Middle School Geometry--Grade 6--

Lesson 1: Area of Polygons

Lesson Summary

In this 50-minute lesson, students learn how to find the area of triangles, quadrilaterals, and other polygons by composing and decomposing shapes into simpler geometric figures. The lesson emphasizes conceptual understanding through visual models, vocabulary development, and multiple problem-solving strategies. Students apply their skills in both mathematical and real-world contexts. Multimedia resources from Media4Math.com are included to support visual learning. A 10-question quiz and answer key are provided at the end of the lesson.

Lesson Objectives

- Find the area of polygons by composing or decomposing them into rectangles and triangles.
- Apply these techniques to solve real-world and mathematical problems.

Common Core Standards

Common Core Standards

CCSS.MATH.CONTENT.6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Prerequisite Skills

- Calculate the area of rectangles and right triangles.
- Identify basic geometric figures (triangles, rectangles, parallelograms).
- Understand coordinate plane plotting and labeling.
- Use multiplication and division with whole numbers and fractions.

Key Vocabulary

- **Polygon**: A closed figure with straight sides. Examples include triangles, quadrilaterals, and pentagons.
  Multimedia Resource: https://www.media4math.com/library/45580/asset-preview
- **Decompose**: To break a shape into smaller, more manageable parts such as triangles or rectangles.
- **Compose**: To build a complex shape by combining simpler shapes.
- **Area**: The number of square units that cover a surface.
  Multimedia Resource: https://www.media4math.com/library/21965/asset-preview
  Multimedia Resource: https://www.media4math.com/library/43277/asset-preview
  Multimedia Resource: https://www.media4math.com/library/42963/asset-preview
  Multimedia Resource: https://www.media4math.com/library/42962/asset-preview
- **Base**: The bottom edge of a triangle or parallelogram used to calculate area.
  Multimedia Resource: https://www.media4math.com/library/42943/asset-preview
- **Height**: The perpendicular distance from the base to the top of a shape.
  Multimedia Resource: https://www.media4math.com/library/42944/asset-preview
- **Right Triangle**: A triangle with one 90° angle.
  Multimedia Resource: https://www.media4math.com/library/21941/asset-preview

- **Parallelogram**: A quadrilateral with opposite sides that are parallel and equal in length.
  Multimedia Resource: https://www.media4math.com/library/43255/asset-preview

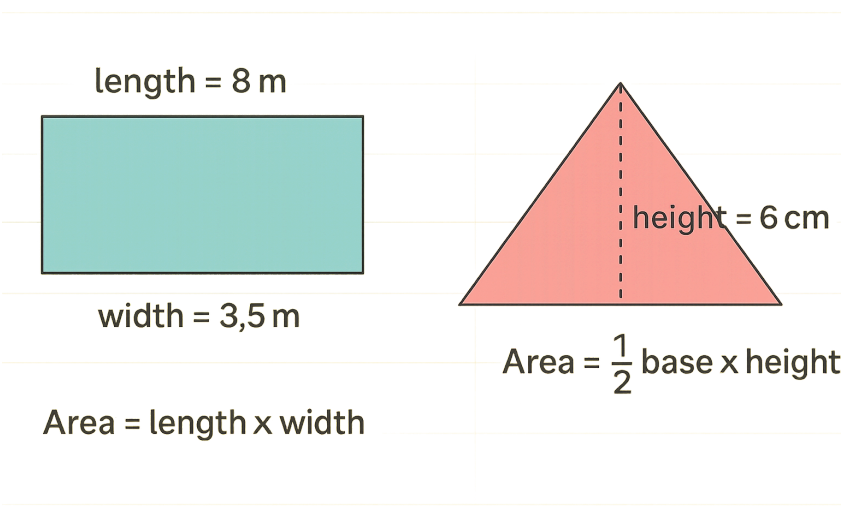
Multimedia Resources

- **Math Definitions Collection: Polygons**
  https://www.media4math.com/library/64859/asset-preview
- **Math Definitions Collection: Quadrilaterals**
  https://www.media4math.com/library/64865/asset-preview
- **Math Definitions Collection: Geometry Theorems and Postulates**
  https://www.media4math.com/library/64854/asset-preview

Warm Up

Activity 1: Area Formula Flash Review

On the board, display a diagram of a rectangle and a triangle side-by-side. Ask students to recall and write down the area formulas for each:



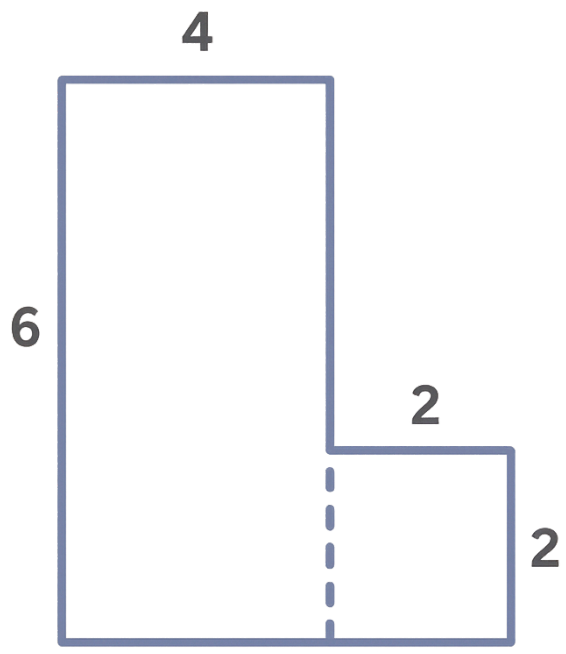
- Rectangle → *Area = length × width*
- Triangle → *Area* = $\frac{1}{2}$ *× base × height*

