PHYSICS

VECTORS

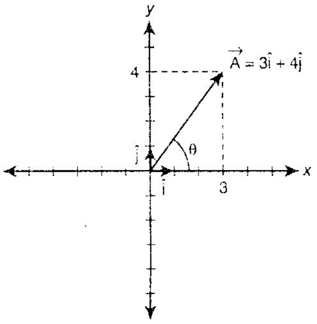
A. VECTORS AND SCALARS

1. Scalar: A quantity that has a magnitude but no direction. Examples: mass, length, time, density, energy, and temperature.
2. Vector: A quantity that has both magnitude and direction. Examples: displacement, velocity, force, acceleration, momentum, electric and magnetic field strength.

B. COMPONENTS OF A VECTOR, 2-DIMENSIONS

A vector in a 2-dimensional Cartesian plane is represented by an x-component and a y-component.

1. Vector components are specified by a scalar, which determines the magnitude of the component, and a unit vector, which determines the direction of the component. The unit vectors $\hat{i}$ and $\hat{j}$ have a magnitude of 1 and point along the x-axis and y-axis respectively.
2. A vector, A, can be represented by: $\mathbf{A} = A_x\hat{i} + A_y\hat{j}$, where A_x and A_y are scalars.
3. The magnitude of a vector is a scalar quantity (length) given by:
   $A = |\mathbf{A}| = \sqrt{A_x^2 + A_y^2}$
4. The direction of the vector (θ) is given by:
   $\tan\theta = \frac{A_y}{A_x}$, or $\theta = \tan^{-1}\frac{A_y}{A_x}$
   Example: $\mathbf{A} = 3\hat{i} + 4\hat{j}$



Magnitude:
$|\mathbf{A}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
Direction:
$\tan\theta = \frac{4}{3}$
$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°$

C. DOT PRODUCT

The dot product of two vectors, a and b, is defined as:
$\mathbf{a} \cdot \mathbf{b} = ab\cos\theta$

- The angle θ is the angle between the vectors.
- The dot product between any two vectors is a scalar quantity. It is sometimes called the scalar product.
- If two vectors are perpendicular to each other, their dot product is zero.

D. CROSS PRODUCT

The magnitude of the cross product of two vectors, a and b, is defined as:
$|\mathbf{a} \times \mathbf{b}| = ab\sin\theta$

- The cross product between vectors **a** and **b** yields a third vector that points perpendicular to the plane in which a and b are oriented.
- To determine the direction of the cross product, use the right-hand rule. Point the fingers of your right hand along vector **a**, and then curl them toward vector **b**. Your thumb now points in the direction of the cross product.
- If two vectors are parallel to each other, their cross product is zero.
- $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$

KINEMATICS

A. ONE-DIMENSIONAL MOTION

Average velocity: $v = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$ (m/s)

Instantaneous velocity: $v(t) = dx/dt$

Average acceleration: $a = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1}$ (m/s²)

Instantaneous acceleration: $a(t) = dv/dt$

Uniformly accelerated motion:

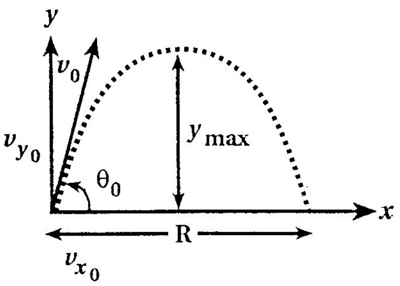
velocity: $v = v_0 + at$; $v^2 = v_0^2 + 2a(x - x_0)$

position: $x = x_0 + v_0t + \frac{1}{2}at^2$

$$x = x_0 + \frac{1}{2}(v_0 + v)t$$

B. MOTION IN A PLANE WITH CONSTANT ACCELERATION

a_x = constant, a_y = constant
$v_x = v_{x_0} + a_xt$, $v_y = v_{y_0} + a_yt$
$v_x^2 = v_{x_0}^2 + 2a_x(x - x_0)$
$v_y^2 = v_{y_0}^2 + 2a_y(y - y_0)$
$r = x\hat{i} + y\hat{j} = r_0 + v_0t + \frac{1}{2}at^2$



C. PROJECTILE MOTION

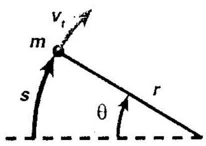
Here, $a_x = 0$, $a_y = -g$
$v_{x_0} = v_0\cos\theta_0$, $v_{y_0} = v_0\sin\theta_0$
$v_x = v_{x_0}$ = constant, $v_y = v_0\sin\theta_0 - gt$
$x = (v_0\cos\theta_0)t$, $y = (v_0\sin\theta_0)t - \frac{1}{2}gt^2 =$
$(\tan\theta_0)x - \frac{g}{2(v_0\cos\theta_0)^2}x^2$

The range: $R = \frac{v_0^2}{g}\sin2\theta_0$

Maximum height: $y_{max} = \frac{v_0^2\sin^2\theta_0}{2g}$

D. UNIFORM CIRCULAR MOTION

Angular displacement: The angle through which a body rotates is given by $\theta = \frac{s}{r}$



Average angular velocity: A body rotates a distance r from the axis of rotation with a tangential velocity v_t. During uniform circular motion, the magnitude of the tangential velocity is constant. The body rotates through an angle θ and the average angular velocity is given by $\omega = \frac{\Delta\theta}{\Delta t} = \frac{v_t}{r}$

Instantaneous angular velocity: $\omega(t) = \frac{d\theta}{dt}$

Average angular acceleration: $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{a_t}{r}$

Instantaneous angular acceleration:
$a(t) = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

Centripetal acceleration: $a_c = \frac{v_t^2}{r}$ (directed toward the axis of rotation)

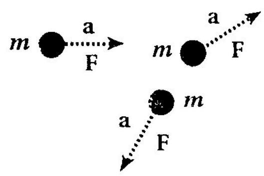
Period: $T = \frac{2\pi}{\omega} = \frac{2\pi r}{v_t}$

Rotational motion with constant acceleration:
$\omega = \omega_0 + \alpha t$
$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$\theta = \theta_0 + \omega_0t + \frac{1}{2}\alpha t^2$
$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$

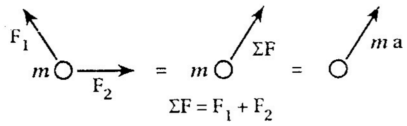
DYNAMICS

A. NEWTON'S LAWS

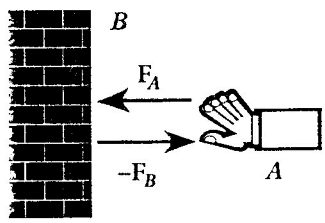
Second law: $F_{net} = ma$



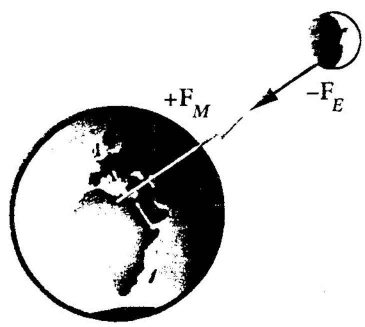
The acceleration is in the direction of the applied force.
Newton's second law also holds for several applied forces.



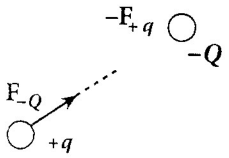
Third law: To every action, there is always an equal and opposite reaction: If body A exerts a force F_A on body B, then body B exerts an equal and opposite force $-F_B$ on body A. Hence, $F_B = -F_A$ or $F_A = -F_B$.



A wall pushes you with the same force with which you push it, but in the opposite direction.



The gravitational force that the Earth exerts on the moon is equal and opposite to the force that the moon exerts on the Earth.



The electrostatic force that the $+q$ charge exerts on the $-Q$ charge is equal and opposite to the force that the $-Q$ charge exerts on the $+q$ charge.

B. SYSTEMS OF PARTICLES

Center of mass in vector notation:

$r_{c.m.} = \frac{1}{M}\sum_{i=1}^{n} m_i r_i = 1$, where $M = \sum m_i$ is the total mass.

