

**Arc** - Given any two points on the outside edge of a circle, an arc is the part of the circumference containing the two points and all the points between them. Two points on a circle are always the endpoints of two arcs. It is customary to identify an arc by three points to avoid ambiguity.

**Average (arithmetic mean)** - To calculate the average of  $n$  numbers, take the sum of the  $n$  numbers and divide it by  $n$ .

**Circular cylinder** - A circular cylinder consists of two bases that are congruent circles and a lateral surface made of all line segments that join points on the two circles and that are parallel to the line segment joining the centers of the two circles. The latter line segment is called the axis of the cylinder. A right circular cylinder is a circular cylinder whose axis is perpendicular to its bases.

**Combination** - In contrast with permutation, this is the number of ways in which a set of values can be ordered but without counting different orders for the same values. Formula: number of ways to select with order / number of ways to order =

**Composite number** - An integer greater than 1 that is not a prime number

**Compound interest** - In the case of compound interest, interest is added to the principal at regular time intervals, such as annually, quarterly, and monthly. Each time interest is added to the principal, the interest is said to be compounded. After each compounding, interest is earned on the new principal, which is the sum of the preceding principal and the interest just

added. If the amount  $P$  is invested at an annual interest rate of  $r$  percent, compounded annually, then the value  $V$  of the investment at the end of  $t$  years is given by the formula  $v = p (1 + r/100)^t$

**Compound interest** (compounded more than once annually) - If the amount  $P$  is invested at an annual interest rate of  $r$  percent, compounded  $n$  times per year, then the value  $V$  of the investment at the end of  $t$  years is given by the formula  $v = p (1 + r/100n)^{nt}$

**Continuous probability distribution** - relative frequency distributions are often approximated using a smooth curve—a distribution curve or density curve—for the tops of the bars in the histogram. The region below such a curve represents a distribution, called a continuous probability distribution. There are many different continuous probability distributions, but the most important one is the normal distribution, which has a bell-shaped curve

**Cumulative percent change** - Must calculate each successive percent change by using the result of the previous change as the new original

**Equilateral triangle** - A triangle with three congruent sides is called an equilateral triangle. The measures of the three interior angles of such a triangle are also equal, and each measure is 60 degrees.

**Frequency/count** - The frequency, or count, of a particular category or numerical value is the number of times that the category or value appears in the data. A frequency distribution is a table or graph that presents



the categories or numerical values along with their associated frequencies.

**Function** - An algebraic expression in one variable can be used to define a function of that variable. Usually denoted by letters such as  $f$ ,  $g$ , and  $h$ . For example, the algebraic expression  $3x+5$  can be used to define a function  $f$  by:  $f(x) = 3x+5$

**Graph of a quadratic equation** - The graph of a quadratic equation of the form  $y = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are constants and  $a \neq 0$  is a parabola

**Graph of an equation** - Equations in two variables can be represented as graphs in the coordinate plane. In the  $xy$ -plane, the graph of an equation in the variables  $x$  and  $y$  is the set of all points whose ordered pairs  $(x, y)$  satisfy the equation.

**Graphing linear inequalities** - Graphs of linear equations can be used to illustrate solutions of systems of linear equations and inequalities. Solve each equation for  $y$  in terms of  $x$ , then graph each. The solution of the system of equations is the point at which the two graphs intersect.

**Greatest common divisor** (or greatest common factor) - the greatest positive integer that is a divisor of both  $a$  and  $b$ . For example, the greatest common divisor of 30 and 75 is 15. This is because the positive divisors of 30 are 1, 2, 3, 5, 6, 10, 15, and 30, and the positive divisors of 75 are 1, 3, 5, 15, 25, and 75. Thus, the common positive divisors of 30 and 75 are 1, 3, 5, and 15, and the greatest of these is 15.

**Identity** - A statement of equality between two algebraic expressions that is true for all possible values of the variables involved

**Integer** - is a number composed of whole units. It may be positive or negative; zero is an integer

**Interquartile range** - The difference between the scores (or estimated scores) at the 75th percentile and the 25th percentile. Used more than the range because it eliminates extreme scores. Formula:  $IQR = Q3 - Q1$

**Interval** - The set of all real numbers that are between, say, 5 and 8 is called an interval, and the double inequality is often used to represent that interval:  $5 < x < 8$

**Isosceles triangle** - A triangle with at least two congruent sides is called an isosceles triangle. If a triangle has two congruent sides, then the angles opposite the two sides are congruent. The converse is also true.