Then solve:

1. Area of 8 m × 3.5 m rectangle → 28 m²
2. Area of triangle with base 10 cm and height 6 cm → 30 cm²

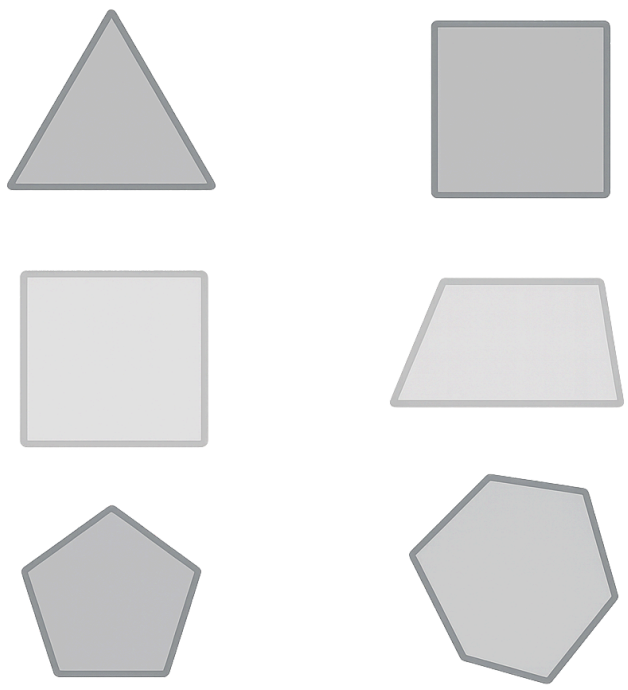
Activity 2: Decompose This Shape

Show an L-shaped figure on the board or handout and ask: “How could you break this shape into rectangles or triangles?” Have students sketch and shade their decomposed version, labeling each part. Then have volunteers share their reasoning. Emphasize that there are often multiple valid decomposition strategies.



Activity 3: Polygon Sorting Challenge

Provide students with index cards or a digital drag-and-drop activity containing various shapes: squares, rectangles, triangles, trapezoids, and irregular polygons. Ask them to sort the shapes into three categories:



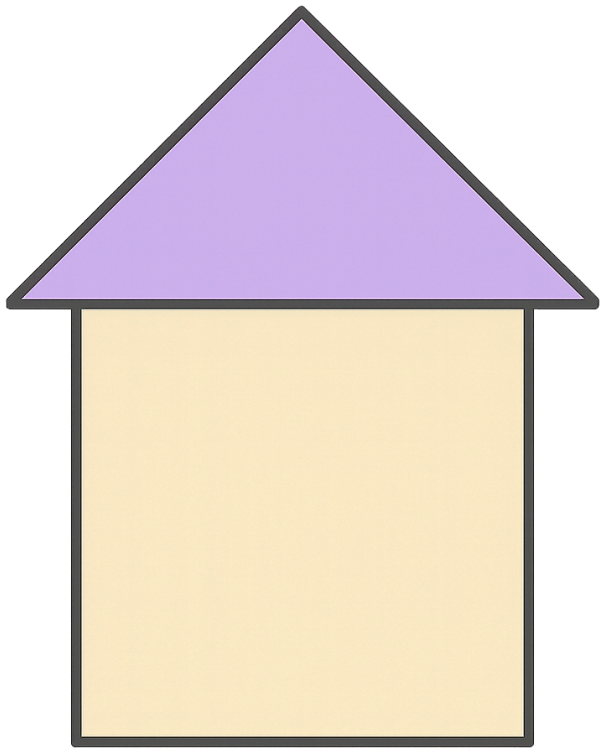
- ○ Shapes with a direct area formula
- ○ Shapes that must be decomposed
- ○ Irregular shapes requiring multiple steps

Discuss the results and reinforce which shapes require decomposition.

Downloadable Activity:
Click here to download the Polygon Sorting Challenge (PDF)

Activity 4: Estimate and Reason

Draw a complex figure such as a birdhouse shape (rectangle + triangle roof) without measurements.



Ask students to:

- Estimate the total area
- Describe how they would calculate the area if measurements were provided
- Identify the shapes

Teach

To find the area of complex polygons, we break them into simpler shapes whose areas we already know how to find — like rectangles and triangles. This process is called *decomposition*. By analyzing a figure and applying known formulas, we can calculate the total area of irregular or compound polygons.