Newton's second law for a system of (n) particles: $F_{ext} = Ma_{c.m.}$

C. WEIGHT, NORMAL FORCE, AND FRICTION

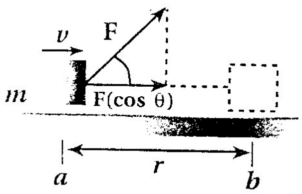
1. **Weight:** The gravitational force exerted on a body of mass m by the Earth. $W = mg$.
2. **Normal Force:** A force, N, exerted on a body by another surface. This force acts perpendicular to the body. When a body is at rest on a level surface, $N = mg$.
3. **Friction:** A force between two surfaces that opposes the motion of a body.
   - **Static friction:** Frictional force (F_s) exerted when a body is at rest relative to another surface. The maximum force of static friction is: $F_s = u_s N$, where u_s is the coefficient of static friction between the two surfaces.
   - **Kinetic friction:** Friction force (F_k) exerted when a body slides across another surface. $F_k = u_k N$, where u_k is the coefficient of kinetic friction between the two surfaces.
   - Between any two surfaces, $u_k < u_s$

D. WORK AND ENERGY

Work done by a force:

$W = \int_a^b F \cdot dr = \int_a^b F\cos\theta\, dr$

(Joules), where θ = angle that force makes with direction of motion.



Work done on a block, mass m.

Instantaneous power: $P = dW/dt = F \cdot v$ (Watts)

Kinetic energy: $K = \frac{1}{2}mv^2$ (Joules)

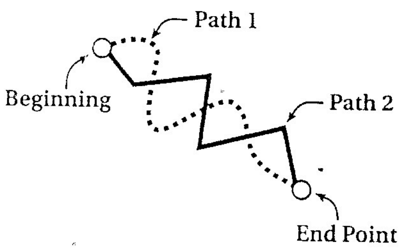
E. WORK-ENERGY THEOREM

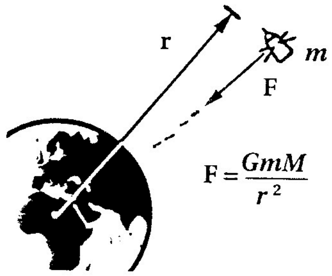
The work done by the resultant external force on a particle (or a system) is equal to the change in kinetic energy of the particle (or system).

$W = K_2 - K_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

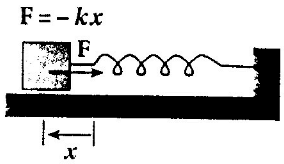
F. CONSERVATION OF ENERGY

1. Conservative force: A force such that the work it does on a particle is independent of the path over which the force is exerted.

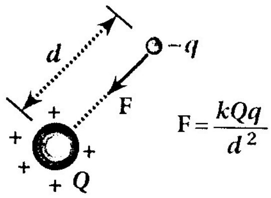




The gravitational force that the Earth exerts on a satellite is an example of a conservative force.



The elastic force F that an ideal spring exerts on an attached mass is an example of a conservative force.



The elastic force that is exerted on the $-q$ charge by the $+Q$ charge is an example of a conservative force.

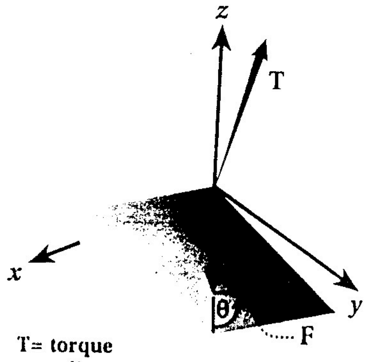
2. **Potential energy:**
   $\Delta U = -W$,
   where W is work done by a conservative force.
3. **Conservation of mechanical energy:** The total mechanical energy of a particle subject to a conservative force is constant.
   $E = K + U = \frac{1}{2}mv^2 + U = \text{constant}$.
4. **Non-conservative forces:** The work done by a non-conservative force (e.g., friction) is equal to the change in internal energy: $W_{fr} = \Delta E = -\Delta U_{int}$, and the sum of the system's mechanical and internal energy remains constant: $\Delta E + \Delta U_{int} = \Delta K + \Delta U + \Delta U_{int} = 0$.

G. LINEAR MOMENTUM AND COLLISIONS

1. **Linear momentum of a particle:** $p = mv$; Newton's Second Law can be rewritten $F = dp/dt$
2. **Total linear momentum of a system:**
   $p = \sum_i P_i = \sum_i m_i v_i$
3. **Conservation of momentum:** If the net external force on a system is zero, momentum is conserved in the system. If $F_{ext} = 0$, then p = constant.
4. **Impulse:** The force applied to a body over an interval of time. When the force is constant, $J = F\Delta t = \Delta p$
5. **Collisions:** Mass m_1 traveling at v_{1i} collides with mass m_2 traveling at v_{2i}. After the collision, m_1 travels at v_{1f} and m_2 travels at v_{2f}. If $F_{ext} = 0$, linear momentum is conserved.
   $m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$
   - During an **elastic collision**, both momentum and kinetic energy are conserved. This generally occurs when bodies collide and bounce away in different directions. Conservation of kinetic energy states
     $\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$
   - During an **inelastic collision**, kinetic energy is not conserved. This generally occurs when bodies collide and stick together.

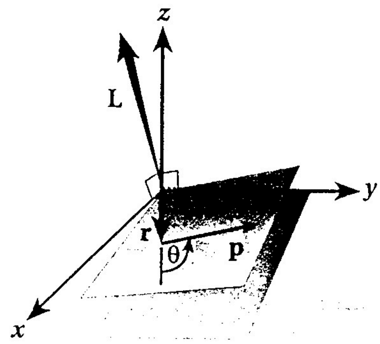
H. ROTATIONAL DYNAMICS

Torque causes a body to rotate. It is de... by $\tau = r \times F$ (vector form) with magnit... $\tau = rF\sin\theta$.



T= torque
r = radius vector
F = applied force
θ = the angle formed by r and F

Angular momentum: $L = r \times p$ (vector form) and magnitude $L = rp\sin\theta$.
L = angular momentum
r = radius vector
p = linear momentum
θ = angle formed by r and p



Relation between torque and angular momentum:

$\tau = \frac{dL}{dt}$

Kinetic energy of rotation: $\frac{1}{2}(\sum m_i r_i^2)\omega^2$

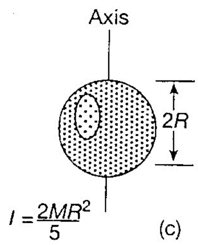
Moment of inertia: The rotational analog of mass.

$I = \sum_i m_i r_i^2$

Angular momentum and torque can also be written $L = I\omega$ and $\tau = I\alpha$

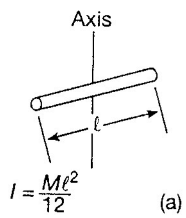
Common moment of inertia quantities:

Solid sphere



$\frac{2}{5}MR^2$

Thin rod of length ℓ rotating about its center.



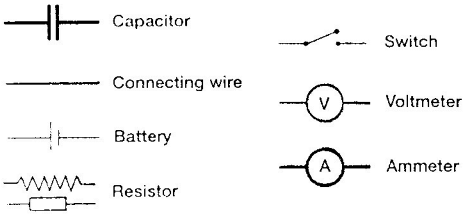
$\frac{1}{12}M\ell^2$

Electric Potential of a point charge: $V = \frac{kq}{r}$

Electric Potential of a continuous charge distribution: $V = k\int\frac{dq}{r}$

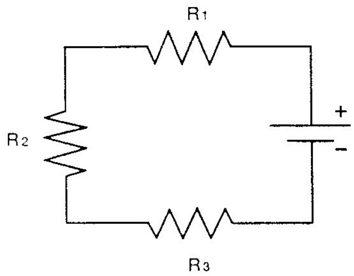
E. CURRENT AND CIRCUITS

1. **Current:** The rate at which charge, Q, flows past an area in a circuit.
   $I = \frac{dQ}{dt}$
2. **Resistance:** Resistors decrease the rate at which charge flows through a circuit. The resistivity, ρ, of a conductor is a property of the material. For a cylindrical conductor with length L, cross-sectional area A, and resistivity ρ, $R = \rho\frac{L}{A}$
3. **Ohm's Law:** When a resistor in a circuit is constant over a range of voltage inputs, the resistor is Ohmic. For a circuit with resistance R, current I, and potential difference ΔV, Ohm's Law states $\Delta V = IR$
4. **Circuit:** A closed path around which charge flows. Circuit elements are

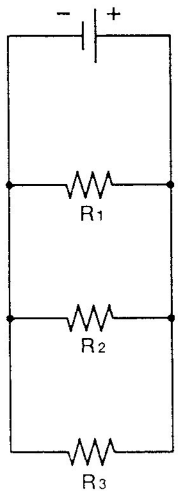


An ammeter measures current, and a voltmeter measures voltage drop.

