

Architectural Prisms

Constant cross-sections, nets, and scale models at the Flatiron Building

What do boxes and wedges have in common? Both are prisms. The Flatiron Building is one of the purest examples ever built — and its most famous feature is a corner that isn't there.

GRADE BAND

6–8, extensions for HS Geometry

TIME

Two class periods

FORMAT

Video, net construction, scale model, calculation

Related lessons in this series: **Rectangular Prisms** (Part 2) and **Antiprisms** (Part 3), both in the Media4Math library.

WHAT STUDENTS WILL BE ABLE TO DO

- Define a prism by the property that matters — a constant cross-section — and name prisms by their base.
- Fold a net into a triangular prism and count its faces, edges, and vertices.
- Compute the volume and surface area of a triangular prism from real measurements.
- Build a scale model, and predict how area and volume change when lengths are scaled.
- Use similar triangles to find a vertex that was never built.

STANDARDS

6.G.A.4 nets and surface area · **7.G.A.1** scale drawings and scale models · **7.G.B.6** area, volume, and surface area of three-dimensional objects · **8.G.B.7** the Pythagorean Theorem · **HSG-GMD.A.1** and **A.3** volume formulas and their justification · **HSG-MG.A.1** modeling real objects with geometric shapes · **HSG-SRT.C.8** right-triangle trigonometry (*extension*)

START HERE

Open the Voyager story and look down on the Flatiron Building from directly overhead (best viewed in Chrome). Students are looking at its floor plan — say so, because the connection between an aerial view and a base shape is not automatic.

Then zoom to the northern point and ask: *is the tip actually a point?* Collect answers and leave them up; §2 and the closing panel settle it.

earth.google.com/web/data=Mj8KPQo7CiExSXdlcWVTNXpzdXNoS2l1bzNXcEJ4S2VuLU4zR1dHcVQSFgoUMDJGNDZFNUVFMzEzMTBDRTVBNTI