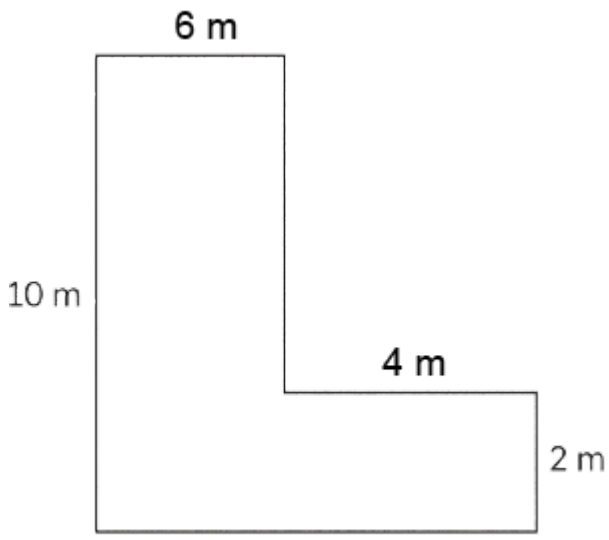
Use these formulas:

- **Area of a rectangle** = length × width
- **Area of a triangle** = (1/2) × base × height

Fractions and multiplication symbols will be rendered using LaTeX in the final version.

Example 1: Decompose an L-Shaped Polygon

Find the area of an L-shaped polygon made from two rectangles:
- Large rectangle: 10 m by 6 m
- Smaller rectangle (cut out): 4 m by 2 m



Step 1:
Break the shape into two rectangles.

Step 2:

Area of large rectangle = 10 × 6 = 60 m²

Step 3:

Area of small rectangle = 4 × 2 = 8 m²

Step 4:

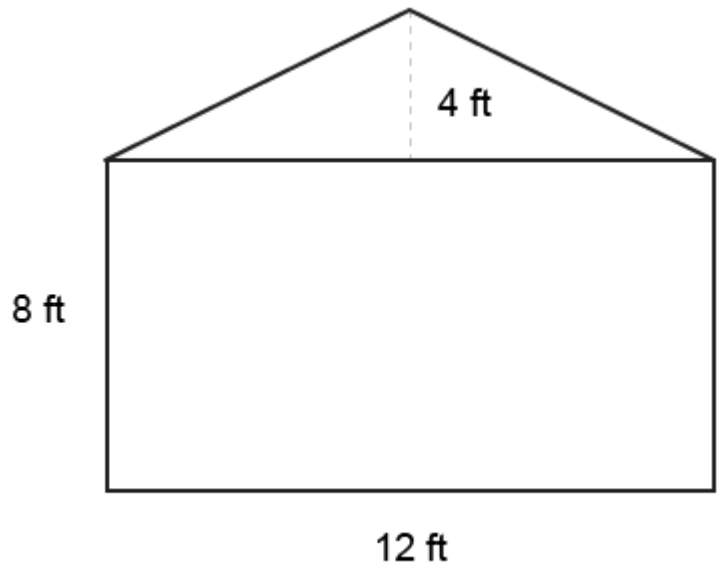
Total area = 60 − 8 = 52 m²

Final Answer: The area is 52 m²

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

Example 2: Calculate Area of a House-Shaped Figure

Find the area of the house shape made from a rectangle and a triangle:



- Rectangle: 12 ft × 8 ft
- Triangle on top: base = 12 ft, height = 4 ft

Step 1:

Area of rectangle = 12 × 8 = 96 ft²

Step 2:

Use the formula for the area of a triangle: $\mathrm{A} = \frac{1}{2} \times \text{base} \times \text{height}.$

Step 3:

Total area = 96 + 24 = 120 ft²

Final Answer: The total area is 120 ft²

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

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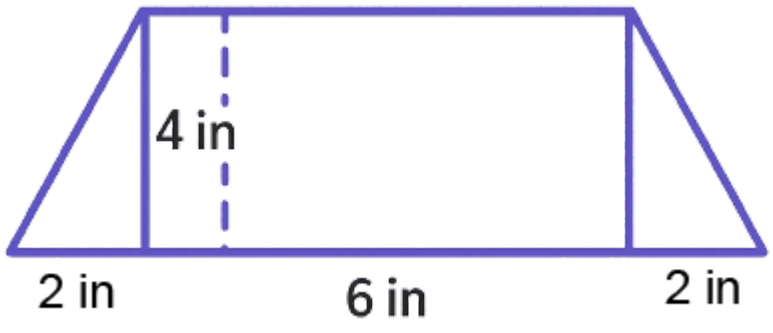
Example 3: Decompose a Trapezoid into Basic Shapes

Find the area of a trapezoid formed from a rectangle and two right triangles:



- Rectangle: 6 in × 4 in
- Each triangle: base = 2 in, height = 4 in

Step 1:

Area of rectangle = 6 × 4 = 24 in²

Step 2:

Use the formula for the area of a triangle: $A = \frac{1}{2} \times \text{base} \times \text{height}$.