Resistors in series: $R_{eq} = R_1 + R_2 + R_3$



Resistors in parallel: $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$



Power: $P = I\Delta V = I^2R = \frac{V^2}{R}$

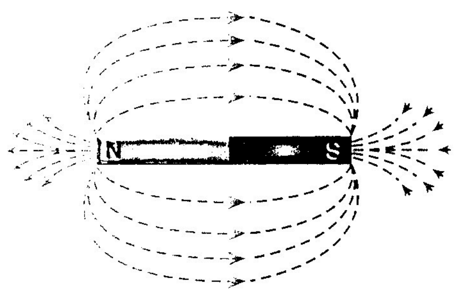
5. **Kirchhoff's rules:**
   Loop Theorem: For a complete circuit loop, $\Delta V_1 + \Delta V_2 + \Delta V_3 + \ldots = 0$
   Junction Rule: The sum of the current entering a junction equals the sum of the current exiting the junction.

6. **Capacitor:** A capacitor is a set of two oppositely charged conductors (each with charge Q), separated by an insulating material. If the potential difference between the two plates is ΔV, then
   $C = \frac{Q}{\Delta V}$
   Parallel Plate Capacitor: Two charged plates with surface area A, separated by a distance d.
   $C = \frac{\varepsilon_0 A}{d}$

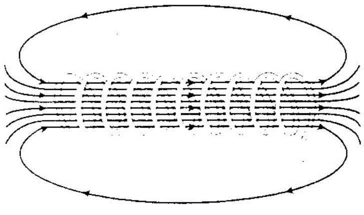
MAGNETISM AND INDUCTION

A. MAGNETIC FIELDS

1. **Bar magnets:** The magnetic field lines of a bar magnet exit the north pole and enter the south pole. Like poles repel each other, and unlike poles attract each other.



2. **Magnetic force on a moving charge:** A particle with charge q moving at speed v through a magnetic field B will experience a force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ with magnitude $F = qvB\sin\theta$. To determine the direction of the force on the particle, use the right-hand rule. The fingers point in the direction of **v** and curl toward the direction of **B**. The thumb will point in the direction of **F**.
3. **Magnetic force on a current-carrying wire:** A wire of length and direction ℓ and current I traveling through a magnetic field will experience a force $\mathbf{F} = I\ell \times \mathbf{B}$ with magnitude $F = I\ell B\sin\theta$.
4. **Magnetic field due to a current-carrying wire:** The strength of the magnetic field a distance d from a long wire with current I is $B = \frac{\mu_0 I}{2\pi d}$. To determine the direction of the magnetic field, use the right-hand rule. The thumb points in the direction of the current, and the fingers will curl around the wire in the direction of magnetic field.
5. **Magnetic field due to a solenoid:** The magnetic field due to a solenoid is concentrated almost entirely along a straight line at the center of the loops. The strength of the magnetic field B due to a solenoid with n loops and current I is $B = \mu_0 nI$.



6. **Biot-Savart law:** The magnetic field B a distance $r\hat{r}$ from a piece of wire $d\ell$ carrying current I is given by $d\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{(d\ell \times \hat{\mathbf{r}})}{r^2}$
7. **Ampere's Law:** The magnetic field B produced by a current-carrying wire of length $d\ell$ is related to the tc
   $\oint \mathbf{B} \cdot d\ell = \mu_0 I$

B. MAGNETIC INDUCTION

1. **Magnetic flux:** The amount field that travels perpendi an area A.
   $\phi = \int \mathbf{B} \cdot d\mathbf{A}$
2. **Faraday's Law:** A varying magnetic through a coil of wire will induce an ele tromotive force (ε) in the wire.
   $\varepsilon = -N\frac{d\phi}{dt}$

C. MAXWELL'S EQUATIONS

1. Gauss's Law for electricity:
   $\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{enclosed}}{\varepsilon_0}$
2. Gauss's Law for magnetism:
   $\oint \mathbf{B} \cdot d\mathbf{A} = 0$
3. Faraday's Law of induction:
   $\oint \mathbf{E} \cdot ds = -\frac{d\phi}{dt}$
4. Ampere-Maxwell Law:
   $\oint \mathbf{B} \cdot ds = \mu_0 I + \mu_0\varepsilon_0\frac{d}{dt}\int \mathbf{E} \cdot d\mathbf{A}$

THERMODYNAMICS

A. HEAT

1. **Specific heat:** The amount of heat, Q, required to change the temperature of a substance of mass m by an amount ΔT is given by $Q = mc\Delta T$ (c is the specific heat of the substance).
2. **Latent heat:** The heat that does work to cause a phase change is latent heat (L), and it is related to the heat added or removed in the system (Q) by $Q = mL$.
3. **Thermal conduction:** The rate at which heat flows ($\Delta Q/\Delta t$) through an area A over a distance d is given by $\frac{\Delta Q}{\Delta t} = \frac{kA\Delta T}{d}$ (where ΔT is the temperature difference over the distance d).
4. **Thermal radiation:** The rate at which a body with emissivity e and surface area A radiates energy (P) is $P = \sigma AeT^4$

B. LAWS OF THERMODYNAMICS

First Law of Thermodynamics: The net heat added to a system (Q) equals the change in internal energy of the system (ΔU) plus the work done on the system (W). $Q = \Delta U + W$

Second Law of Thermodynamics:

1. Heat does not flow spontaneously from a colder body to a warmer body.
2. Heat energy is never completely transformed into mechanical work.
3. The total entropy in the universe increases during all natural processes.

Entropy: The change in entropy (ΔS) is related to the amount of heat added or removed from a system (Q) at constant temperature (T).

$\Delta S = \frac{Q}{T}$

Thermal efficiency: $\varepsilon = \frac{\text{work in}}{\text{heat out}} = \frac{W_{out}}{Q_{in}} = 1 - \frac{Q_{cold}}{Q_{hot}}$

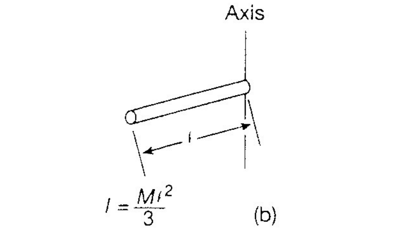
Carnot efficiency: $\varepsilon_C = 1 - \frac{T_{cold}}{T_{hot}}$

C. GASES

Gases are described by their pressure p, volume V, and temperature T.

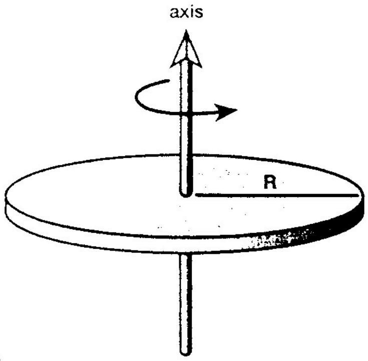
1. **Boyle's Law:** $P_1V_1 = P_2V_2$
2. **Charles's Law:** $\frac{V_1}{T_1} = \frac{V_2}{T_2}$
3. **Ideal gas law:** For a gas containing n number of moles, $PV = nRT$ (where R is the universal gas constant).
4. **Kinetic energy per molecule of an ideal gas:** $K = \frac{1}{2}mv^2 = \frac{3}{2}RT$
5. **Total internal energy of a gas:** $U = \frac{3}{2}nRT$

Thin rod of length ℓ rotating about one end.



$\frac{1}{3}M\ell^2$

Solid disk rotating about an axis perpendicular to its surface.



