The Geometry of Sustainable Architecture

Hidden faces, the rule that decides every addition, and what Halley VI is really optimized for

Bolt an extra piece onto a building and you add surface area, which ought to make it harder to heat. Halley VI's wedge-shaped ends do the opposite. Working out why gives you a rule that decides every addition you will ever make.

GRADE BAND	TIME	FORMAT
9–12 · Algebra 2 through Precalculus	**Two to three class periods**	**Video, composite-solid analysis, two Desmos investigations**

Part 1 (Citigroup Center) establishes S/V for a square prism, the size-versus-shape distinction, and the cube optimum. This part assumes that groundwork and extends it to composite solids.

WHAT STUDENTS WILL BE ABLE TO DO

- Write surface area and volume for a rectangular prism and a trapezoidal prism in terms of one variable.
- Combine two solids correctly, remembering that joining *hides* two faces.
- State the rule deciding whether an addition improves or worsens the ratio.
- Predict what happens as more modules are joined, and find the limit.
- Explain what Halley VI's shape is actually optimized for — and it isn't warmth.

STANDARDS

HSA-CED.A.2 creating equations in two or more variables · **HSA-SSE.B.3** rewriting expressions to reveal properties · **HSF-IF.B.4** and **F-BF.A.1** interpreting and building functions · **HSG-GMD.A.3** volume formulas for prisms · **HSG-MG.A.1, A.2** and **A.3** modeling, per-unit reasoning, and design problems

START HERE

Open the Voyager story in Chrome and find Halley VI on the Brunt Ice Shelf. Then get a written prediction before any calculation:

The modules have sloped wedge ends instead of flat ones, adding extra outside surface in the coldest inhabited place on Earth. Does that make the station easier or harder to keep warm? Classes overwhelmingly say harder. The answer is easier, and §4 shows why — it is the best moment in the lesson, so protect it by collecting predictions first.

earth.google.com/web/data=Mj8KPQo7CiExRGs1N3VKZUpsZVVRVHlfcTk5eTZ6UzllbmhnX080emISFgoUMEE4RDc3OUI5NDEwMDk4REIxOEY

01 The basic design of Halley VI

VIDEO **Halley VI research station**
youtube.com/embed/dhR-JZLtzvQ
Third-party embed — worth checking it still plays before class.

Designed by Hugh Broughton Architects with AECOM, opened 2013, replacing Halley V. The first four stations on this site were all lost to the ice.

FEATURE	DETAIL
Modules	8 — seven blue, one red
Module size	10 m × 20 m
Blue module floor area	152 m^2 each
Red module (double height)	479 m^2
Legs	4 m high, on steel skis
Design temperature	−56 °C, winds over 160 km/h
Annual snowfall	about 1 m per year

Three problems drove the design: snow buries things (so the modules climb on hydraulic legs), the ice shelf flows seaward (so the legs carry skis — the station was towed 23 km inland in 2017), and drifting snow piles behind obstacles (so the modules stand in a line perpendicular to the wind, which accelerates underneath and scours the snow away).

02 A schematic view of Halley VI

VIDEO **Halley VI animated build-up of the composite shape**
s3.amazonaws.com/Media4MathPlus/Videos/GoogleVoyagerVideos/HalleyAnimation.mp4

IMAGE **Cutaway of a science module**
hbarchitects.co.uk/media/thumbnails/halley-vi-british-antarctic-research-station/ScienceModule02_rMGhk1b.jpg.1200x1200_q90.jpg
Hot-linked from the architects' own site — an external dependency outside Media4Math's control.

BUILDING THE MODEL FROM TWO PUBLISHED NUMBERS

The architects say the modules are **10 m × 20 m**; the engineering press says each blue module has **152 m²** of floor. Together those pin the shape down.

152 ÷ 10 = 15.2 m, so the rectangular core is 10 × 15.2 m. That leaves 4.8 m for two end wedges of **2.4 m** each. Take the core 4 m high and the tips 2 m, and the model is complete.

Worth narrating as a modeling move: two independent published figures, one consistent shape. This is reconstruction from evidence, not assumption.