Step 3:

Two triangles = 4 × 2 = 8 in²

Step 4:

Total area = 24 + 8 = 32 in²

Final Answer: Area is 32 in²

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

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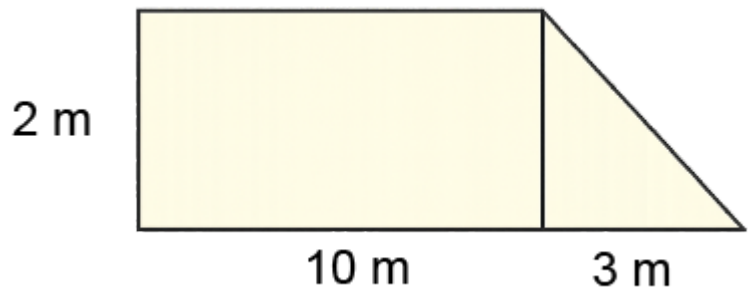
Example 4: Sidewalk Shape – Rectangle with Triangle Extension

Find the area of a sidewalk shape consisting of a rectangle and a triangle:



- Rectangle: 10 m × 2 m
- Triangle: base = 3 m, height = 2 m

Step 1:

Area of rectangle = 10 × 2 = 20 m²

Step 2:

Area of triangle = $\frac{1}{2} \times 3 \times 2$ = 3 m²

Step 3:

Total area = 20 + 3 = 23 m²

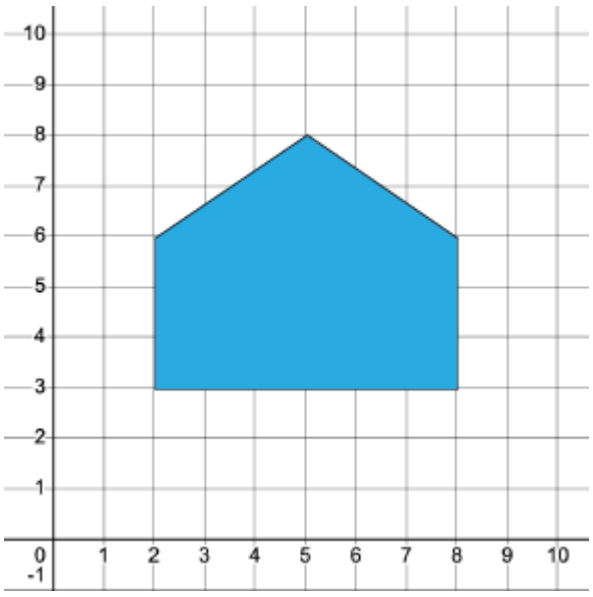
Final Answer: The total area is 23 m²

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

Multimedia Resource: https://www.media4math.com/library/21941/asset-preview

Example 5: Break Down a House-Shaped Polygon on a Coordinate Grid

A polygon on a coordinate grid has these vertices:
A(2, 3), B(8, 3), C(8, 6), D(5, 8), E(2, 6).
The shape resembles a house: a rectangle with a triangle on top. Find the total area by decomposing the figure.



Step 1:

Decompose the figure into two parts:

- A rectangle with base 6 units and height 3 units (from A to B to C to E)
- A triangle on top with base 6 units and height 2 units (from E to C with peak at D)

Step 2:

Area of the rectangle = 6 × 3 = 18 square units

Step 3:

Area of the triangle = $\frac{1}{2} \times 6 \times 2$ = 6 square units

Step 4:

Total area = 18 + 6 = 24 square units

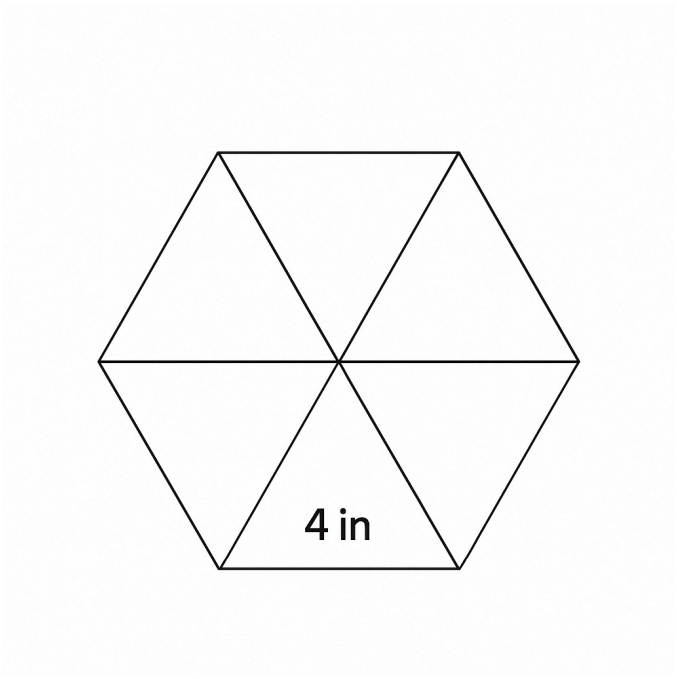
Final Answer: The total area of the polygon is 24 square units.