$\frac{1}{2}MR^2$

Rotational kinetic energy: $\frac{1}{2}I\omega^2$

Conservation of angular momentum: When no external torque is acting on a system, angular momentum is conserved. If $\tau_{ext} = 0$, L is constant.

Work: $W = \tau\theta$

Power: $P = \tau\omega$

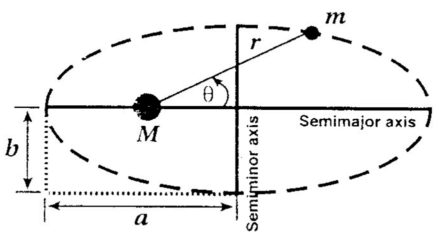
I. GRAVITATION

Newton's law: $F = \frac{GMm}{r^2}$ (r from center-of-mass of M)

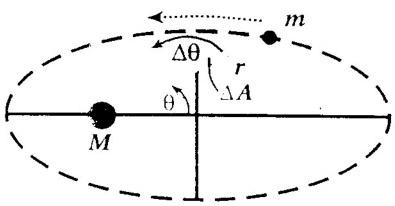
Potential energy: $U = -\frac{GMm}{r}$

Kepler's laws:

1. All planets move in elliptical orbits with the sun as one focus.



2. A line joining any planet to the sun sweeps out equal areas in equal times: $dA/dt = \frac{1}{2}r^2\omega = L/2m$ = constant (L = angular momentum).



3. Law of periods: $\tau^2 = \frac{4\pi^2}{GM}a^3$ (τ = period; a = semi major axis) for $M >> m$.

SIMPLE HARMONIC MOTION

A. SPRING WITH A MASS

The force on a mass in simple harmonic motion is always directed opposite to the direction in which the mass is moving.

Force: $F = -kx$ (k = spring constant; x = distance from equilibrium position)

Frequency: The number of oscillations the spring completes in a period of time.

$f = \frac{1}{\tau} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Angular frequency: The number of radians the spring oscillates through in a period of time.

$\omega = 2\pi f = \sqrt{\frac{k}{m}}$

Period: The time it takes for the spring to complete one oscillation.

$\tau = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Equation of motion: If the mass is initially compressed a distance A and released from rest, the subsequent equation of motion is given by: $x(t) = A\cos(\omega t)$. In this case, $x_{max} = A$, where A is the amplitude of oscillation.

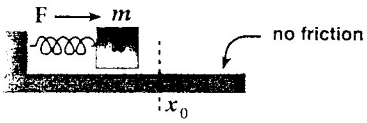
Velocity: $x(t) = \frac{dx}{dt} = -\omega A\sin(\omega t)$

where $v_{max} = \omega A$

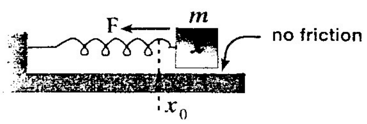
Acceleration: $a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t) = -\omega^2 x = -\frac{k}{m}xw$ where $a_{max} = \omega^2 A$

Potential energy: $U = \frac{1}{2}kx^2$

Total energy: $E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$



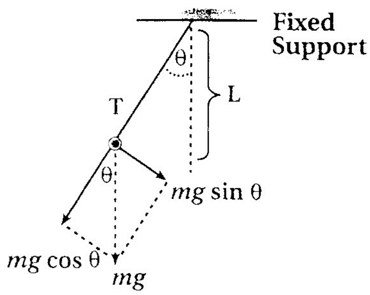
x_0 = block's center at rest



Oscillation of a spring with mass m

B. PENDULUMS

1. Simple pendulum: $F = -mg\sin\theta$; for small θ, $\sin\theta \approx \theta$. Thus, $F = -mg\theta = mL\theta$, $\omega^2 = g/L$ and $\tau = 2\pi\sqrt{L/g}$ is the period.
   - point mass (m)
   - inextensible, weightless string



Simple pendulum in a plane

SOUND WAVES

Speed of sound waves (in an ideal gas):

$c, v = \sqrt{\gamma\frac{P_0}{\rho_0}}$, $\gamma = \frac{c_p}{c_v}$ = ratio of specific heats,

P_0 = pressure, ρ_0 = density.

Relation to frequency and wavelength:

$V = f\lambda$

Pressure wave: $p = p_0\sin(kx - \omega t - 90°)$,

$k = 2\pi/\lambda$, $\omega = 2\pi f = 2\pi/\tau$

Standing waves: Examples are an organ pipe open at both ends or a string fixed at both ends.

$f_n = \frac{n}{2L}v$ (L = length of pipe or string)

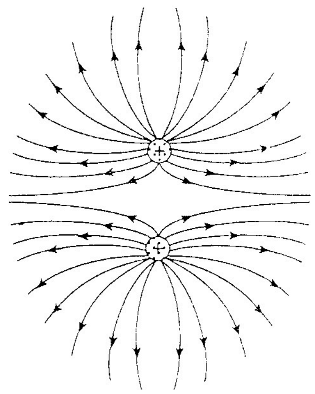
Doppler effect: $f' = f\left(\frac{v \pm v_0}{v \mp v_s}\right)$; f' and f are the observed and actual frequencies, v the speed of sound, v_o the speed of the observer, and v_s the speed of source.

ELECTRICITY

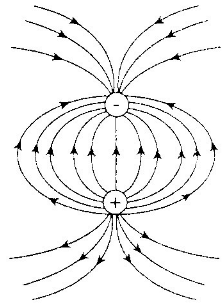
A. ELECTRIC CHARGE

Electric charge is a fundamental property of nature. The charge on a particle can be positive or negative.

Two particles with like charges repel each other.



Two particles with unlike charges attract each other.



Coulomb's Law: Two particles of charge q_1 and q_2 a distance r away from each other will exert a force on each other.

$F = k\frac{q_1 q_2}{r^2}$

The force that q_1 exerts on q_2 is equal and opposite to the force that q_2 exerts on q_1.

B. ELECTRIC FIELD

All electrically charged particles create an electric field. This is defined as the force a test charge q_0 would experience if it were placed in a particular location near the charged particle.

$E = \frac{F}{q_0}$

Electric field due to a point charge: The strength of the electric field produced by a point charge, q, at a distance r from the charge is given by

$E = k\frac{q}{r^2}$

C. GAUSS'S LAW

Electric Flux: The amount of the electric field that passes perpendicularly through an area A.

$\phi = \int \mathrm{E}\cdot d\mathrm{A}$

Gauss's Law: The total electric flux through a closed surface equals the amount of charge enclosed in that surface divided by the permittivity constant, ε_0.

$\int \mathrm{E}\cdot d\mathrm{A} = \frac{Q_{enclosed}}{\varepsilon_0}$

D. ELECTRIC POTENTIAL

The electric potential, also known as voltage, is defined as the electric potential energy per unit charge. The potential difference between two points, A and B, in an electric field is defined as the work done by an external force to move a test charge, q_0, from point A to point B divided by the test charge.

$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0}$

When work is done by the electric field in moving q_0 from point A to point B, the potential difference is given by $\Delta V = -\frac{W_{AB}}{q_0}$

Potential difference in a uniform electric field: $\Delta V = V_B - V_A = Ed$ (where d is the distance between points A and B)

Electric Potential Energy of a point charge: The potential energy of a test charge q_0 in an electric field produced by the charge q is

$U = \frac{kqq_0}{r}$

PHYSICS EQUATIONS & ANSWERS

PHYSICAL CONSTANTS

Acceleration due to gravity	g	9.81 m/s
Gravitational constant	G	$6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$
Coulomb's constant	k	$9 \times 10^9 \frac{N \cdot m^2}{C^2}$
Speed of light	c	3.0×10^8 m/s
Charge of electron	e	-1.60×10^{-19} C
Planck's constant	h	6.63×10^{-34} J·s
Permittivity of free space	ε_0	$8.85 \times 10^{-12} \frac{C}{V \cdot m}$
Permeability of free space	μ_0	$1.26 \times 10^{-6} \frac{m \cdot kg}{s^2 \cdot A^2}$
Mass of Earth	m_E	5.97×10^{24} kg
Radius of Earth	r_E	6.37×10^6 m
Mass of sun	m_{sun}	1.99×10^{30} kg

VECTORS

A. VECTORS AND SCALARS

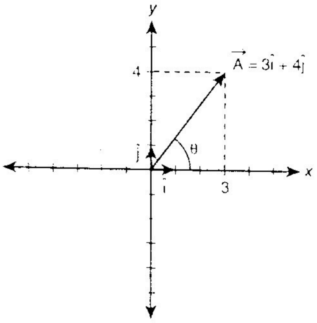
1. **Scalar:** A quantity that has a magnitude but no direction. Examples: mass, length, time, density, energy, and temperature.
2. **Vector:** A quantity that has both magnitude and direction. Examples: displacement, velocity, force, acceleration, momentum, electric and magnetic field strength.