01 What makes a prism a prism

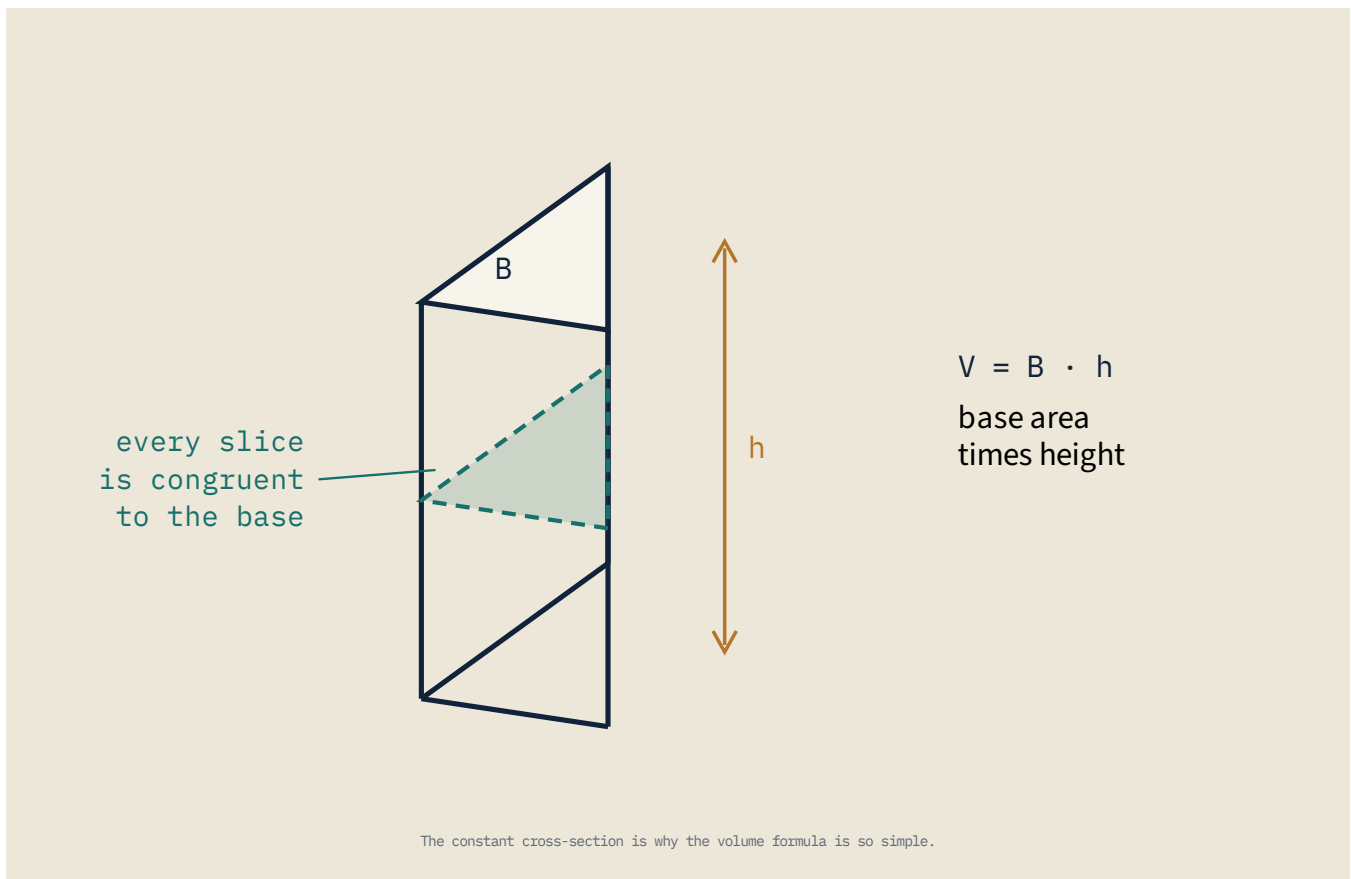
VIDEO

What Are Prisms?

s3.amazonaws.com/Media4MathPlus/Videos/GoogleVoyagerVideos/WhatArePrisms.mp4

Prism	A solid with two congruent, parallel faces (the bases) joined by parallelogram faces.
Base	One of the two congruent parallel faces. A prism is named for its base.
Lateral face	Any face that isn't a base. In a right prism these are rectangles.
Right prism	Lateral edges perpendicular to the bases — it stands straight up.
Oblique prism	A prism that leans. Bases still congruent and parallel; lateral faces are non-rectangular parallelograms.
Cross-section	The shape produced by slicing the solid. In a prism, every slice parallel to the base is congruent to the base.

That last one is the definition worth carrying around, and the one to press on. A cube, a wedge, a hexagonal pencil, and a length of steel I-beam are all prisms for exactly the same reason: **the cross-section never changes**.



$$V = B \cdot h$$

Compare to a pyramid, where the cross-section shrinks all the way up and the volume picks up a factor of $\frac{1}{3}$. No shrinking here, so no $\frac{1}{3}$. Students who did *The Mathematics of Pyramids* will have the contrast ready.



02 The Flatiron Building

VIDEO

Triangular Prisms — the Flatiron Building

s3.amazonaws.com/Media4MathPlus/Videos/GoogleVoyagerVideos/TriangularPrisms.mp4

Designed by Daniel Burnham and Frederick Dinkelberg with structural engineers Purdy & Henderson, the Flatiron opened in 1902 as one of New York's first steel-skeleton skyscrapers: 22 stories, 285 feet. Broadway cuts diagonally across Manhattan's grid, and the leftover wedge between Broadway, Fifth Avenue, and East 22nd Street is the plot Burnham had to work with.

What makes it such a clean example: most skyscrapers step inward as they rise, so their cross-section shrinks. The Flatiron has **no setbacks at all** — the same triangle on all 22 floors. It is very nearly a textbook triangular prism at the scale of a city block.