**Least common multiple** - the least positive integer that is a multiple of both  $a$  and  $b$ . For example, the least common multiple of 30 and 75 is 150. This is because the positive multiples of 30 are 30, 60, 90, 120, 150, 180, 210, 240, 270, 300, etc., and the positive multiples of 75 are 75, 150, 225, 300, 375, 450, etc. Thus, the common positive multiples of 30 and 75 are 150, 300, 450, etc., and the least of these is 150.

**Linear equation** - A linear equation is an equation involving one or more variables in which each term in the equation is either a constant term or a variable multiplied by a coefficient. None of the variables are multiplied together or raised to a power greater than 1

**List** - A list is like a finite set, having members that can all be listed, but with two



differences. In a list, the members are ordered; that is, rearranging the members of a list makes it a different list. Thus, the terms "first element," "second element," etc., make sense in a list. Also, elements can be repeated in a list and the repetitions matter. For example, the lists 1, 2, 3, 2 and 1, 2, 2, 3 are different lists, each with four elements, and they are both different from the list 1, 2, 3, which has three elements

**Multiplication principle** - Suppose there are two choices to be made sequentially and that the second choice is independent of the first choice. Suppose also that there are  $k$  different possibilities for the first choice and  $m$  different possibilities for the second choice. The multiplication principle states that under those conditions, there are  $km$  different possibilities for the pair of choices.

**Non-integer** – is a number that cannot be expressed in whole units.

**Parabola** - The graph of a quadratic equation of the form  $y = ax^2 + bx + c$ , where  $a$ ,  $b$ , and  $c$  are constants and  $a \neq 0$  is a parabola. The  $x$ -intercepts of the parabola are the solutions of the equation  $ax^2 + bx + c = 0$ . If  $a$  is positive, the parabola opens upward and the vertex is its lowest point. If  $a$  is negative, the parabola opens downward and the vertex is the highest point. Every parabola is symmetric with itself about the vertical line that passes through its vertex. In particular, the two  $x$ -intercepts are equidistant from this line of symmetry.

**Parallelogram** - A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram. In a parallelogram, opposite sides are congruent and opposite angles are congruent

**Percent change** - If a quantity increases from 600 to 750, then the percent increase is found by dividing the amount of increase, 150, by the base, 600, which is the initial number given

**Percent change formula** -  $\frac{\text{difference}}{\text{original}} (100) = \% \text{ increase}$

**Percentage** -  $\frac{\text{part}}{\text{whole}} (100) = \%$

**Permutation** - The number of ways in which a set of values can be ordered. Formula:  $n(n-1)(n-2)(n-3)$  etc. Symbolized by  $n!$

**Prime factorization** - Every integer greater than 1 either is a prime number or can be uniquely expressed as a product of factors that are prime numbers, or prime divisors

**Prime number** - an integer greater than 1 that has only two positive divisors: 1 and itself

**Probability** - probability of event occurring is defined by the ratio  $P(E) = \frac{\text{number of outcomes that satisfy event } E}{\text{the number of possible outcomes}}$

**Proportion** - A proportion is an equation relating two ratios; for example,  $9/2 = 3/4$ . To solve a problem involving ratios, you can often write a proportion and solve it by cross multiplication

**Quadratic equation** - An equation that can be written in the form  $ax^2 + bx + c = 0$ , where  $a, b$ , and  $c$  are real numbers and  $a \neq 0$

**Quadratic formula** -  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$   
Use this to determine the value of variables in quadratic equations. Quadratic equations have at most two real solutions

**Ratio** - The ratio of one quantity to another is a way to express their relative sizes, often



in the form of a fraction, where the first quantity is the numerator and the second quantity is the denominator. Thus, if  $s$  and  $t$  are positive quantities, then the ratio of  $s$  to  $t$  can be written as the fraction  $\frac{s}{t}$ . The notation " $s$  to  $t$ " or " $s : t$ " is also used to express this ratio. For example, if there are 2 apples and 3 oranges in a basket, we can say that the ratio of the number of apples to the number of oranges is  $\frac{2}{3}$  or that it is 2 to 3 or that it is 2:3.