3. COMPONENTS OF A VECTOR, 2-DIMENSIONS – A vector in a

2-dimensional Cartesian plane is represented by an x-component and a y-component.

1. **Vector components** are specified by a scalar, which determines the magnitude of the component, and a unit vector, which determines the direction of the component. The **unit vectors** $\hat{i}$ and $\hat{j}$ have a magnitude of 1 and point along the x-axis and y-axis, respectively.
2. A vector, A, can be represented by: $A = A_x\hat{i} + A_y\hat{j}$, where A_x and A_y are scalars.
3. The **magnitude** of a vector is a scalar quantity (length) given by:
   $A = |A| = \sqrt{A_x^2 + A_y^2}$
4. The direction of the vector (θ) is given by:
   $\tan\theta = \frac{A_y}{A_x}$, or $\theta = \tan^{-1}\frac{A_y}{A_x}$

Example: $A = 3\hat{i} + 4\hat{j}$



Magnitude:
$|A| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$
Direction:
$\tan\theta = \frac{4}{3}$
$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53°$

C. DOT PRODUCT – The dot product of two vectors, a and b, is defined as:

$a \cdot b = ab\cos\theta$

- The angle θ is the angle between the vectors.
- The dot product between any two vectors is a scalar quantity. It is sometimes called the **scalar product.**
- If two vectors are perpendicular to each other, their dot product is zero.

D. CROSS PRODUCT – The magnitude of the cross product of two vectors, a and b, is defined as:

$|a \times b| = ab\sin\theta$

- The cross product between vectors **a** and **b** yields a third vector that points perpendicular to the plane in which **a** and **b** are oriented.
- To determine the direction of the cross product, use the **right-hand rule.** Point the fingers of your right hand along vector **a**, and then curl them toward vector **b**. Your thumb now points in the direction of the cross product.
- If two vectors are parallel to each other, their cross product is zero.
- $a \times b = -b \times a$

KINEMATICS

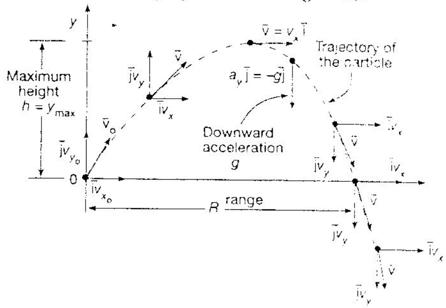
A. ONE-DIMENSIONAL MOTION

1. **Displacement:** The distance between the initial and final locations of a body in motion: $\Delta x = x - x_0$
   - Units: meter (m)
   - Displacement is a vector quantity.
   - Note: The total distance an object travels may not be the same as its displacement. If an object travels one full cycle around a 100–m circular track, the total distance traveled is 100 meters, but the displacement is zero.
2. **Velocity:** The rate at which an object changes position.
   Average velocity: $v = \frac{\Delta x}{\Delta t}$
   Instantaneous velocity: $v(t) = \frac{dx}{dt}$
   - Units: m/s
   - Velocity is a vector quantity. It is represented by a body's **speed** *in a particular direction.*

   Note: If the velocity of a body is constant over a period of time, the instantaneous velocity of the body equals the average velocity at every moment over the period.
3. **Acceleration:** The rate at which velocity changes.
   Average acceleration: $a = \frac{\Delta v}{\Delta t}$
   Instantaneous acceleration: $a(t) = \frac{dv}{dt}$
   - Units: m/s²
4. **Kinematic Equations:** Relate displacement, velocity, and acceleration.
   $\Delta x = v_0 t + \frac{1}{2}at^2$
   $\Delta x = \left(\frac{v + v_0}{2}\right)t$
   $v = v_0 + at$
   $v^2 = v_0^2 + 2a\Delta x$
   - **Only applicable when acceleration is constant.**

   Example: An object accelerates at a constant rate from rest to 12 m/s over a distance of 36 m. What was its acceleration?
   $v^2 = v_0^2 + 2a\Delta x \rightarrow$
   $a = \frac{v^2 - v_0^2}{2\Delta x} = \frac{(12\text{ m/s})^2 - 0}{2(36\text{ m})}$
   $a = 2\text{ m/s}^2$

B. TWO-DIMENSIONAL PROJECTILE MOTION – A projectile is a body whose motion is affected only by the force of gravity.



- Ignoring air resistance, a projectile accelerates toward Earth at a constant rate of g, which is the acceleration due to gravity.
- The x- and y-components of motion are independent of each other.

If a projectile is launched with a speed v_0 at an angle θ above the horizontal, the motion is described as follows:

Horizontal motion: Constant velocity
$a_x = 0$
$v_{x0} = v_0\cos\theta =$ constant
$\Delta x = v_{x0}t = v_0 t\cos\theta$

Vertical motion: Accelerates at a rate of g
$a_y = -g$
$v_{y0} = v_0\sin\theta$
$\Delta y = (v_0\sin\theta)t - \frac{1}{2}gt^2$

Range: The total horizontal distance traveled:
$R = \frac{v_0^2}{g}\sin 2\theta$

Maximum height: At this point, $v_y = 0$:
$y_{max} = \frac{v_0^2\sin^2\theta}{2g}$

Example: An object is launched with an initial speed of 20 m/s at a 25° angle above the horizontal. How far away is the object from its initial position after 0.5 seconds?

1. Find the x- and y-positions of the object:
   x-position:
   $\Delta x = v_{x0}t = v_0\cos\theta t$
   $\Delta x = (20\text{ m/s})(\cos 25°)(0.5\text{ s})$
   $\Delta x = 9\text{ m}$
   y-position:
   $\Delta y = (v_0\sin\theta)t - \frac{1}{2}gt^2$
   $\Delta y = (20\text{ m/s})(\sin 25°)(0.5\text{ s}) - \frac{1}{2}(9.8\text{ m/s}^2)(0.5\text{s})^2$
   $\Delta y = 3\text{ m}$
2. The distance r of the object from the origin is described by the vector:
   $r = 9\hat{i} + 3\hat{j}$
   Find the magnitude of this vector:
   $|r| = \sqrt{9^2 + 3^2} = \sqrt{90}$
   $|r| = 9.5\text{ m}$

DYNAMICS

A. NEWTON'S LAWS

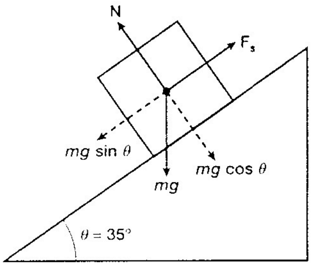
First Law: An object in motion will stay in motion with constant velocity unless acted on by an external force. (Or, an object at rest will stay at rest unless acted on by an external force.)
Second law: $F_{net} = ma$
Third law: To every action there is always an equal and opposite reaction: $F_B = -F_A$ or $F_A = -F_B$.

B. FORCES AND FREE BODY DIAGRAMS –

A force is a push or pull on an object. Newton's 2nd Law states that when an unbalanced force, or **net force**, acts on a body, it causes an acceleration.

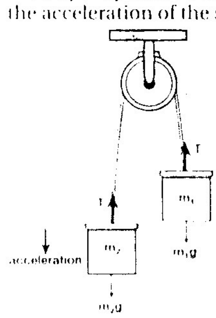
- Units: Newton: $N = \frac{kg \cdot m}{s^2}$
- Net force is a vector that acts in the same direction as the acceleration of a body.