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

Multimedia Resource: https://www.media4math.com/library/21965/asset-preview

Example 6: Use Triangles to Find the Area of a Hexagon

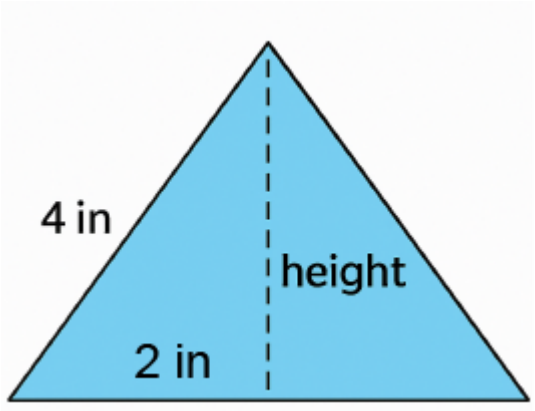
A regular hexagon is made up of 6 identical equilateral triangles. Each triangle has a side length of 4 inches. Find the total area of the hexagon.



Step 1:

To find the area of the hexagon, first calculate the area of one triangle. Since it's equilateral, all sides are 4 inches. We split the triangle in half to form two right triangles. Each right triangle has:

- One leg = 2 inches (half the base)
- Hypotenuse = 4 inches (the side of the equilateral triangle)



Step 2:

Use the Pythagorean Theorem to find the height of the triangle:

$$h^2 + 2^2 = 4^2 \Rightarrow h^2 + 4 = 16 \Rightarrow h^2 = 12 \Rightarrow h = \sqrt{12} = 2\sqrt{3}$$

Step 3:

Now use the formula for the area of a triangle:

$$A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 2\sqrt{3} = 4\sqrt{3}$$

Step 4:

Since the hexagon is made of 6 of these triangles:

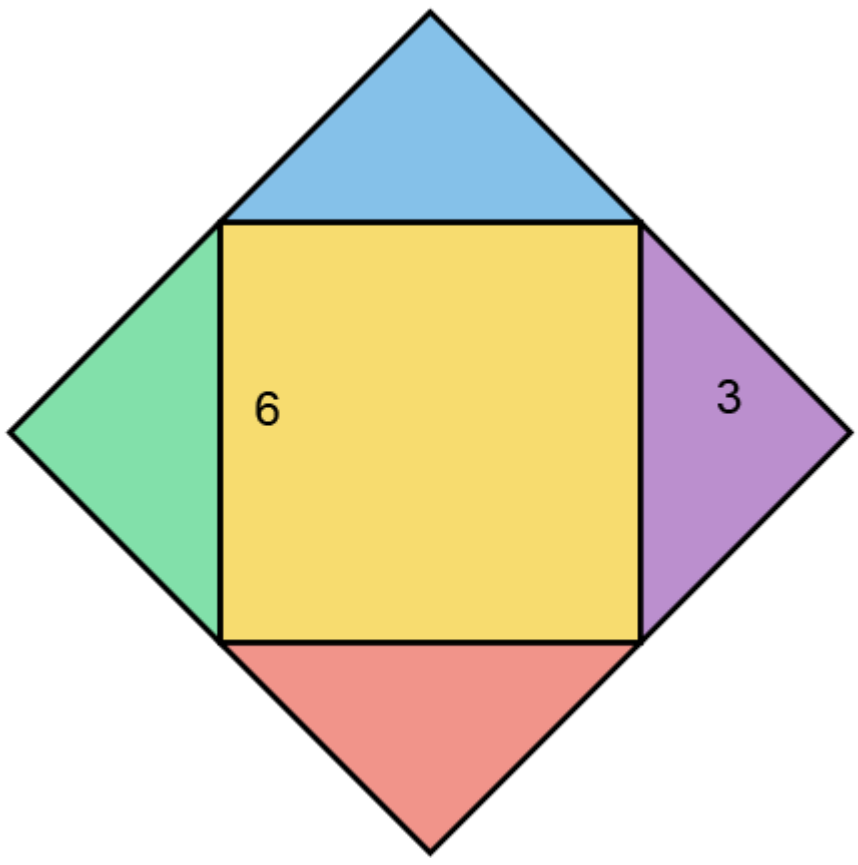
Total area = $6 \times 4\sqrt{3} = 24\sqrt{3} \approx 41.6$ square inches

Final Answer: The area of the hexagon is approximately 41.6 in²

Multimedia Resource: https://www.media4math.com/library/45580/asset-preview

Example 7: Kite Shape – Square with Triangular Wings

A kite-shaped figure is made from a 6 ft × 6 ft square and four identical triangles (base = 6 ft, height = 3 ft). Find its area.