**Rectangle / square** - A quadrilateral with four right angles is called a rectangle. Opposite sides of a rectangle are parallel and congruent, and the two diagonals are also congruent. A rectangle with four congruent sides is called a square.

**Rectangular solid** - A rectangular solid has six rectangular surfaces called faces, as shown in the figure below. Adjacent faces are perpendicular to each other. Each line segment that is the intersection of two faces is called an edge, and each point at which the edges intersect is called a vertex. There are 12 edges and 8 vertices. The dimensions of a rectangular solid are the length  $l$ , the width  $w$ , and the height  $h$ .

**Relative frequency** - The relative frequency of a category or a numerical value is the associated frequency divided by the total number of data. Relative frequencies may be expressed in terms of percent's, fractions, or decimals. A relative frequency distribution is a table or graph that presents the relative frequencies of the categories or numerical values

**Right triangle** - A triangle with an interior right angle is called a right triangle. The side opposite the right angle is called the

hypotenuse; the other two sides are called legs.

**Sample standard deviation** - computed by dividing the sum of the squared differences by instead of  $n$ . The sample standard deviation is only slightly different from the standard deviation but is preferred for technical reasons for a sample of data that is taken from a larger population of data. Sometimes the standard deviation is called the population standard deviation to help distinguish it from the sample standard deviation

**Set** - The objects of a set are called members or elements. Some sets are finite, which means that their members can be completely counted. Finite sets can, in principle, have all of their members listed, using curly brackets, such as the set of even digits  $\{0, 2, 4, 6, 8\}$ .

**Simple interest** - Simple interest is based only on the initial deposit, which serves as the amount on which interest is computed, called the principal, for the entire time period. If the amount  $P$  is invested at a simple annual interest rate of  $r$  percent, then the value  $V$  of the investment at the end of  $t$  years is given by the formula  $v = p(1 + rt / 100)$  ( $v$  and  $p$  in dollars)

**Standard deviation** - The standard deviation of a group of  $n$  numerical data is computed by (1) calculating the mean of the  $n$  values, (2) finding the difference between the mean and each of the  $n$  values, (3) squaring each of the differences, (4) finding the average of the  $n$  squared differences, and (5) taking the nonnegative square root of the average squared difference



**Weighted average/mean** - When several values are repeated in a list, it is helpful to think of the mean of the numbers as a weighted mean of only those values in the

list that are different. The number of times a value appears in the list, or the frequency, is called the weight of that value.

## CONVERSIONS:

|                    |                       |
|--------------------|-----------------------|
| 1 °F = -17.22 °C   | 1 °C = 33.8 °F        |
| 1 °K = -457.87 °F  | 1 °F = 255.928 °K     |
| 1 °K = -272.15 °C  | 1 °C = 274 °K         |
| 1 yd = 3 ft        | 1 ft = 0.333333 yd    |
| 1 ft = 0.3048 m    | 1 m = 3.28084 ft      |
| 1 m = 0.001 km     | 1 km = 1000 m         |
| 1 km = 0.621371 mi | 1 mi = 1.60934 km     |
| 1 ft = 12 in       | 1 in = 0.0833333 ft   |
| 1 ton = 2000 lbs   | 1 lb = 0.0005         |
| 1 ton = 907.185 kg | 1 kg = 0.00110231 ton |
| 1 lb = 0.453592 kg | 1 kg = 2.20462 lbs    |
| 1 L = 0.264172 gal | 1 gal = 3.78541 L     |

## FORMULAS / EQUATIONS

$$\sqrt{a}\sqrt{b} = \sqrt{ab}$$

$$(\sqrt{a})^2 = a$$

$$\sqrt{a^2} = a$$

$$\sqrt{a}/\sqrt{b} = \sqrt{a/b}$$

### Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### Slope (m)

$$\text{rise/run, } y_2 - y_1 / x_2 - x_1$$

### Equation of a line

$$y = mx + b$$

b is the y-intercept, y is the point on the y axis, x is the point on the x axis.