1. **Weight:** The gravitational force exerted on a body of mass m by Earth. $W = mg$.
2. **Components of weight:** The weight of a body on an inclined plane that makes an angle θ with the horizontal is resolved into parallel and perpendicular components along the plane:
   $F_{\perp} = mg\cos\theta$
   $F_{\parallel} = mg\sin\theta$
3. **Normal Force:** A force, N, exerted on a body by another surface. This force acts perpendicular to the interface between the body and the surface. When a body is at rest on a level surface, $N = mg$.
4. **Friction:** A force between two surfaces that opposes the motion of a body.
   a. **Static friction:** Frictional force (F_s) exerted when a body is at rest relative to another surface. The maximum force of static friction is $F_s = \mu_s N$, where μ_s is the coefficient of static friction between the two surfaces.
   Example: A box with mass 15 kg is at rest on a 35° incline. What is the coefficient of static friction between the box and the incline?



   1. Write Newton's 2nd Law for the forces that act perpendicular and parallel to the incline. Since the box is at rest, $a = 0$:
      Perpendicular:
      $F_{net} = ma = 0 = N - mg\cos\theta$
      $N = mg\cos\theta$
      Parallel:
      $F_{net} = ma = 0 = F_s - mg\sin\theta$
      $F_s = mg\sin\theta$
   2. Substitute the definition of static friction into the parallel equation. Then substitute the expression for normal force and solve:
      $\mu_s N = mg\sin\theta$
      $\mu_s = \frac{mg\sin\theta}{N}$
      $\mu_s = \frac{mg\sin\theta}{mg\cos\theta}$
      $\mu_s = \tan\theta = \tan(35°)$
      $\mu_s = 0.70$

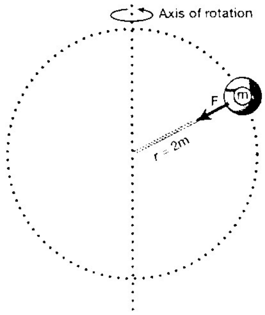
   b. **Kinetic friction:** Friction force (F_k) exerted when a body slides across another surface. $F_k = \mu_k N$, where μ_k is the coefficient of kinetic friction between the two surfaces.
5. **Tension:** The force that a string or rope exerts on an object.
   Example: The Atwood Machine. Two masses, $m_1 = 10$ kg and $m_2 = 20$ kg, are suspended from a pulley and released from rest. What is the acceleration of the system?



   1. Note that m_2 will accelerate downward, and m_1 will accelerate upward. Assuming the rope does not stretch, the acceleration of each mass and the tension on each mass are the same. Write Newton's 2nd Law for each mass:
      m_1: $F_{net} = m_1a = T - m_1g$
      m_2: $F_{net} = m_2a = m_2g - T$
   2. Solve each equation for T and set them equal to each other:
      $T = m_1a + m_1g$
      $T = m_2g - m_2a$
      $m_1a + m_1g = m_2g - m_2a$
   3. Solve for a:
      $m_1a + m_2a = m_2g - m_1g$
      $a(m_1 + m_2) = g(m_2 - m_1)$
      $a = \frac{g(m_2 - m_1)}{(m_1 + m_2)}$
      $a = \frac{(9.8\ m/s^2)(20\ kg - 10\ kg)}{(20\ kg + 10\ kg)}$
      $a = 3.3\ m/s^2$

6. **Centripetal Force:** The force that causes a body to move in a circle. For a body of mass m traveling with linear speed v a distance r from the axis of rotation:
   $F_c = m\frac{v^2}{r}$
   - Centripetal force is directed inward, toward the axis of rotation.
   - Centripetal force is an example of $F = ma$, where a equals $\frac{v^2}{r}$. This acceleration is called **centripetal acceleration**, and it is also directed toward the axis of rotation.

   Example: A person swings a 2-m rope with a 0.25-kg ball attached to the end. The person exerts a force of 5 N on the rope as she swings the ball in a circle. What is the speed of the ball?

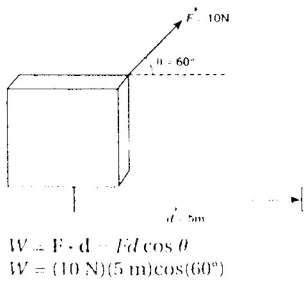


   1. Write Newton's 2nd Law for the ball:
      $F_{net} = ma = m\frac{v^2}{r}$
   2. Solve for v:
      $F_{net} = m\frac{v^2}{r} \rightarrow$
      $v = \sqrt{\frac{(F_{net})r}{m}} = \sqrt{\frac{(5\ N)(2\ m)}{(0.25\ kg)}}$

C. WORK AND ENERGY

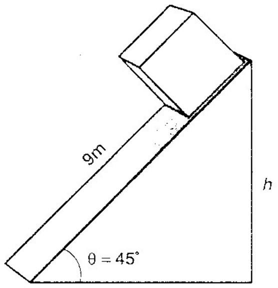
1. **Work done by a force:** When a force moves an object over some distance, d, the force does **work** on the object:
   $W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta$
   - Units: Joules (J): $\frac{kg \cdot m^2}{s^2} = N \cdot m$

   Example: A 10-N force is exerted on a box at a 60° angle above the horizontal. The box slides a horizontal distance of 5 m. Calculate the work done by the force.



   $W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\theta$
   $W = (10\ N)(5\ m)\cos(60°)$
   $W = 25\ N \cdot m = 25\ J$

2. **Conservative force:** A force such t[...] work it does on a particle is independ[...] the path over which the force is exerte[...]
   Examples: gravity, springs, electric force
3. **Work-Energy Theorem:**
   a. The work done by the resultant external force on a particle (or a system) is equal to the change in **kinetic energy** of the particle (system):
      $W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$
   b. The work done by a conservative force on an object is equal and opposite to the change in **potential energy** of the object:
      $W = -\Delta U$
      $U(x) = -\int F \cdot dx$
      - Gravitational potential energy:
        $U_g = mgh$
4. **Conservation of mechanical energy:** The total mechanical energy of a particle subject to a conservative force is constant:
   $K_i + U_i = K_f + U_f = E_{total} = \text{constant}$
   Example: A box slides from rest down a ramp 9 m long with a 45° angle. Assuming there is no friction between the box and ramp, how fast is the box traveling at the bottom of the ramp?



   1. Calculate the height of the ramp:
      $\sin\theta = \frac{h}{\ell} \rightarrow$
      $h = \ell\sin\theta = (9\ m)\sin(45°)$
      $h = 6.4\ m$
   2. Initially, the mass has only gravitational potential energy. At the bottom of the ramp, it has only kinetic energy. Apply conservation of energy:
      $E_i = E_f$
      $mgh = \frac{1}{2}mv_f^2 \rightarrow$
      $v_f = \sqrt{2gh} = \sqrt{2(9.8\ m/s^2)(6.4\ m)}$
      $v_f = 11\ m/s$
5. **Non-conservative forces:** When non-conservative forces (such as friction or applied forces) do work on an object (W_{other}):
   $\Delta E = W_{other}$
   $K_i + U_i + W_{other} = K_f + U_f$
6. **Power:** The rate of change of energy over time:
   1. **Average power:**
      $P = \frac{\Delta E}{\Delta t} = \frac{\Delta W}{\Delta t}$
   2. **Instantaneous power:**
      $P(t) = \frac{dW}{dt} = F(t) \cdot v(t)$
   - Units: Watts (W): J/s

D. LINEAR MOMENTUM AND COLLISIONS

1. **Linear momentum of a particle:**
   $p = mv$
   - Newton's Second Law can be rewritten:
     $F = ma = \frac{dp}{dt}$

   Units: kg · m/s
2. **Total linear momentum of a system:**
   $p = \sum_i P_i = \sum_i m_i v_i$
3. **Conservation of momentum:** If the net external force on a system is zero, momentum is conserved in the system: If $F_{net} = 0$, then p = constant.
4. **Collisions:**
   Mass m_1 traveling at v_{1i} collides with mass m_2 traveling at v_{2i}. After the collision, m_1 travels at v_{1f} and m_2 travels at v_{2f}. If $F_{net} = 0$, linear momentum is conserved:
   $m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$

a. During an **elastic collision**, both momentum and kinetic energy are conserved. This generally occurs when bodies collide and bounce away in different directions.
b. During an **inelastic collision**, kinetic energy is not conserved. This generally occurs when bodies collide and stick together.