03 Mathematical analysis: the rectangular core

$$V = ab\,x^3 \qquad S = 2(a + b + ab)x^2 \qquad S/V = 2(a + b + ab)/(ab\,x)$$

As in Part 1, x sits alone in the denominator: the ratio is inversely proportional to size, and that effect swamps everything else. Comparing shapes fairly means holding volume constant, $x = (V/ab)^{1/3}$:

$$S/V = 2(a + b + ab)(ab)^{-2/3}\,V^{-1/3}$$

That shape factor is smallest at $a = b = 1$ — the cube, value 6, the same number Part 1 reached.

ACTIVITY **Desmos: the rectangular-prism ratio**
desmos.com/calculator/d6q87hn8v8

THE THREE QUESTIONS, AND THE TRAP IN THE SECOND

1. How does the width affect the ratio? What does that say about the size of the prism?
2. What happens as you increase a relative to b?
3. At what point does increasing a stop decreasing the ratio significantly?

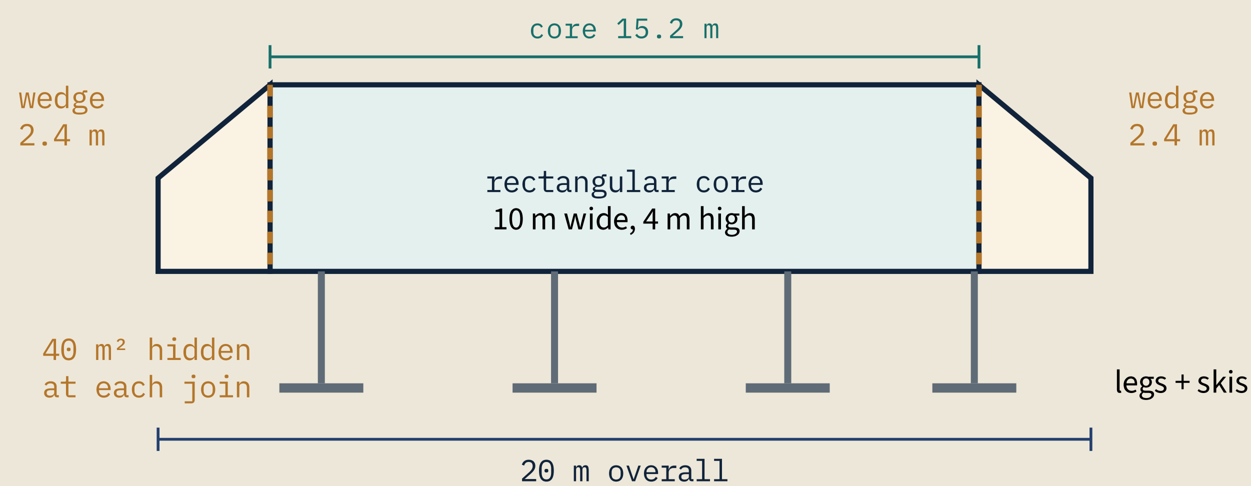
Question 2 carries the same confound as Part 1: stretching a at fixed width makes the ratio fall, but it also builds a *bigger* prism, and bigger always wins. Have students redo it holding volume fixed — set the width to $(V/ab)^{1/3}$ each time — and the effect reverses.

Question 3 has a clean answer either way: the curve approaches a horizontal asymptote, and most of the benefit is gone by about $a = 4$.

04 Adding the trapezoidal ends

$$V = \tfrac{1}{2}(p + q)h \cdot d \qquad S_{\text{total}} = S_1 + S_2 - 2F$$

The shared face F comes off **twice** — once from each solid — because both stop being outside surface the moment the pieces meet. Students who subtract it once get the classic wrong answer.



Each wedge covers up 40 m² of wall that would otherwise face Antarctica.

PIECE	VOLUME	SURFACE	RATIO
Rectangular core alone	608 m³	505.6 m²	0.832
One wedge, exposed faces	72 m³	89.6 m²	–
One wedge, *net* of the 40 m² it hides	72 m³	49.6 m²	0.690
Composite module	**752 m³**	**604.9 m²**	**0.804**

The ratio **improved** — 0.832 to 0.804, a 3.3% gain — even though the module now has more outside surface than before. This is the counterintuitive result the opening prediction sets up.