**Graph of a circle**

$$(x - a)^2 + (y - b)^2 = r^2 \text{ (centre is at point a, b and radius of r)}$$

**Sum of the measures of the interior angles of a triangle**

180 degrees

**Sum of the measures of the interior angles of an n-sided polygon**

$$(n - 2)(180 \text{ degrees})$$

**Pythagorean theorem**

$$a^2 + b^2 = c^2$$

**Area of a triangle**

$$A = \frac{1}{2}bh \text{ or } bh/2$$

**Area of a quadrilateral**

$A = bh$  (or  $lw$ ): the base times height or length times width

**Area of a trapezoid**

half the product of the sum of the lengths of the two parallel sides  $b_1$  and  $b_2$  and the corresponding height  $h$ :  $A = \frac{1}{2} (b_1 + b_2)(h)$

**Area of a circle**

$$A = \pi r^2$$

**Area of a sector**

$$A = \frac{\pi r^2}{360} (c), \text{ where } c = \text{the central angle}$$

**Area of square calculated in relation to its diagonal**

$$a = \frac{1}{2}d^2$$

**Area of a parallelogram**

$A = bh$  (do not mistake with the formula for the height of a triangle. note also that "height" must be a straight line drawn from the base, not one of the diagonal sides)



**Area of an equilateral triangle**

$$a = 1/4s^2\sqrt{3}$$

**Area of a trapezoid**

$a = 1/2h(B + b)$ , where B and b represent the "bases" (i.e. typically the straight lines at the bottom and the top of the figure, between which the height is drawn and measured)

**Area of an equilateral triangle**

$$\sqrt{3}s^2 / 4$$

**Area of a hexagon**

$$a = (3\sqrt{3} / 2)t \text{ (where } t \text{ is the side length)}$$

**Area of a triangle**

$ab \sin C / 2$ , where a and b are any two sides and C is the angle between them

**Area of an equilateral triangle with side s**

$$A = s^2\sqrt{3} / 4$$

**Area of a circle**

$$A = (\pi * r^2) / 2$$

**Circumference**

$$C = 2(\pi)r$$

**Length of an Arc**

$$\text{Length of an Arc} = 1 (n/360) (2\pi r)$$

**Volume of rectangular solid**

$$V = lwh$$

**Surface area of rectangular solid**

$A = 2(lw + lh + wh)$  -- the sum of the areas of the six faces



**Length of diagonal in rectangular prism**

$$A^2 + B^2 + C^2 = D^2 \text{ or } L^2 + W^2 + H^2 = D^2 \text{ (A is not area, just a side length)}$$

**Volume of a right circular cylinder**

$$V = (\pi)r^2h$$

**Surface area of a right circular cylinder**

$$A = 2(\pi r^2) + 2\pi rh$$

**Number of permutations of n objects taken k at a time**

$$n! / (n-k)!$$

**Number of combinations of n objects taken k at a time**

$$n! / k!(n-k)!, \text{ sometimes notated as } nCk$$

**Probability**

$$P(E) = \text{number of outcomes that satisfy event E} / \text{the number of possible outcomes}$$

**Probability of two or more events BOTH occurring**

$$P(A \text{ and } B) = P(A) \times P(B)$$

**Probability of EITHER one or another event occurring**

$$P(A) + P(B) - P(AB)$$

**Probability of neither of multiple events occurring**

$$\text{the product of } 1 - P(A), 1 - P(B), \text{ etc.}$$

**Probability of event E AND F**

$$E \times F \text{ (if E and F are independent)}$$

**Probability of event E OR F**

$$E + F \text{ (if E and F are mutually exclusive)}$$

**Probability of event E OR F but not both**

$$E + F - P(E \text{ and } F)$$



**Length of a diagonal in a parallelogram**

$$p^2 + q^2 = 2((a^2) + (b^2))$$

**Distance formula**

$$\text{speed} \times \text{time} = \text{distance}$$

**Work formula**

$$\text{rate} \times \text{time} = \text{work/output}$$

**Mixture formula**

$$\text{concentration} \times \text{amount of solution} = \text{amount of ingredient}$$