Step 1:

Area of square = 6 × 6 = 36 ft²

Step 2:

Area of one triangle = $\frac{1}{2} \times 6 \times 3$ = 9 ft²

Step 3:

Total triangle area = 9 × 4 = 36 ft²

Step 4:

Total area = 36 + 36 = 72 ft²

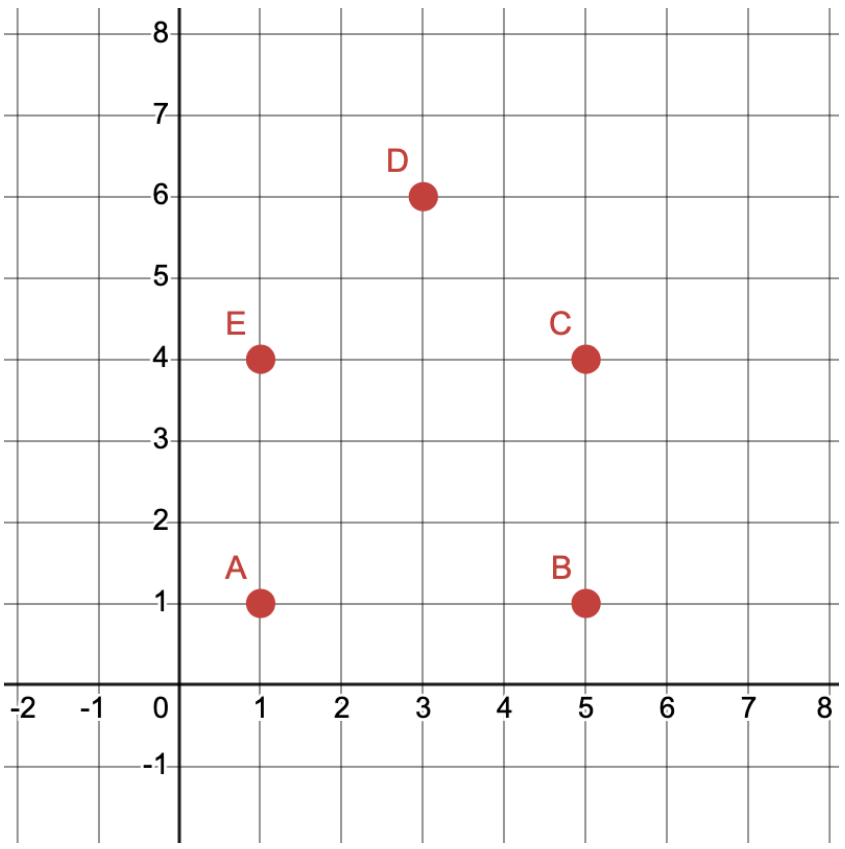
Final Answer: Area = 72 ft²

Multimedia Resource: https://www.media4math.com/library/42962/asset-preview

Multimedia Resource: https://www.media4math.com/library/21965/asset-preview

Example 8: Irregular Polygon on a Coordinate Grid

A polygon on a coordinate grid has the points A(1,1), B(5,1), C(5,4), D(3,6), and E(1,4).
Find its area by decomposing it into a rectangle and a triangle.



Step 1:

Rectangle: 4 units wide × 3 units tall = 12 units²

Step 2:

Triangle above rectangle: base = 4, height = 2 → $\frac{1}{2} \times 4 \times 2$ = 4 units²

Step 3:

Total area = 12 + 4 = 16 units²

Final Answer: The total area is 16 square units

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

Multimedia Resource: https://www.media4math.com/library/21965/asset-preview

Summary: In this section, we explored how to decompose polygons into rectangles and triangles. Each example showed how to apply known area formulas and combine results to find the area of complex shapes, including real-world and coordinate-based figures.

Review

In this lesson, students learned how to find the area of polygons by decomposing them into simpler shapes—mainly rectangles and triangles. This strategy builds on their prior knowledge of basic area formulas and encourages flexible thinking about geometry.

Students practiced:

- Identifying how complex shapes can be split into parts
- Applying formulas for rectangles and triangles
- Using logical reasoning to solve multi-step problems
- Checking results by re-composing the shape or applying alternate formulas

Key Definitions Revisited

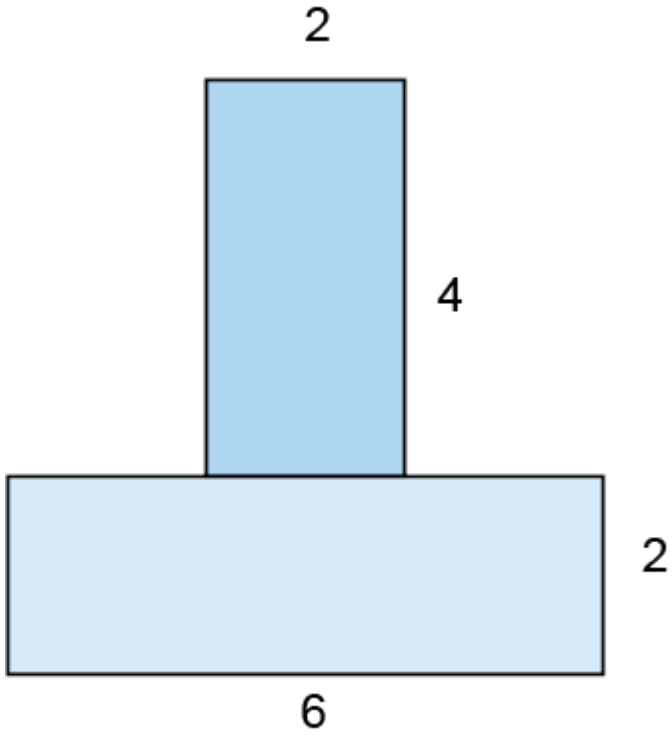
- **Polygon**: A closed figure with straight sides.
  Multimedia Resource: https://www.media4math.com/library/45580/asset-preview