THE RULE THAT DECIDES EVERY ADDITION

Each wedge brings 72 m³ of volume but only 49.6 m² of genuinely new surface, because 40 m² of the core's end wall vanishes behind it. Its own net ratio, 49.6/72 = **0.690**, beats the core's 0.832.

$$\text{improves} \Leftrightarrow (S_{\text{new}} - 2F)/\,V_{\text{new}} < \text{current } S/V$$

Add something whose net ratio beats what you have and the average falls; add something worse and it rises. A thin spindly extension is nearly all surface and hurts; a fat blunt one plugging a large opening helps. Practice questions 4 and 5 are a matched pair testing exactly this.

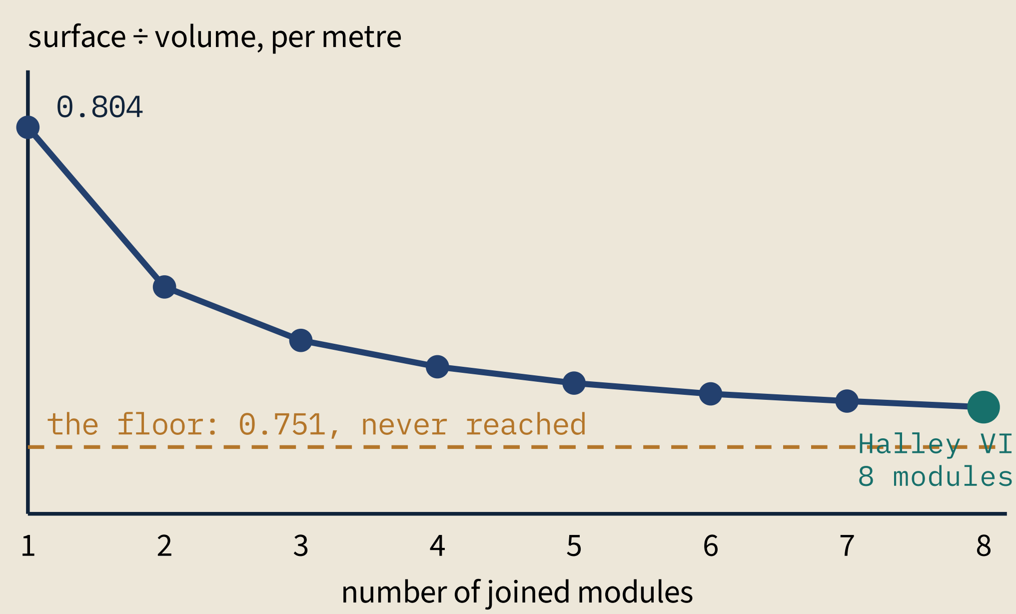
05 Joining the modules

ACTIVITY **Desmos: the composite ratio, graphed against the original**
desmos.com/calculator/4tu0qncuwv

Join *N* identical modules and each junction hides two faces. Splitting the result into two terms makes the behaviour obvious:

$$S/V = (S_1 - 2F)/\,V_1 + 2F/(N\,V_1)$$

A constant plus a term shrinking like $1/N$ — so the ratio falls toward a floor it never reaches. This is a nice, concrete encounter with a horizontal asymptote in a context where the asymptote means something.



The first junction buys most of the benefit. The eighth buys almost none.

MODULES	RATIO	GAIN OVER THE PREVIOUS
1	0.804	—
2	0.778	3.3%
3	0.769	1.1%
4	0.765	0.6%
6	?	?
8 — Halley VI	0.758	0.1%
→ ∞	0.751	0

Blank row: 0.760, a gain of about 0.2%. Eight modules land within 1% of the theoretical floor — there is essentially nothing left for a ninth to gain, which is a useful thing to establish before anyone proposes one.

06 What the shape is actually for

The student lesson's last question asks how Halley VI's shape keeps the structure warm. The honest answer is that mostly *it doesn't*, and saying so is more valuable than a tidy affirmative.