**Cost**

$$\text{rate} \times \text{number of items} = \text{value}$$

**Perimeter of a semicircle**

$$P = d(1/2\pi + 1)$$

**Volume of a cube**

$$V = e^3 \text{ (where } e \text{ is any edge of the cube)}$$

**Surface area of a cube**

$$S = 6e^2 \text{ (where } e \text{ is any edge of the cube)}$$

**Surface area of a cylinder (bases incl.)**

$$S = 2\pi rh(h + r)$$

**Surface area of a cylinder (without bases)**

$$S = 2\pi rh$$

**Volume of a sphere**

$$V = 4/3\pi r^3$$

**Surface area of a sphere**

$$S = 4\pi r^2$$



**Volume of a hemisphere**

$$V = \frac{2}{3}\pi r^2$$

**Surface area of a hemisphere**

$$S = 2\pi r^2 \text{ (without base); } S = 3\pi r^2 \text{ (with base)}$$

**Diagonal of a square**

$$d = s\sqrt{2} \text{ (where } s \text{ equals the length of a side)}$$

**Perimeter of a hexagon**

$$p = 6r \text{ (where } r \text{ is a given radius)}$$

**Formula for distance between two points on a coordinate graph**

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \text{ (NB that the sqrt sign extends across the entire formula)}$$

**Converting diameter to radius**

$$d = 2r; \text{ so } d^2 = 4r^2$$

**Diagonal of a square**

$$d = s\sqrt{2}$$

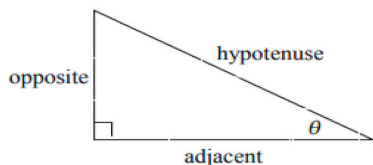


## TRIGONOMETRY

### Right triangle definition

For this definition we assume that

$$0 < \theta < \frac{\pi}{2} \text{ or } 0^\circ < \theta < 90^\circ.$$



$$\begin{aligned}\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} & \csc \theta &= \frac{\text{hypotenuse}}{\text{opposite}} \\ \cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} & \sec \theta &= \frac{\text{hypotenuse}}{\text{adjacent}} \\ \tan \theta &= \frac{\text{opposite}}{\text{adjacent}} & \cot \theta &= \frac{\text{adjacent}}{\text{opposite}}\end{aligned}$$

### Domain

The domain is all the values of  $\theta$  that can be plugged into the function.

$\sin \theta$ ,  $\theta$  can be any angle

$\cos \theta$ ,  $\theta$  can be any angle

$\tan \theta$ ,  $\theta \neq \left(n + \frac{1}{2}\right)\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

$\csc \theta$ ,  $\theta \neq n\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

$\sec \theta$ ,  $\theta \neq \left(n + \frac{1}{2}\right)\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

$\cot \theta$ ,  $\theta \neq n\pi$ ,  $n = 0, \pm 1, \pm 2, \dots$

### Range

The range is all possible values to get out of the function.

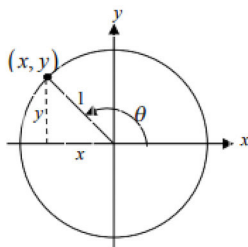
$$-1 \leq \sin \theta \leq 1 \quad \csc \theta \geq 1 \text{ and } \csc \theta \leq -1$$

$$-1 \leq \cos \theta \leq 1 \quad \sec \theta \geq 1 \text{ and } \sec \theta \leq -1$$

$$-\infty < \tan \theta < \infty \quad -\infty < \cot \theta < \infty$$

### Unit circle definition

For this definition  $\theta$  is any angle.



$$\begin{aligned}\sin \theta &= \frac{y}{1} = y & \csc \theta &= \frac{1}{y} \\ \cos \theta &= \frac{x}{1} = x & \sec \theta &= \frac{1}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y}\end{aligned}$$

### Period

The period of a function is the number,  $T$ , such that  $f(\theta + T) = f(\theta)$ . So, if  $\omega$  is a fixed number and  $\theta$  is any angle we have the following periods.

$$\sin(\omega \theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega \theta) \rightarrow T = \frac{2\pi}{\omega}$$