- **Decompose**: Break a complex shape into simpler parts.
- **Compose**: Combine simple shapes to form a complex figure.
- **Area**: The number of square units needed to cover a shape.
  Multimedia Resource: https://www.media4math.com/library/43277/asset-preview
- **Rectangle**: A quadrilateral with four right angles.
- **Triangle**: A polygon with three sides and three angles.
  Multimedia Resource: https://www.media4math.com/library/21965/asset-preview

Example 1: Decompose a T-Shaped Polygon into Rectangles

A polygon is composed of two rectangles:

- Rectangle A is 6 m × 2 m
- Rectangle B is 2 m × 4 m and is attached to the top center of Rectangle A



Step 1:
Find the area of Rectangle A: 6 × 2 = 12 m²

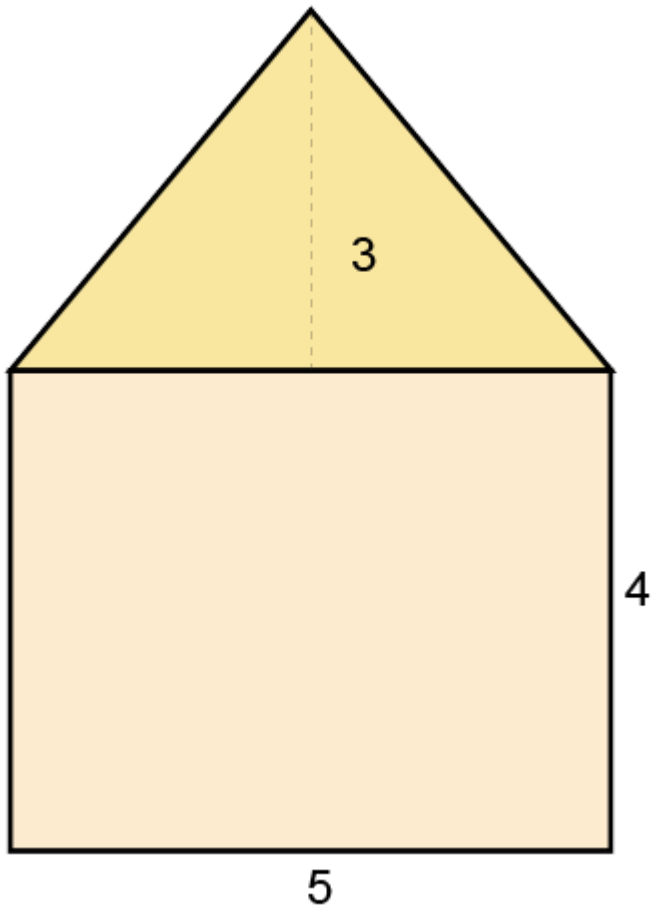
Step 2:
Find the area of Rectangle B: 2 × 4 = 8 m²

Step 3:
Add the two areas together: 12 + 8 = 20 m²

Final Answer: The total area of the polygon is 20 m²

Example 2: Break Down a Tent-Shaped Figure (Triangle + Rectangle)

A tent-shaped figure has a 5 ft × 4 ft rectangle and a triangle on top with a base of 5 ft and a height of 3 ft.



Step 1:

Decompose the figure into a rectangle and a triangle.

Step 2:

Area of the rectangle = 5 × 4 = 20 ft²

Step 3:

Area of the triangle = $\frac{1}{2} \times 5 \times 3$ = 7.5 ft²

Step 4:

Total area = 20 + 7.5 = 27.5 ft²

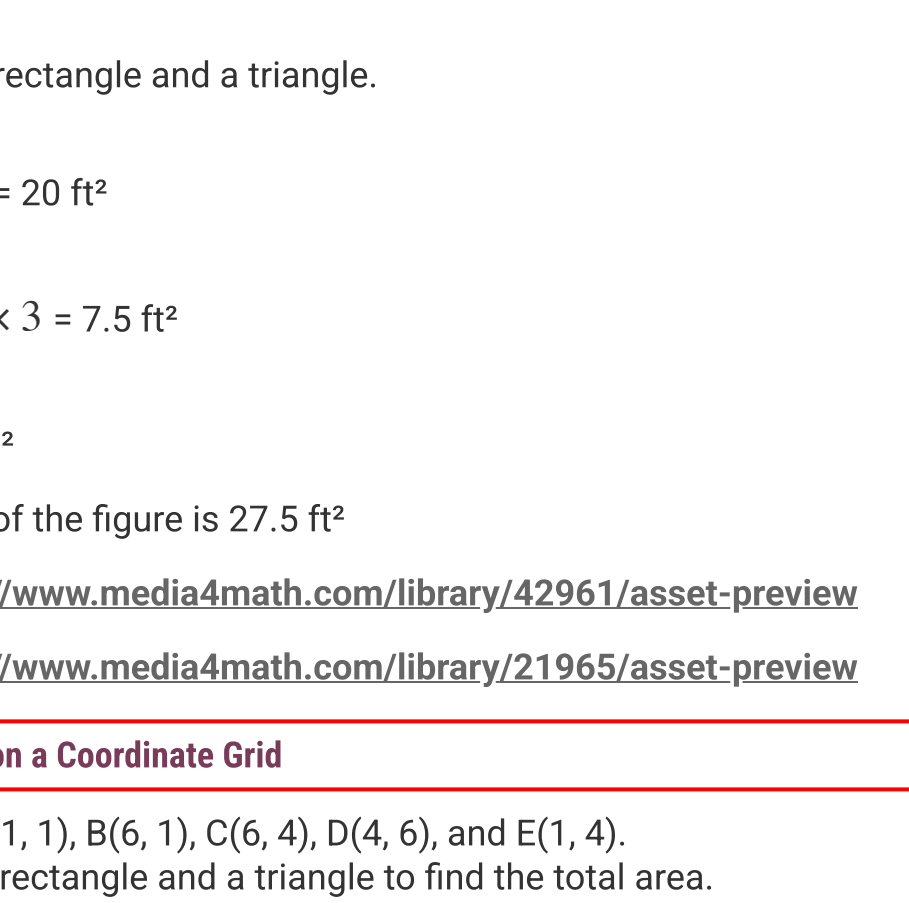
Final Answer: The total area of the figure is 27.5 ft²

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

Multimedia Resource: https://www.media4math.com/library/21965/asset-preview

Example 3: Analyze a Polygon on a Coordinate Grid

A polygon has the vertices A(1, 1), B(6, 1), C(6, 4), D(4, 6), and E(1, 4).
Decompose the shape into a rectangle and a triangle to find the total area.



Step 1:

Identify the rectangle: A(1, 1), B(6, 1), C(6, 4), and E(1, 4).
The width is 5 units (from x = 1 to x = 6), and the height is 3 units (from y = 1 to y = 4).

Step 2:

Area of the rectangle = 5 × 3 = 15 square units

Step 3:

Identify the triangle on top: C(6, 4), D(4, 6), and E(1, 4).
The base is 5 units (from x = 1 to x = 6), and the height is 2 units (from y = 4 to y = 6).

Step 4:

Area of the triangle = $\frac{1}{2} \times 5 \times 2$ = 5 square units

Step 5:

Total area = 15 + 5 = 20 square units

Final Answer: The area of the polygon is 20 square units

Multimedia Resource: https://www.media4math.com/library/42961/asset-preview

Multimedia Resource: https://www.media4math.com/library/21965/asset-preview

Quiz

Directions: Find the area of each polygon. Show your work. All units are in centimeters (cm) or meters (m) unless otherwise noted.

1. A rectangle is 7 m long and 3 m wide. What is its area?
2. A triangle has a base of 10 cm and a height of 6 cm. What is its area?
3. A shape is made of a 5 m × 4 m rectangle and a triangle on top with base = 5 m and height = 3 m.
   What is the total area?
4. A T-shaped polygon is made from two rectangles:
   - Rectangle A: 6 m × 2 m
   - Rectangle B: 2 m × 2 m
   What is the total area?
5. A regular hexagon is decomposed into six equilateral triangles with side lengths of 4 cm.
   Approximate the total area.
6. A kite shape is made from a 4 ft × 4 ft square with four identical triangles attached (base = 4 ft, height = 2 ft).
   What is the total area?
7. An L-shaped polygon is made by subtracting a 2 m × 3 m rectangle from a 7 m × 5 m rectangle.
   What is the area?
8. A trapezoid is made of a rectangle (6 in × 4 in) and two triangles (base = 2 in, height = 4 in).
   What is the total area?
9. A house-shaped figure consists of a 10 ft × 6 ft rectangle and a triangle on top (base = 10 ft, height = 5 ft).
   What is the area?
10. A polygon on a coordinate grid has the points A(2, 2), B(6, 2), C(6, 5), D(4, 7), and E(2, 5).
    Decompose the shape and find the area.

Answer Key

1. 21 m²
2. 30 cm²
3. Area = (5 × 4) + (1/2 × 5 × 3) = 20 + 7.5 = 27.5 m²
4. Area = (6 × 2) + (2 × 2) = 12 + 4 = 16 m²
5. Area of 1 triangle ≈ 6.93 cm² → Total ≈ 6 × 6.93 ≈ 41.6 cm²
6. Area = 4 × 4 = 16 ft²; each triangle = 4 ft² → Total = 16 + (4 × 4) = 32 ft²
7. Area = (7 × 5) − (2 × 3) = 35 − 6 = 29 m²
8. Area = (6 × 4) + 2 × (1/2 × 2 × 4) = 24 + 8 = 32 in²
9. Area = (10 × 6) + (1/2 × 10 × 5) = 60 + 25 = 85 ft²
10. Rectangle: 4 × 3 = 12; Triangle: (1/2 × 4 × 2) = 4 → Total = 16 units²