

Greek Symbols for Quantities and Units

a-A α **Alpha** angular acceleration, coefficient of linear expansion,
b-B β **Beta** coefficient of volume expansion, lorentz transformation factor,
c-X χ **Chi**
d- $\Delta\delta$ **Delta** Δ =Change in a variable,
e-E ϵ **Epsilon** ϵ_0 = permittivity of free space,
f- $\Phi\phi$ (j) **Phi** Magnetic Flux, angle,
g- $\Gamma\gamma$ **Gamma** surface tension = F / L , $1 / \gamma$ = Lorentz transformation factor,
h-H η **Eta**
i-I ι **Iota**
k-K κ **Kappa** dielectric constant,
l- $\Lambda\lambda$ **Lambda** wavelength of a wave, rate constant for Radioactive decay $=1/\tau=\ln 2 / \text{half-life}$,
m-M μ **Mu** friction, μ_0 = permeability of free space, micro-,
n-N ν **Nu** alternate symbol for frequency,
o-O $\omicron$ **Omicron**
p- $\Pi\pi$ **Pi** 3.1415926536...,
q- $\Theta\theta$ (J) **Theta** angle between two vectors,
r-P ρ **Rho** density of a solid or liquid, resistivity,
s- $\Sigma\sigma$ **Sigma** Summation, standard deviation,
t-T τ **Tau** torque, time constant for any exponential process; eg $\tau=RC$ or $\tau=L/R$ or $\tau=1/k=1/\lambda$,
u-Y υ **Upsilon**
w- $\Omega\omega$ (v) **Omega** angular speed or angular velocity, Ohms,
x- $\Xi\xi$ **Xi**
y- $\Psi\psi$ **Psi**
z-Z $\zeta\varsigma$ (V) **Zeta**
 (Four Greek letters have alternate lower-case forms. Use the letter in () and change its font to Symbol to get the alternate version of the letter.)

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Values of Trigonometric Functions for 1st Quadrant Angles

(simple, mostly-rational approximations)

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
10°	1/6	65/66	11/65
15°	1/4	28/29	29/108
20°	1/3	16/17	17/47
29°	15 ^{1/2} /8	7/8	15 ^{1/2} /7
30°	1/2	3^{1/2}/2	1/3^{1/2}
37°	3/5	4/5	3/4
42°	2/3	3/4	8/9
45°	2^{1/2}/2	2^{1/2}/2	1
49°	3/4	2/3	9/8
53°	4/5	3/5	4/3
60°	3^{1/2}/2	1/2	3^{1/2}
61°	7/8	15 ^{1/2} /8	7/15 ^{1/2}
70°	16/17	1/3	47/17
75°	28/29	1/4	108/29
80°	65/66	1/6	65/11
90°	1	0	∞

(Memorize the **Bold** rows for future reference.)

Derivatives of Polynomials

For polynomials, with individual terms of the form Ax^n , we define the derivative of each term as

$$\frac{d}{dx}(Ax^n) = nAx^{n-1}$$

To find the derivative of the polynomial, simply add the derivatives for the individual terms:

$$\frac{d}{dx}(3x^2 + 6x - 3) = 6x + 6$$

Integrals of Polynomials

For polynomials, with individual terms of the form Ax^n , we define the indefinite integral of each term as

$$\int (Ax^n) dx = \frac{1}{n+1} Ax^{n+1}$$

To find the indefinite integral of the polynomial, simply add the integrals for the individual terms and the constant of integration, C .

$$\int (6x + 6) dx = [3x^2 + 6x + C]$$

Prefixes

Factor	Prefix	Symbol	Example
10^{18}	exa-	E	38 Es (Age of the Universe in Seconds)
10^{15}	peta-	P	
10^{12}	tera-	T	0.3 TW (Peak power of a 1 ps pulse from a typical Nd-glass laser)
10^9	giga-	G	22 G\$ (Size of Bill & Melissa Gates' Trust)
10^6	mega-	M	6.37 Mm (The radius of the Earth)
10^3	kilo-	k	1 kg (SI unit of mass)
10^{-1}	deci-	d	10 cm
10^{-2}	centi-	c	2.54 cm (=1 in)
10^{-3}	milli-	m	1 mm (The smallest division on a meter stick)
10^{-6}	micro-	μ	
10^{-9}	nano-	n	510 nm (Wavelength of green light)
10^{-12}	pico-	p	1 pg (Typical mass of a DNA sample used in genome studies)
10^{-15}	femto-	f	
10^{-18}	atto-	a	600 as (Time duration of the shortest laser pulses)