5. **Impulse:** The force applied to a body over an interval of time. When the force is constant: $J = F\Delta t = \Delta p$

E. GRAVITATION

Universal Law of Gravitation: Two massive bodies, m_1 and m_2, exert attractive gravitational forces on one another. This attraction is inversely proportional to the square of the distance r between them.

$$F_g = \frac{Gm_1m_2}{r^2}$$

- G is called the gravitational constant and is equal to
  $6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}\left(\text{or } \frac{m^3}{kg \cdot s^2}\right)$
- The distance r is measured from the center of mass of each body.

Acceleration due to gravity (*also called "gravitational field"*): The acceleration that any body experiences due to the gravitational pull of another body, M, is given by:

$$g = \frac{GM}{r^2}$$

Potential energy: $U = -\frac{Gm_1m_2}{r}$.

Escape velocity: The minimum speed at which a body, m_1, can escape from Earth's (m_2) gravitational pull without falling back toward the Earth occurs when its kinetic energy $K = U_g$:

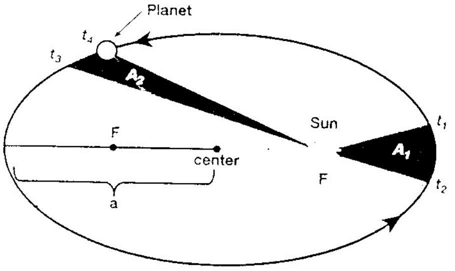
$$K = U_g$$

$$\frac{1}{2}m_1v^2 = \frac{Gm_1m_2}{r} \rightarrow$$

$$v_{esc} = \sqrt{\frac{2Gm_2}{r}}$$

Kepler's laws:

1. All planets move in elliptical orbits with the sun as one focus.
2. A line joining any planet to the sun sweeps out equal areas in equal times:



The time it takes a planet to travel from t_1 to t_2 is the same as the time it takes to travel from t_3 to t_4.

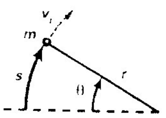
3. **Law of periods:** The square of the period of a planet's orbit around the sun (T) is proportional to the cube of its semi-major axis (a):

$$T^2 = \frac{4\pi^2a^3}{GM}$$

ROTATIONAL DYNAMICS

A. UNIFORM CIRCULAR MOTION

Angular displacement: The angle through which a body rotates is given by $\theta = \frac{s}{r}$



Average angular velocity: A body rotates a distance r from the axis of rotation with a tangential velocity v_t. If it rotates through an angle θ, its angular velocity (ω) is given by:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{v_t}{r}$$

- During uniform circular motion, the magnitude of the tangential velocity is constant. However, the direction of the tangential velocity is constantly changing, due to the centripetal force.

Instantaneous angular velocity: $\omega(t) = \frac{d\theta}{dt}$

Period: The period is the time it takes a body to complete one revolution. It is the reciprocal of frequency.

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi r}{v_t}$$

- Units: seconds (s)

Average angular acceleration: $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{a_t}{r}$

Instantaneous angular acceleration:

$$\alpha(t) = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

Centripetal acceleration: $a_c = \frac{vt^2}{r}$ (directed toward the axis of rotation)

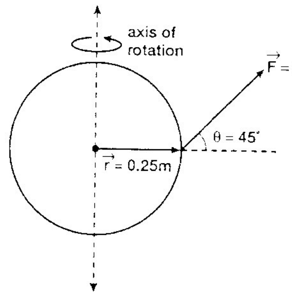
Rotational motion with constant acceleration:

$$\omega = \omega_0 + \alpha t$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$$

Torque causes a body to rotate.

- When an external force, F, is applied to a body a distance r from a pivot point (or axis of rotation), the torque on the body is defined as:
- $\tau = r \times F$ with magnitude $\tau = rF\sin\theta$.
- Torque is the rotational analog to force, and it has an analog to Newton's 2[nd] Law: $\tau_{net} = I\alpha$
- Torque always points perpendicular to the plane in which a body is rotating.
- Counter-clockwise rotation is usually defined as "positive torque," and clockwise rotation is usually defined as "negative torque."
- Units: N · m

Example: A 5-N force is applied at a 45° angle to the edge of a 0.25-m disc. What is the torque on the disc?



$$\tau = r \times F$$
$$|\tau| = rF\sin\theta = (0.25\text{ m})(5\text{ N})(\sin 45°)$$
$$|\tau| = 0.9\text{ N} \cdot \text{m}$$

Using the right-hand rule, we can see that the torque vector points upward along the axis of rotation.

Angular momentum is analogous to linear momentum. A particle rotating with linear momentum p a distance r from the axis of rotation has angular momentum defined by:

- $L = r \times p$ (vector form) and magnitude $L = rmv\sin\theta$.
- For a rigid rotating body, $L = I\omega$, where I is the moment of inertia of the body.
- Relation between torque and angular momentum: $\tau = \frac{dL}{dt}$

Conservation of angular momentum: When no external torque is acting on a system, angular momentum is conserved. If $\tau_{net} = 0$, L is constant.

- Note: When the torque acting on a system is parallel to the vector, r, between the body and the axis of rotation, the net torque on the system is zero.

Moment of inertia: The rotational analog of mass.

$$I = \sum_i m_ir_i^2$$

Common moment of inertia quantities: (*Assume all bodies lie flat in the x-y plane and rotate about the z-axis, unless otherwise specified;* m *is mass and* R *is radius*)

Type of Body or Bodies	Moment of Inertia
Single particle, thin cylinder, thin hoop	mR^2
Solid cylinder, solid disc, thin hoop rotating about the x- or y-axis	$\frac{1}{2}mR^2$
Thin disc rotating about x or y axis	$\frac{1}{4}mR^2$
Solid sphere	$\frac{2}{5}mR^2$
Hollow sphere	$\frac{2}{3}mR^2$
Thin rod of length ℓ rotating about its center	$\frac{1}{12}m\ell^2$
Thin rod of length ℓ rotating about one end	$\frac{1}{3}m\ell^2$

Rotational kinetic energy:

$$K = \frac{1}{2}\left(\sum m_ir_i^2\right)\omega^2 = \frac{1}{2}I\omega^2$$

Work: $W = \tau\theta$

Power: $P = \tau\omega$

Rotational and Translational Analogs

Quantity	Translational	Rotational
mass	m	I
displacement	x	θ
velocity	v	ω
acceleration	a	α
force	F	τ
Newton's 2[nd] Law	$F = ma$	$\tau = I\alpha$
Kinetic energy	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$
Work	$W = Fd$	$W = \tau\theta$
Power	$P = Fv$	$P = \tau\omega$
Momentum	$p = mv$	$L = I\omega$

SIMPLE HARMONIC MOTION

A. SPRING WITH A MASS

The force on a mass in simple harmonic motion is always directed opposite to the direction in which the mass is moving.

Force: $F = -kx$ (k = spring constant; x = distance from equilibrium position)

Frequency: The number of oscillations the spring completes in a period of time:

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- **Units:** cycles/s, or s^{-1}, or hertz (Hz)

Angular frequency: The number of radians the spring oscillates through in a period of time:

$$\omega = 2\pi f = \sqrt{\frac{k}{m}}$$

- Units: rad/s

Period: The time it takes for the spring to complete one oscillation. $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Equation of motion: If the spring is initially compressed a distance A and released from rest, the subsequent motion is given by $x(t) = A\cos(\omega t)$. In this case, $x_{max} = A$, where A is the amplitude of oscillation.

Velocity: $v(t) = \frac{dx}{dt} = -\omega A\sin(\omega t)$

where $v_{max} = \omega A$

Acceleration:

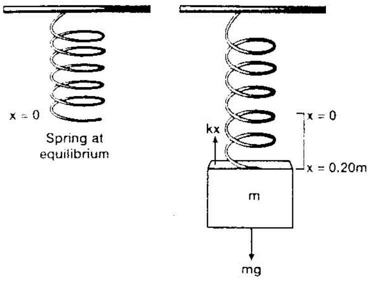
$$a(t) = \frac{dv}{dt} = -\omega^2 A\cos(\omega t) = -\omega^2 x = -\frac{k}{m}x$$

where $a_{max} = \omega^2 A$

Potential energy: $U = \frac{1}{2}kx^2$

Total energy: $E = \frac{1}{2}kA^2 = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

Example: A 5-kg object is attached to a vertical spring and stretches the spring 0.20 m. What is the natural frequency of oscillation of this system?