The modules are not butted together — they are separated and linked by short flexible corridors so each can be unhitched and towed, which gives up most of the junction saving above. And the station stands on 4 m legs, so its underside faces −56 °C air at 160 km/h rather than relatively still, insulating snow. That downward-facing surface is **200 m² of the module's 605 m² — a third of the whole envelope.**

> OPTIMIZED FOR SOMETHING ELSE ENTIRELY
>
> Raising the station and spacing the modules both make it harder to heat. They were done anyway:
>
> - A building sitting on the snow is buried in a few years — four of the five earlier Halley stations were lost that way.
> - A building that can't move is stranded when the shelf calves; Halley VI was towed 23 km inland in 2017.
> - Modules in a line perpendicular to the wind let it accelerate underneath and carry snow away rather than drifting it into a dune.
>
> So the shape is chosen to survive, and the thermal penalty is paid back through insulation — thick composite panels, not clever geometry. Same lesson as Part 1 in a harsher setting: Citigroup gave up 32% on its ratio for daylight; Halley gives up more to avoid burial. The geometry never chooses the building. It tells you exactly what the other priorities cost.

07 Practice, with answers

1. Complete the blank row: six joined modules.

> ANSWER $S = 6(604.9) - 2(5)(20) = 3{,}429$ m²; $V = 4{,}512$ m³; ratio **0.760** — a gain of about **0.2%** over five. The returns have all but vanished.

2. A prism has $x = 6$ m, $a = 3$, $b = 0.5$. Find V, S, and the ratio.

> ANSWER 6 × 18 × 3 m. V = **324 m³**; $S = 2(5)(36)$ = **360 m²**; ratio **1.111**.

3. A trapezoidal prism has parallel sides 5 m and 3 m, trapezoid height 2 m, depth 8 m. Find its volume.

> ANSWER Section ½(5 + 3)(2) = 8 m²; volume **64 m³**. The trapezoid's height is the perpendicular distance between the parallel sides, not the slanted edge — the same trap as in the parallelogram lesson.

4. Two solids joined on a 30 m^2 face. A: 400 m^3, 350 m^2. B: 120 m^3, 140 m^2. Find the composite ratio; did joining help?

> ANSWER $V = 520\ m^3$; $S = 350 + 140 - 60 =$ **430 m^2**; ratio **0.827** against A's 0.875 — **yes**. Rule check: B's net ratio is $(140 - 60)/120 = 0.667$, well below 0.875.

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5. Repeat with a spindly solid B: 20 m^3, 90 m^2, same face.

> ANSWER B's net ratio $= (90 - 60)/20 = 1.50$, above A's 0.875, so it should hurt. Check: $V = 420\ m^3$, $S = 380\ m^2$, ratio **0.905** — worse, as predicted. Run 4 and 5 together; the pair is what makes the rule stick.

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6. Halley's module is 0.804; an equal-volume cube is 0.660. What's the penalty, and is that the main thermal problem?

> ANSWER **22% more skin**. But no — the bigger issue is *which way the skin faces*. A third of the envelope points down into moving air at −56 °C where a ground-founded building would have still, insulating snow. Shape and exposure are separate problems; Halley accepts penalties on both to avoid burial.

> BACK TO THE OPENING QUESTION
>
> Do the wedges make the station easier or harder to keep warm? **Easier.** Each adds 72 m^3 of volume but only 49.6 m^2 of new surface, because it hides 40 m^2 of the wall it covers. Net ratio 0.690 beats the core's 0.832, so the module improves by 3.3% despite growing.
>
> The rule worth taking away: an addition helps whenever its own net surface-to-volume, counting the hidden face as a saving, beats what you already have. It explains the wedges, the diminishing returns from joining modules, and why a thin spindly extension would have made everything worse.

Teacher's Guide · Google Earth Voyager Story: The Geometry of Sustainable Architecture, Part 2
Companion to the online lesson. Videos, the Desmos activities, and the Voyager story are linked by URL above; the interactive version carries them inline.