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Linear Equivalent Mass

Rotating systems can be handled using the linear forms of the equations of motion. To do so, however, you must use a mass equivalent to the mass of a non-rotating object. We call this the Linear Equivalent Mass (LEM). (See Example I)

For objects that are both rotating and moving linearly, you must include them twice; once as a linearly moving object (using m) and once more as a rotating object (using LEM). (See Example II)

The LEM of a rotating mass is easily defined in terms of its moment of inertia, I .

$$LEM = I/r^2$$

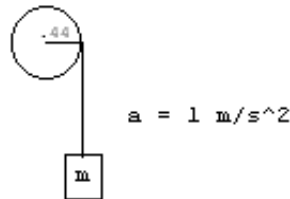
For example, using a standard table of Moments of Inertia, we can calculate the LEM of some standard rotating objects as follows:

	<u>I</u>	<u>LEM</u>
Cylindrical hoop	mr^2	m
Solid disk	$\frac{1}{2}mr^2$	$\frac{1}{2}m$
Hollow sphere	$\frac{2}{5}mr^2$	$\frac{2}{5}m$
Solid sphere	$\frac{2}{5}mr^2$	$\frac{2}{5}m$

Example I

A flywheel, $M = 4.80$ kg and $r = 0.44$ m, is wrapped with a string. A hanging mass, m , is attached to the end of the string.

When the hanging mass is released, it accelerates downward at 1.00 m/s^2 . Find the hanging mass.



To handle this problem using the linear form of Newton's Second Law of Motion, all we have to do is use the LEM of the flywheel. We will assume, here, that it can be treated as a uniform solid disk.

The only external force on this system is the weight of the hanging mass. The mass of the system consists of the hanging mass plus the linear equivalent mass of the fly-wheel. From Newton's 2nd Law we have

$$F_{\text{EXT}} = m_{\text{SYS}}a_{\text{SYS}}, \text{ so, } mg = [(m + (LEM = \frac{1}{2}M))] a_{\text{SYS}}$$

$$mg = [m + \frac{1}{2}M] a_{\text{SYS}}$$

$$(mg - ma_{\text{SYS}}) = \frac{1}{2}M a_{\text{SYS}}$$

$$m(g - a_{\text{SYS}}) = \frac{1}{2}M a_{\text{SYS}}$$

$$m = \frac{1}{2} \cdot M \cdot a_{\text{SYS}} / (g - a_{\text{SYS}})$$

$$m = \frac{1}{2} \cdot 4.8 \cdot 1.00 / (9.80 - 1)$$

$$m = 0.273 \text{ kg}$$

$$\text{If } a_{\text{SYS}} = g/2 = 4.90 \text{ m/s}^2, \quad m = 2.40 \text{ kg}$$

$$\text{If } a_{\text{SYS}} = \frac{3}{4}g = 7.3575 \text{ m/s}^2, \quad m = 7.23 \text{ kg}$$

Note, too, that we do not need to know the radius unless the angular acceleration of the fly-wheel is requested. If you need α , and you have r , then $\alpha = a/r$.

Example II

Find the kinetic energy of a disk, $m = 6.7$ kg, that is moving at 3.2 m/s while rolling without slipping along a flat, horizontal surface.

The total kinetic energy consists of the linear kinetic energy, $\frac{1}{2}mv^2$, plus the rotational kinetic energy, $\frac{1}{2}(\frac{1}{2}m)v^2$.

$$KE = \frac{1}{2}mv^2 + \frac{1}{2} \cdot (LEM = \frac{1}{2}m) \cdot v^2$$

$$KE = \frac{1}{2} \cdot 6.7 \cdot 3.2^2 + \frac{1}{2} \cdot (\frac{1}{2} \cdot 6.7) \cdot 3.2^2$$

$$KE = 34.304 + 17.152 = 51 \text{ J}$$

Final Note:

This method of incorporating rotating objects into the linear equations of motion works in every situation I've tried; even very complex problems. Work your problem the classic way and this way to compare the two. Once you've verified that the LEM method works for a particular type of problem, you can confidently use it for solving other problems of the same type.

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T-Pots

For the functional form

$$\frac{1}{A} = \frac{1}{B} + \frac{1}{C}$$

You may use "The **Product over the Sum**" rule.

$$A = \frac{B \cdot C}{B + C}$$

For the Alternate Functional form

$$\frac{1}{A} = \frac{1}{B} - \frac{1}{C}$$

You may substitute T-Pot-d

$$A = \frac{B \cdot C}{C - B} = -\frac{B \cdot C}{B - C}$$

Three kinds of strain: unit-less ratios

I. Linear: strain = $\Delta L / L$

II. Shear: strain = $\Delta x / L$

III. Volume: strain = $\Delta V / V$