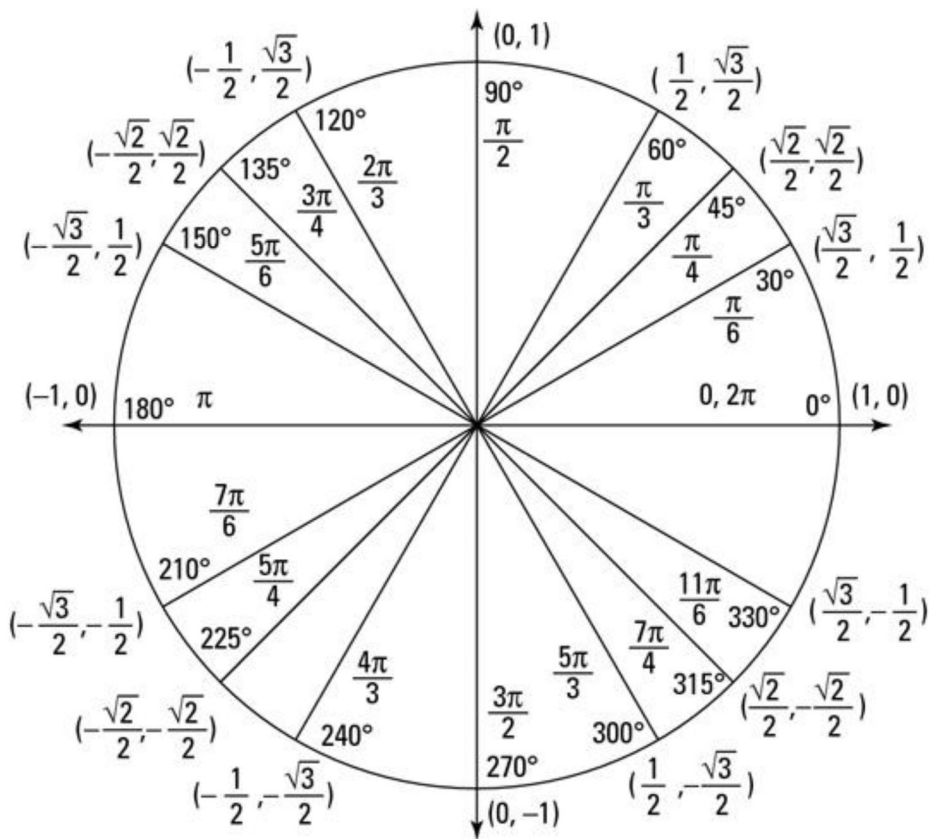
$$\tan(\omega \theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega \theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega \theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega \theta) \rightarrow T = \frac{\pi}{\omega}$$

# UNIT CIRCLE



For any ordered pair on the unit circle ( x y, ) : cosq = x and sinq = y

## Inverse Trig Functions

### Definition

$y = \sin^{-1} x$  is equivalent to  $x = \sin y$

$y = \cos^{-1} x$  is equivalent to  $x = \cos y$

$y = \tan^{-1} x$  is equivalent to  $x = \tan y$

### Inverse Properties

$$\cos(\cos^{-1}(x)) = x \quad \cos^{-1}(\cos(\theta)) = \theta$$

$$\sin(\sin^{-1}(x)) = x \quad \sin^{-1}(\sin(\theta)) = \theta$$

$$\tan(\tan^{-1}(x)) = x \quad \tan^{-1}(\tan(\theta)) = \theta$$

### Domain and Range

| Function          | Domain                 | Range                                      |
|-------------------|------------------------|--------------------------------------------|
| $y = \sin^{-1} x$ | $-1 \leq x \leq 1$     | $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$ |
| $y = \cos^{-1} x$ | $-1 \leq x \leq 1$     | $0 \leq y \leq \pi$                        |
| $y = \tan^{-1} x$ | $-\infty < x < \infty$ | $-\frac{\pi}{2} < y < \frac{\pi}{2}$       |

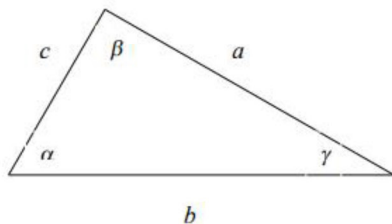
### Alternate Notation

$$\sin^{-1} x = \arcsin x$$

$$\cos^{-1} x = \arccos x$$

$$\tan^{-1} x = \arctan x$$

## Law of Sines, Cosines and Tangents



### Law of Sines

$$\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$$

### Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

### Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos \frac{1}{2}(\alpha - \beta)}{\sin \frac{1}{2}\gamma}$$

### Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(\alpha - \beta)}{\tan \frac{1}{2}(\alpha + \beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan \frac{1}{2}(\beta - \gamma)}{\tan \frac{1}{2}(\beta + \gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan \frac{1}{2}(\alpha - \gamma)}{\tan \frac{1}{2}(\alpha + \gamma)}$$