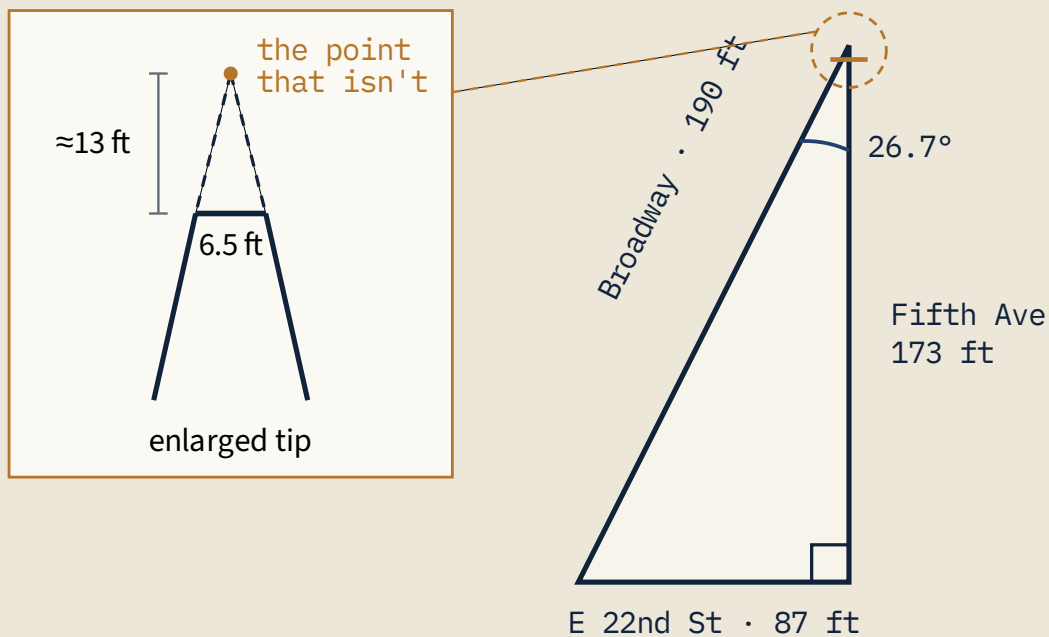
SIDE	STREET	LENGTH
Short leg	East 22nd Street	87 ft
Long leg	Fifth Avenue	173 ft
Hypotenuse	Broadway	190 ft
Prism height	—	285 ft

The base is a *right* triangle, not the isosceles one most people picture — Fifth Avenue and 22nd Street meet at a square corner and Broadway slices across as the hypotenuse. Worth correcting explicitly; the "flatiron" name pushes students toward isosceles.

CHECK IT WITH THE PYTHAGOREAN THEOREM

$\sqrt{(87^2 + 173^2)} = \sqrt{37,498} \approx 193.6$ ft against a surveyed 190 ft — within 2%.

Not exact, and it shouldn't be: street frontages are measured along building faces with rounded corners and a truncated tip. Two percent is a good match, and knowing what counts as a good match is part of the skill. Resist the urge to let students "fix" the discrepancy.



The plan is a right triangle. The famous "point" is a flat wall 6.5 ft across.

Finding a vertex nobody built

The apex angle comes straight from the legs — short leg opposite, long leg adjacent:

$$\tan \theta = 87/173 = 0.503 \rightarrow \theta \approx 26.7^\circ$$

Published descriptions put the tip at "about 25 degrees," so the model holds up. But the tip is not a point — it is a flat wall 6.5 feet wide. Similar triangles find where the building stops:

$$6.5/x = 87/173 = 0.503 \rightarrow x \approx 12.9 \text{ ft}$$

The Flatiron stops about **13 feet** short of its own corner. That missing wedge is the most photographed non-existent piece of geometry in New York — and it is the same reasoning as the Eiffel Tower's virtual apex in *Triangular Structures*, if students have met that lesson.



03 Constructing a triangular prism

VIDEO

3D Geometry Animation: Triangular Prism · and · Net for a Triangular Prism

s3.amazonaws.com/Media4MathPlus/Videos/GeometryVideos/3DGeometryTriangularPrism.mp4
s3.amazonaws.com/Media4MathPlus/Videos/GeometryVideos/TriangularPrismNet.mp4

HANDOUT

Printable Nets (PDF) — two nets per page

media4math.com/sites/default/files/library_asset/files/PrintableNets2.pdf

COUNT THE PARTS

A triangular prism has **5 faces**, **9 edges**, and **6 vertices**. Against Euler's formula: $6 - 9 + 5 = 2$ — the same 2 as a cube, a pyramid, or any Platonic solid. Euler's formula does not care what shape the base is.

Steps for the build, as given in the student version: print the net; cut along the outline including flaps; fold along all lines; glue one side at a time, attaching flaps to the underside of the opposite side; repeat; let the glue dry before handling.

ASK BEFORE THEY FOLD

Have students find the three rectangles in the flat net and notice that all three share one common length. Why must that be?

Because that shared length is the height of the prism, and every lateral face runs the full height. That gives lateral surface area in a single step — perimeter of the triangle $\times$ height — instead of three separate rectangles. Question 3 in the practice set depends on this.



04 A scale model of the Flatiron Building

Students build a scale model from these two ratios: the triangular base is a right triangle with legs in a 2:1 ratio, and the short leg to prism height is 1:3.5. The net image in the student version shows the target shape. Flaps make gluing easier.