1. When the mass/spring system hangs at rest, the forces acting on it are balanced:
   $F_{net} = ma = 0 = mg - kx$
   $mg = kx$
   $\frac{k}{m} = \frac{g}{x}$
2. Plug in the expression for k/m into the equation for the frequency of oscillation:
   $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
   $f = \frac{1}{2\pi}\sqrt{\frac{g}{x}} = \frac{1}{2\pi}\sqrt{\frac{9.8\ m/s^2}{0.2\ m}}$
   $f = 1.1$ Hz

B. PENDULUMS – The period of a simple pendulum with length ℓ is given by

$T = 2\pi\sqrt{\frac{\ell}{g}}$

ELECTRICITY

A. ELECTRIC CHARGE – Electric charge is a fundamental property of nature. The charge on a particle can be positive or negative.

- **Two particles with like charges repel each other.**
- **Two particles with unlike charges attract each other.**

Units: Coulomb (C)

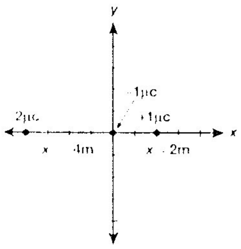
Coulomb's Law: Two particles of charge q_1 and q_2 a distance r away from each other will exert a force on each other:

$F = k\frac{|q_1 q_2|}{r^2}$

The force that q_1 exerts on q_2 is equal and opposite to the force that q_2 exerts on q_1.

Note: To determine the direction of the electric force, decide whether the force is attractive or repulsive, given the signs of the charges.

Example: A charge of 1 μC is placed 2 m to the right of the origin. A charge of −2 μC is placed 4 m to the left of the origin. What is the force on a charge of −1 μC located at the origin?



1. Calculate the force that each charge exerts on $-q$. The positive charge ($+q$) will exert an attractive force since opposites attract, so it will be directed to the right. The negative charge ($-2q$) will exert a repulsive force because like charges repel, so it will be directed to the right also.
   $F_{+q} = k\frac{|(q)(-q)|}{2^2}$
   $F_{+q} = \frac{kq^2}{4}$ directed to the right
   $F_{-2q} = k\frac{|(-2q)(-q)|}{4^2}$
   $F_{-2q} = \frac{kq^2}{8}$ directed to the right
2. Add up the forces.
   $F_{net} = F_{+q} + F_{-2q}$
   $F_{net} = \frac{kq^2}{4} + \frac{kq^2}{8} = \frac{3kq^2}{8}$
   $F_{net} = \frac{3(8.99 \times 10^9\ N \cdot m^2/C^2)(1.0 \times 10^{-9}\ C)^2}{8m^2}$
   $F_{net} = 3.4 \times 10^{-9}$ N directed to the right

B. ELECTRIC FIELD – All electrically charged particles create an electric field. This is defined as the force a test charge q_0 would experience if it were placed in a particular location near the charged particle.

$E = \frac{F}{q_0}$

C. GAUSS'S LAW

Electric Flux: The amount of the electric field that passes perpendicularly through an area A.

$\phi = \int E \cdot dA$

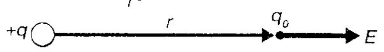
Gauss's Law: The total electric flux through a closed surface equals the amount of charge enclosed in that surface divided by the permittivity constant, ε_0.

$\oint E \cdot dA = \frac{Q_{enclosed}}{\varepsilon_0}$.

Common electric field configurations. In all cases, q_0 is a test charge in the field.

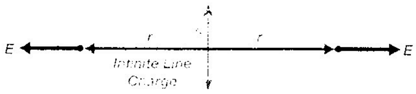
Charge configuration: *Point charge, q+*

E-field: $E = k\frac{q}{r^2}$



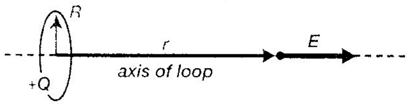
Charge configuration: *Infinite line charge with uniform density, λ*

E-field: $E = 2k\frac{\lambda}{r}$



Charge configuration: *Ring with charge Q and radius R.*

E-field: $E = kQ\frac{r}{\sqrt{(r^2 + R^2)^3}}$



Charge configuration: *Infinite charged plane with charge density σ*

E-field: $E = \frac{\sigma}{2\varepsilon_0}$

D. ELECTRIC POTENTIAL – The electric potential, also known as voltage, is defined as the electric potential energy per unit charge. The potential difference between two points, A and B, in an electric field is defined as the work done by an external force to move a test charge, q_0, from point A to point B divided by the test charge:

$\Delta V = V_B - V_A = \frac{W_{AB}}{q_0}$

- When work is done by the electric field in moving q_0 from point A to point B, the potential difference is given by
  $\Delta V = -\frac{W_{AB}}{q_0}$

Note: Electric potential energy is analogous to gravitational potential energy.

Note: Electric field lines point from high to low potential. A positive charge placed in an electric field will accelerate from high to low potential, while a negative charge in an electric field will accelerate from low poten... high potential.

Potential difference in a uniform electric

$\Delta V = V_B - V_A = Ed$ (where d is the distan... tween points A and B)

Electric Potential of a point charge: $V =$

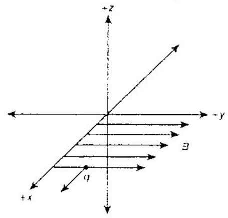
Electric Potential of a continuous charg... tribution: $V = k\int\frac{dq}{r}$

MAGNETISM AND INDUCTION

A. MAGNETIC FIELDS

1. **Bar magnets:** The magnetic field lines of a bar magnet exit the north pole and enter the south pole. Like poles repel each other, and unlike poles attract each other.
   Magnetic field units: Tesla (T)
   $T = \frac{N}{C \cdot m/s} = \frac{N}{A \cdot m}$
2. **Magnetic force on a moving charge:** A particle with charge q moving at velocity v through a magnetic field B will experience a force: $F = qv \times B$ with magnitude $F = qvB\sin\theta$.
   - To determine the direction of the force on the particle, use the right hand rule: The fingers point in the direction of v and curl toward the direction of B. The thumb will point in the direction of F.

   Example: A magnetic field of strength 1.5×10^{-2} T points along the positive y-axis. A proton travels through the field at a speed 3.0×10^6 m/s along the positive x-axis. What is the force on the proton?

   1. Calculate the magnitude of the magnetic force. The charge on a proton is 1.60×10^{-19} C. The angle between the magnetic field and the velocity vector is 90°.
      $F = (1.60 \times 10^{-19}\ C)(3.0 \times 10^6\ m/s)\left(1.5 \times 10^{-2}\frac{N}{C \cdot m/s}\right)\sin(90°)$
      $F = 7.2 \times 10^{-15}$ N
   2. Determine the direction of the force using the right-hand rule. The fingers point along the direction of the particle's velocity (x-axis) and curl toward the magnetic field (y-axis). The thumb points along the positive z-axis. Thus, the magnetic field points in the positive z-direction.
3. **Magnetic force on a current-carrying wire:** A wire of length and direction ℓ and current I traveling through a magnetic field will experience a force $F = I\ell \times B$ with magnitude $F = I\ell B\sin\theta$.
4. **Magnetic field around a current-carrying wire:** The strength of the magnetic field a distance d from a long wire with current I is
   $B = \frac{\mu_0 I}{2\pi d}$.
5. **Magnetic field around a solenoid:** The magnetic field around a solenoid is concentrated almost entirely along a straight line at the center of the loops. The strength of the magnetic field B around a solenoid with n loops and current I is $B = \mu_0 nI$.

B. MAGNETIC INDUCTION

1. **Magnetic flux:** The amount of the magnetic field that travels perpendicularly through an area A:
   $\phi = \int B \cdot dA$
2. **Faraday's Law:** A varying magnetic field through a coil of wire will induce an electromotive force (ε) in the wire:
   $\varepsilon = -N\frac{d\phi}{dt}$