TEST THE RATIOS AGAINST THE REAL BUILDING

- **Legs 2:1.** $173/87 = 1.99$. Almost exactly 2 — the model nails this one.
- **Short leg to height 1:3.5.** $285/87 = 3.28$, so the real ratio is nearer 1:3.3. A model built at 1:3.5 comes out about 7% too tall.

Neither ratio is wrong; they are *simplifications* chosen to be easy to measure with a ruler. Every model trades accuracy for something else — here, for whole numbers a student can cut out. Naming the trade is more useful than pretending it isn't there, and it prepares students for the discrepancy in practice question 2.

What happens when you shrink a building

With a 5 cm short leg against a real 87 ft = 2,652 cm, the scale factor is $5/2,652 = 1 : 530$. Lengths shrink by 530. Areas and volumes do not.

QUANTITY	SCALES BY	MODEL : REAL	MEANING
Any length	k	1 : 530	edges, heights, street frontages
Any area	k ²	1 : 281,000	floor space, wall surface, paper used
Any volume	k ³	1 : 149,000,000	enclosed space, material, weight

The paper model holds about 438 cm³ of air. Multiplied by 149 million that is roughly 65,000 m³, against 60,700 m³ computed directly from $\frac{1}{2} \times 87 \times 173 \times 285$. The two routes agree, and the gap between them is the 7% height ratio doing exactly what you'd predict — a satisfying place to end the modeling thread.

WHY K³ MATTERS OUTSIDE OF MATH CLASS

Double every dimension and you get four times the wall to clad and eight times the volume to heat. Costs that follow area and costs that follow volume pull apart fast, which is why a scaled-up design is never just a bigger version of the same budget.



05 Practice, with answers

1. Find the area of one Flatiron floor, then the volume of the whole building.

ANSWER Floor area = $\frac{1}{2} \times 87 \times 173 = 7,525.5$ sq ft. Volume = $7,525.5 \times 285 \approx 2,144,800$ cu ft (about 60,700 m³). Across 22 stories, roughly 165,600 sq ft of floor space.

2. The published floor area is 255,000 sq ft, but question 1 gives about 165,600. Where did the extra space come from?

ANSWER Not from an arithmetic error — from the model's assumptions. The prism model counts only the 22 stories of the triangular tower; the published figure also includes basement levels and the ground-floor "cowcatcher" extension. **A model reports on the thing you modeled**, so when a real figure disagrees, check what you left out before checking your arithmetic. This is the most valuable question in the set.

3. How much exterior wall does the Flatiron have?

ANSWER Lateral area = perimeter $\times$ height = $(87 + 173 + 190) \times 285 = 450 \times 285 = 128,250$ sq ft. This is where the shared-length observation from §3 pays off — one multiplication instead of three.

4. A different wedge building has legs of 60 ft and 150 ft. Find its apex angle, and how far from the vertex it is 5 ft wide.

ANSWER $\tan \theta = 60/150 = 0.4$, so $\theta \approx 21.8^\circ$ — sharper than the Flatiron. For a 5 ft width: $5/x = 0.4$, so $x = 12.5$ ft.

5. A student builds the model with a 4 cm short leg instead of 5 cm. What are the scale factor and the volume ratio?

ANSWER $4/5 = 0.8$; $0.8^3 = 0.512$. Volume scales by $0.512 \approx 1 : 1.96$. Volume scales by $663^3 \approx 1 : 291,000,000$ — nearly double the ratio of the 5 cm model, from a 1 cm change. Cubing is unforgiving.

6. Verify Euler's formula for a hexagonal prism.

ANSWER Faces $2 + 6 = 8$; vertices $6 + 6 = 12$; edges $6 + 6 + 6 = 18$. Then $12 - 18 + 8 = 2$. In general an n -gonal prism has $n + 2$ faces, $2n$ vertices, and $3n$ edges, and $2n - 3n + n + 2 = 2$ for every n .

BACK TO THE OPENING QUESTION

Is the tip a point? No. The Flatiron's northern end is a flat wall 6.5 feet across, and similar triangles put it about 13 feet short of where the two street lines would cross. Burnham never built the last thirteen feet — that wedge would have been too narrow to stand in, let alone rent.

The building is famous for a corner it doesn't have. But the geometry of that corner is exactly what tells you where the walls had to stop.

Teacher's Guide · Google Earth Voyager Story: Architectural Prisms, Part 1

Companion to the online lesson. Videos, nets, and the Voyager story are linked by URL above; the interactive version carries them inline.

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